Dynamic Fuzzy Sets

Andrzej Buller
Faculty of Electronics, Telecommunications and Informatics
Technical University of Gda□sk
ul. Gabriela Narutowicza 11/12, 80-952 Gda□sk, Poland
e-mail: buller@pg.gda.pl

Abstract: This paper presents a way of fuzzy inferencing that allows for the resulting judgment to change in time. The aim of the research is to explain the empirical evidence that people's feelings about a person or a social situation sometimes oscillate between a highly positive value and a highly negative value even in the absence of new data. The proposed method of judgment generation employs the concept of fuzzy set, however, the idea of dynamic fuzzy sets, for which membership values are allowed to change in time, is introduced. A set of constant membership values has been used as the control parameter **a** in a dynamic model $X_{t+1} = F(\mathbf{a}, X_t)$ with the proposed function F implemented as a feed-forward neural network. X_t is the set of state variables. One of the state variables was interpreted as the resulting judgment. Simulation showed that the judgment produced by the proposed model sometimes oscillates like the social judgment produced by a human subject.

Introduction

Let us assume that for an object H there is the set of its features {P, Q, ... } perceived by a subject. These features are used by the subject to evaluate the object. The result of the evaluation is the set {J, K, ... }of more or less consciously concluded features called here judgments. There are several possible approaches to modeling judgment generation. The simplest, propositional calculus-based approach employs production rules, such as, "if P and Q then J". This approach works only when the occurrence of the features P and Q is obvious to the subject and his/her way of evaluation is based on social stereotypes.

In the real life, the notions describing features are often fuzzy. Also, as it was experimentally confirmed, subjects' certainty that a perceived object belongs to the given category depends on the objects [6]. We can assume that P and Q are fuzzy sets of objects with the properties P and Q respectively. Fuzzy set theory offers the concept of membership of an object to a fuzzy set. The membership value is assumed to be a real number in the range from 0 to 1 [8]. Let $\mu_P(H)$ and $\mu_Q(H)$ be values of memberships of the object H to the fuzzy sets P and Q, respectively. Fuzzy calculus lets us find $\mu_J(H)$, i.e. the value of membership of the object H to the fuzzy set J interpreted as the set of objects for which judgment J is adequate. When J is assumed to result from the conjunction of the perceived features P and Q, the classic fuzzy set-theoretic formula $\mu_J(H) = \min\{ \mu_P(H), \mu_Q(H) \}$ can be used [9].

The approach based on fuzzy set theory works well for objects emotionally neutral to the subject and only when the considered features are assumed to be equally essential for producing the judgment. When a judgment reflects the subject's feelings, even fuzzy-set-based models are not always adequate [3]. Among features contributing to the judgment some may be more essential than others. Moreover, it was experimentally confirmed that people's feelings about a person or a social situation may oscillate between a highly positive value and a highly negative value even in the absence of new data [5]. The mystery of the cognitive mechanism causing this psychological phenomenon motivated the search for a way of inferencing that the input data are not only fuzzy but also not equally essential. The target way of inferencing should also produce judgments that change in time. Hence, the concept *dynamic fuzzy set* is proposed in this paper. A way of inferencing that allows to determine a function that for a given time t returns the value of membership of object H in *dynamic fuzzy set* J associated with the appropriate judgment is also suggested. The *dynamic membership*, denoted $\mu_{J,t}(H)$, represents the strength of subject's feeling that the judgment J applies to the perceived person or social situation. *Dynamic defuzzification*, consisting in finding a function mapping time onto the set $\{J, \neg J\}$, provides an answer to the question "J or not J?" The answer may depend on time even when the data about the perceived person or social situation remain constant. This applies to both human subjects and the discussed model of mind.

Fuzzy calculus to simulate dynamic mental processes

Let us consider a dynamic system defined by the state equation $X_{t+1} = F(\mathbf{a}, X_t)$, where $X_t = (x_{0,t}, x_{1,t}, \dots, x_{n,t}) \in \mathbf{X}$ is the vector of state variables, $\mathbf{X}$ is the space of such vectors, and $\mathbf{a}$ is a constant control parameter. If perceived features and generated judgments were described in terms of the elements of the state equation, the subject's behavior could be analyzed using the rich tool-set of discrete dynamical system theory [4].

In order to use the dynamic system as a model of mind, let us assume that **a** = ($\mu_P(H)$, $\mu_Q(H)$, $\mu_J(P)$, $\mu_J(Q)$), where P and Q are fuzzy sets of objects with the properties P and Q respectively. $\mu_P(H)$ and $\mu_Q(H)$ are values of memberships of the object H to the fuzzy sets P and Q, respectively, while the values of $\mu_J(P)$ and $\mu_J(Q)$ depend on how essential are the features P and Q when the subject generates the judgment J or ¬J. The target function $\mu_{J,\,t}(H)$ for a given time t returns a value of membership of a given object H to a fuzzy set J associated with an appropriate judgment. The target function is defined by consecutive values of state variable $x_{n-2,t}$. While $x_{n-2,t}$ can be interpreted as a representation of feelings of a subject to the perceived object H, the last state variable $x_{n,t}$ is reserved for the proper judgment that can oscillate between two contradictory values J and ¬J.

Let us take an example of a judgment produced based on the proposed operation on fuzzy notions. A subject looks for a co-worker. The subject's view is that a good candidate should look neat and energetic, that the latter feature is more important. Since the candidate looks "rather energetic", a dense stream of copies of patterns representing the pair ⟨P, H⟩, where P represents the feature "energetic", starts flowing into the subject's working memory. However, because of the "rather", at the same time a weak stream of copies of ⟨¬P, H⟩ also starts flowing into the subject's working memory. It is hardly to say that the candidate H looks neat. Hence, an intermediately dense stream of copies of ⟨Q, H⟩ and a bit denser stream of copies of ⟨¬Q, H⟩ flow into the subjects working memory, where Q represents the feature "neat". The subject's semantic memory releases patterns ⟨J, P⟩ and ⟨J, Q⟩, where J represents the imperative "Employ him!". Since "neatness" is of lesser importance, the stream of copies of ⟨J, P⟩ is appropriately denser than the stream of copies of ⟨J, Q⟩. Every pattern moves through the working memory, meets other patterns, and interact with them. As a result, after a period of time the working memory may be dominated by the population of copies of ⟨J, H⟩. This would mean the subject's positive feelings about the idea to employ the candidate H. Suddenly, the population of copies of ⟨J, H⟩ loses to the population of copies of ⟨¬J, H⟩. This means that the subject's feelings turned negative. This phenomenon may be a natural consequence of dynamic character of human mind represented in the discussed model as the function F.

In order to define the proposed function F, let us introduce the set **M** of pairs of notions used in the discussed example of judgment. Let **M** = {⟨P, H⟩, ⟨¬P, H⟩, ⟨Q, H⟩, ⟨¬Q, H⟩, ⟨J, P⟩, ⟨J, Q⟩, ⟨∅, ∅⟩}. Let $m = \mu_P(H) + \mu_{\neg P}(H) + \mu_Q(H) + \mu_{\neg Q}(H) + \mu_J(P) + \mu_J(Q) + \mu_\emptyset(\emptyset)$.

Let us introduce the function $\nu \mid \mathbf{T} \rightarrow \mathbf{M}$ such that for all $\langle b, a\rangle \in \mathbf{M}$, $\Pi(\ \nu(t) = \langle b, a\rangle\ ) = k\mu_b(a)/m$, where for all Z, $\Pi(Z)$ is probability of Z. The function F is such that for $0 \leq i \leq n\text{-}3$:

$$x_{i,t+1} = \nu(t) \text{ if } f(x_{j,\,t}, x_{k,\,t}) = \langle \varnothing, \varnothing \rangle,$$
$$x_{i,\,t+1} = f(x_{j,\,t}, x_{k,\,t}) \text{ if } f(x_{j,\,t}, x_{k,\,t}) \neq \langle \varnothing, \varnothing \rangle.$$

The function *f* is defined by the table:

$x_{j,\,t}$	$x_{k,\,t}$	$f(x_{j,\,t}, x_{k,\,t})$
$\langle b, a\rangle$	$\langle \varnothing, \varnothing\rangle$	$\langle b, a\rangle$
$\langle \varnothing, \varnothing\rangle$	$\langle b, a\rangle$	$\langle \varnothing, \varnothing\rangle$
$\langle b, a\rangle$	$\langle a, H\rangle$	$\langle b, H\rangle$
$\langle b, a\rangle$	$\langle \neg a, H\rangle$	$\langle \neg b, H\rangle$
$\langle a, H\rangle$	$\langle \neg a, H\rangle$	$\langle \varnothing, \varnothing\rangle$

$j = g(i, X_t)$, $k = h(i, X_t)$, while:

$\mu_{J,\,t}(H) = x_{n-2,\,t+1} = N_{\langle J,\,H\rangle}(X_t)/(\ N_{\langle J,\,H\rangle}(X_t)+N_{\langle \neg J,\,H\rangle}(X_t))$

where $N_M(X_t) = C_{M,\,n}$, $C_{M,\,0} = m$, $C_{M,\,i+1} = C_{M,\,i} + m$,

$m = 1$ if $x_{i,\,t} = M$, $m = 0$ if $x_{i,\,t} \neq M$,

$x_{n-1,\,t+1} = x_{n-2,\,t} - 0.5 + r\,x_{n-1,\,t}$,

$x_{n,\,t+1} = J$ if $x_{n,\,t} = J$ and $x_{n-1,\,t}. > -\varepsilon$ or $x_{n,\,t} \neq J$ and $x_{n-1,\,t} > \varepsilon$,

$x_{n,\,t+1} = \neg J$ if $x_{n,\,t} = \neg J$ and $x_{n-1,\,t} < \varepsilon$ or $x_{n,\,t} \neq \neg J$ and $x_{n-1,\,t} > -\varepsilon$,

$x_{n,\,0} = \varnothing$.

The constant parameters r and ε represent subject's emotional stability. The functions $g, h \mid \{i \mid 0 \leq i \leq n\text{-}3\} \times \mathbf{X} \rightarrow \{i \mid 0 \leq i \leq n\text{-}3\}$ are defined in such a way that when the state variables $x_{0,\,t}, x_{1,\,t}, \ldots, x_{n-3,\,t}$ are represented as nodes of a 2-dimentional grid, a given value can be observed as a pattern moving through the grid until, as a result of an interaction with another moving pattern, it changes its informational content.

The preliminary computational results show that for some sets of memberships values constituting the control parameter **a** there exist such functions F that the values of the state variables $x_{n-2,\,t}, x_{n-1,\,t}, x_{n,\,t}$ change in a quasi-chaotic way tending to a two-focal attractor [1][2]. An example of such attractor as well as the judgment oscillations are presented in Figure 1.

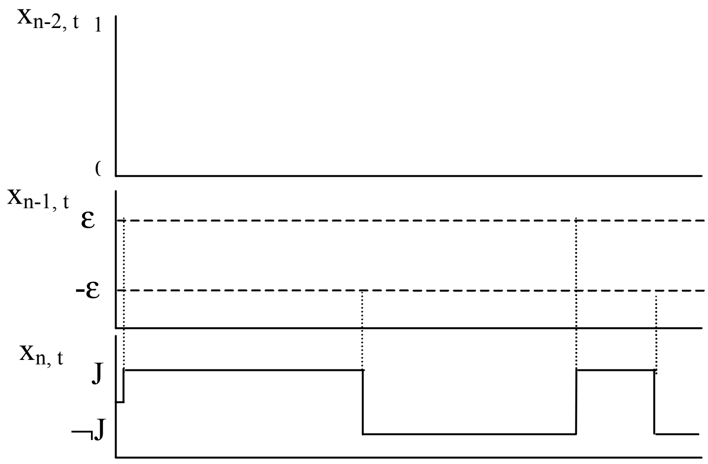

Figure 1. Quasi-chaotic behavior of the dynamic system used as a fuzzy inferencing engine. $x_{n-2,t}$ represents subject's feelings about a perceived object (a person or a social situation). $x_{n,t}$ represents a judgment based on the feelings. The judgment oscillates between two contradictory values J and ¬J. The value of $x_{n-2,t}$ can be interpreted as $\mu_{J,t}(H)$, i.e. a dynamic membership of a perceived object to the fuzzy set J.

Concluding remarks

Recent progress in the field of dynamic social psychology [5] shows that the categorization that takes place in human mind is not only a fuzzy calculus providing a fixed result. Psychologists empirically confirmed that subject's feelings about a perceived person or social situation change in time and there are cases that the feelings oscillate from a highly positive value to a highly negative value and back. Hence the motivation for building an adequate cognitive model that integrates data of various significance and allows for a resulting judgment to change in time.

The solution proposed in this paper adds dynamics to the classic fuzzy-set-theoretic concept of membership. A set of constant membership values is used as the control parameter **a** in a dynamic model $X_{t+1} = F(a, X_t)$, where X_t is a set of state variables. One of the state variables is a dynamic membership of the perceived object to the fuzzy set J interpreted as the resulting judgment. Most of the state variables are represented as nodes of a 2-dimentional grid, where a given value can be observed as a pattern moving through the grid by jumping from one node to another until, as a result of an interaction with one of other patterns it met, it changes its identity (informational content). The grid processing meaningful patterns can be implemented as a feed-forward neural network.

The preliminary computational result shows that for some sets of constant membership values constituting the control parameter there are such functions F that the state of the model changes in a quasi-chaotic way, and that the state variable representing the subject's feelings about the adequacy of a particular judgment tends to a two-focal attractor, which satisfactorily explains the empirically confirmed dynamics of humans' feelings. In other words, simulation showed that the judgment produced by the proposed model sometimes oscillates like the social judgment produced by a human subject. Hence, the described cognitive model seems to be both psychologically and, to certain extent, biologically plausible.

The presented approach adds temporal aspect to the classic fuzzy logic. The introduced concept of dynamic membership may start a new branch of logic--Temporal Fuzzy Logic to be developed as a key to the understanding of mental processes in complex dynamic systems.

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