Dynamic Fuzzy Logic

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Abstract: This paper presents such a way of fuzzy inferencing that a resulting judgment changes in time. The research was motivated by the empirical evidence that people's feelings about a perceived person or a social situation may oscillate between a highly positive value and a highly negative value even in absence of new data. The proposed method of judgment generation employs the classic fuzzy-set-theoretic concept of membership. A set of membership values is used as the control parameter a in a dynamic model $X_{t+1} = F(a, X_t)$, where X_t is a set of state variables. One of the state variables can be interpreted as resulting judgment based on 'feelings' given as a dynamic membership. The function F can be implemented as a feed-forward neural network.

Introduction

Let us assume that for an object H there is a set of its features {P, Q, ... } perceived by a subject. Based on the features the subject evaluates the object. The result of the evaluation is a set of judgments {J, K, ... }. There are several possible approaches to modeling of judgment generation. The simplest, propositional calculus-based approach employs production rules, as, for example, "J if P and Q". This approach works only when for the subject the existence of the features P and Q is obvious and his/her way of evaluation is stereotype-based.

In real life the notions describing features are often fuzzy. Also, as it was experimentally confirmed, subjects' certainty that a perceived object belongs to a given category is different for different objects [6]. In such cases we can assume that P and Q are fuzzy sets of objects having the properties P and Q respectively. Fuzzy set theory offers the notion of membership of an object to a fuzzy set. The membership value is assumed to be a real number ranged from 0 to 1 [8]. Let $\mu_H(P)$ and $\mu_H(Q)$ be values of memberships of the object H to the fuzzy sets P and Q respectively. Fuzzy calculus lets us find $\mu_H(J)$, i.e. the value of membership of the object H to the fuzzy set J that can be interpreted as the set of objects to which the judgment J is adequate. When J is assumed to result from the conjunction of the perceived features P and Q, the classic fuzzy set-theoretic formula $\mu_H(J) = \min\{ \mu_H(P), \mu_H(Q) \}$ can be used [9].

The approach based on fuzzy set theory works well in case of categorization of objects emotionally neutral for a subject and only when the considered features can be assumed as equally essential from the point of view of a generation of a given judgment. When a judgment reflects subject’s feelings, even fuzzy set-based models demonstrate limited adequacy [3]. Among features contributing to a judgment some may be more essential than others. Moreover, it was experimentally confirmed that people’s feelings about a perceived person or a social situation may oscillate between a highly positive value and a highly negative value even in absence of new data [5]. This psychological phenomenon motivated the search for such a way of inferencing that input data are not only fuzzy but also not equally essential, while resulting judgment changes in time. The proposed name of this way of inferencing is DFL (Dynamic Fuzzy Logic). DFC provides, as its result, a function that for a given time t returns a value of a membership of a given object H to a fuzzy set J associated with an appropriate judgment. The dynamic membership can be denoted $\mu_{H,\, t}(J)$. Dynamic defuzzification consists on finding a function mapping time onto the set $\{J, \neg J\}$.

Fuzzy calculus in a dynamic mind

Let us consider a dynamic system defined by the state equation $X_{t+1} = F(a, X_t)$, where $X_t = (x_{0,\, t}, x_{1,\, t}, \ldots , x_{n,\, t}) \in \mathbf{X}$ is a vector of state variables, $\mathbf{X}$ is the space of such vectors, a is a constant control parameter. If perceived features and generated judgments were described in terms of the elements of the state equation, the subject's behavior could by analyzed using a rich tool-set of discrete dynamical system theory [4].

In order to employ the dynamic system as a model of mind, let us assume that $a = (\mu_H(P), \mu_H(Q), \mu_P(J), \mu_Q(J))$, where P and Q are fuzzy sets of objects having the properties P and Q respectively. $\mu_H(P)$ and $\mu_H(Q)$ are values of memberships of the object H to the fuzzy sets P and Q respectively, $\mu_P(J)$ and $\mu_Q(J)$ are values of essentiality of the features P and Q in case of generation of the judgment J. Let us assume that the target function $\mu_{H,\, t}(J)$, that for a given time t returns a value of a membership of a given object H to a fuzzy set J associated with an appropriate judgment, is defined by consecutive values of the state variable $x_{n-2,\, t}$.

While can be interpreted as a representation of feelings of a subject to the perceived object H, the last state variable $x_{n,\,t}$ is reserved for the proper judgment that can oscillate between two contradictory values J and ¬J.

In order to define the proposed function F let us introduce the set **M** of certain pairs of notions used in the discussed example of judgment. Let **M** = {⟨P, H⟩, ⟨¬P, H⟩, ⟨Q, H⟩, ⟨¬Q, H⟩, ⟨J, P⟩, ⟨J, Q⟩}. Let us also introduce the function $\nu_M(\tau_M, \varphi_M, t) = M$ if $t = \varphi_M + k\tau_M$, such that k = 0, 1, 2, ... , $\tau_M = \lfloor T/\mu_a(b)\rfloor$, M = ⟨a, b⟩ ∈ **M**, φ_M is arbitrarily taken. T is a fixed constant value common for all M∈**M**. The function F is such that for $0 \le i \le n\text{-}3$:

$x_{i,\,t+1} = f(x_{j,\,t}, x_{k,\,t})$ if $f(x_{j,\,t}, x_{k,\,t}) \neq \varnothing$,

$x_{i,t+1} = \nu_M(\tau_M, \varphi_M, t)$ if $f(x_{j,\,t}, x_{k,\,t}) = \varnothing$ and $c(M) = i$,

where: $c \mid \mathbf{M} \rightarrow \{i \mid 0 \le i \le n\text{-}3\}$; and the function *f* is defined by the table:

$x_{j,\,t}$	$x_{k,\,t}$	$f(x_{j,\,t}, x_{k,\,t})$
⟨b, a⟩	∅	⟨b, a⟩
∅	⟨ba⟩	∅
⟨b, a⟩	⟨a, H⟩	⟨b, H⟩
⟨b, a⟩	⟨¬a, H⟩	⟨¬b, H⟩
⟨a, H⟩	⟨¬a, H⟩	∅

$j = g(i, X_t)$, $k = h(i, X_t)$, while:

$\mu_{H,\,t}(J) = x_{n-2,\,t+1} = N_{\langle J,\,H\rangle}(X_t)/(\,N_{\langle J,\,H\rangle}(X_t)+N_{\langle\neg J,\,H\rangle}(X_t))$

where $N_M(X_t) = C_{M,\,n}$, $C_{M,\,0} = m$, $C_{M,\,i+1} = C_{M,\,i} + m$,

$m = 1$ if $x_{i,\,t} = M$, $m = 0$ if $x_{i,\,t} \neq M$,

$x_{n-1,\,t+1} = x_{n-2,\,t} - 0.5 + r\,x_{n-1,\,t}$,

$x_{n,\,t+1} = J$ if $x_{n,\,t} = J$ and $x_{n-1,\,t}. > -\varepsilon$ or $x_{n,\,t} \neq J$ and $x_{n-1,\,t} > \varepsilon$,

$x_{n,\,t+1} = \neg J$ if $x_{n,\,t} = \neg J$ and $x_{n-1,\,t} < \varepsilon$ or $x_{n,\,t} \neq \neg J$ and $x_{n-1,\,t} > -\varepsilon$,

$x_{n,\,0} = \varnothing$.

The constant parameters r and ε represent subject's emotional stability. The functions $g, h \mid \{i \mid 0 \le i \le n\text{-}3\} \times \mathbf{X} \rightarrow \{i \mid 0 \le i \le n\text{-}3\}$ are defined in such a way that when the state variables $x_{0,\,t}, x_{1,\,t}, \ldots, x_{n-3,\,t}$ are represented as nodes of a 2-dimentional greed, a given value can be observed as an entity navigating all of the greed (in the course of jumping from one node to another) until, as a result of an interaction with an other entity it meets, it changes its informational identity.

The preliminary computational results shows that for some sets of memberships values constituting the control parameter a there are such functions F that the values of the state variables $x_{n-2, t}$, $x_{n-1, t}$, $x_{n, t}$ change in a chaotic way, suggesting a two-focal strange attractor [1][2] (Figure 1).

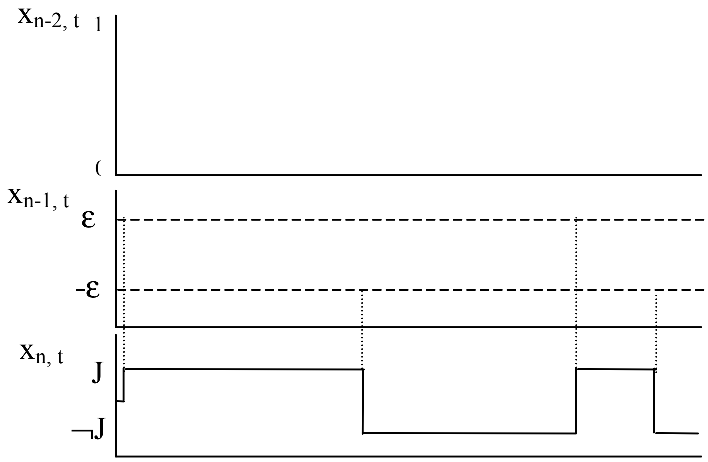

Figure 1. Chaotic behavior of the dynamic system used as a fuzzy inferencing engine. $x_{n-2, t}$ represents subject's feelings about a perceived object. $x_{n, t}$ represents a judgment based on the feelings. The judgment oscillates between two contradictory values J and ¬J. The value of $x_{n-2, t}$ can be interpreted as $\mu_{H, t}(J)$, i.e. a dynamic membership of a perceived object to the fuzzy set J.

Concluding remarks

Recent progress in the field of dynamic social psychology [5] shows that the categorization that takes place in human mind is not only a fuzzy calculus providing a fixed result. As it was empirically confirmed, subject's feelings about a perceived person or social situation change in time and there are cases that the feelings oscillate from a highly positive value to a highly negative value and back. Hence the motivation for building an adequate cognitive model that integrates data of various significance and processes them towards a judgment that can change in time.

The proposed solution employs the classic fuzzy-set-theoretic concept of membership. A set of membership values is used as the control parameter a in a dynamic model $X_{t+1} = F(a, X_t)$, where X_t is a set of state variables. One of the state variables can be interpreted as resulting judgment. Some others are represented as nodes of a 2-dimentional greed, where a given value can be observed as an entity navigating all of the greed in the course of jumping from one node to another until, as a result of an interaction with an other entity it meets, it changes its informational identity. Hence, the function F can be implemented as a feed-forward neural network.

The preliminary computational results shows that for some sets of memberships values constituting the control parameter there are such functions F that the state of the model changes in a chaotic way, suggesting a two-focal strange attractor, which satisfactorily explains the empirically confirmed dynamics of humans' feelings. Hence, the proposed cognitive model seems to be both psychologically and, to certain extent, biologically plausible.

The presented approach adds temporal aspect to the classic fuzzy logic. The introduced concept of dynamic membership has a good chance to become a cornerstone for a new branch of logic--Dynamic Fuzzy Logic (DFL) to be developed as a key to a thought emerging in complex dynamic systems.

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