- 1. Brief Review of Cube Calculus
- 2. Simple Combinational Operators
- 3. Complex Combinational Operators
- 4. Sequential Operators
- 5. Multi-valued Operations
- 6. Patterns of Operations
- 7. General Patterns of Operations and Horizontal Microprogramming
- 8. Why we need CCM?

Cube Calculus for CCM

Overview Of This Lecture

- A Brief Review of Cube Calculus.
- Summary of Cube Calculus operations.
- Positional notation concept for Cube Calculus operations.
- Summary of Positional notation concept for Cube Calculus operations.
- Why we need a Cube Calculus Machine?

Brief Review Of Cube Calculus

Representation of cubes in (Binary logic)

- For the function f(a,b,c,d)= A + B = $a'bc' + ac'd$ we have two cubes ($a'bc'$ and $ac'd$) ⇒ Cube is a Product of literals.
- In binary logic, they can be represented as $A = a^{\{0\}}.b^{\{1\}}.c^{\{0\}}.d^{\{0,1\}}$ and $B = a^{\{1\}}.b^{\{0,1\}}.c^{\{0\}}.d^{\{1\}}$

General Representation of cubes in (n valued logic)

- $A = x_1^{s_1A} . x_2^{s_2A} ....x_n^{s_nA}$
- $B = x_1^{s_1B} . x_2^{s_2B} ....x_n^{s_nB}$
- where $x_1^{s_1A}$ .....$x_n^{s_nA}$.,$x_1^{s_1B}$, ...$x_n^{s_nB}$ are literals.
- ***n*** is the number of variables.
- ***s_i^A, s_i^B*** are true sets of literal ***x_i*** .
- An example of a ternary logic is:

$$A = x_1^{\{0\}} . x_2^{\{0,2\}} . x_3^{\{1\}} . x_4^{\{1,2\}}$$

$$B = x_1^{\{1\}} . x_2^{\{1\}} . x_3^{\{2\}} . x_4^{\{0,1\}}$$

Cube Calculus Operations

Cube Calculus Operations

The cube calculus operations are classified as :

- ***Simple Combinational operations*** (e.g. Intersection, SuperCube).
- ***Complex Combinational operations*** (e.g. Prime, Consensus, Cofactor).
- ***Sequential operations*** (e.g. Crosslink, Sharp(non-disjoint), Sharp (disjoint)).
- Let us discuss these operations..

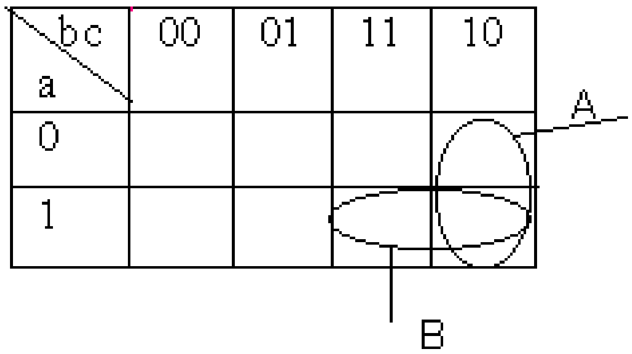
Simple Combinational Cube Operations

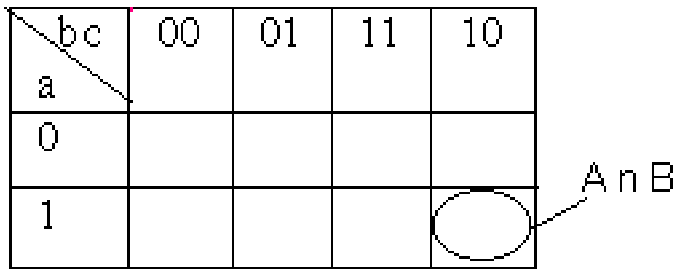
Simple Combinational Operations

- Defined as a ***SINGLE*** set operation on all pairs of true sets and produces one resultant cube.
- ***Intersection*** and ***Supercube*** are simple combinational cube operations.

Example Of Binary Function

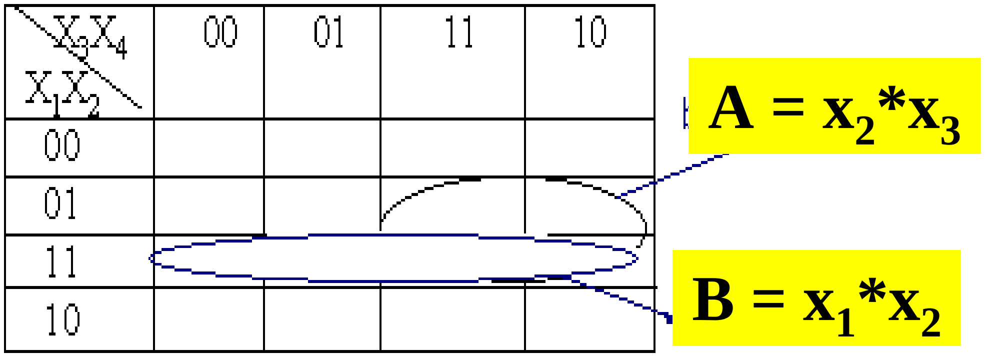
Kmap representation of Intersection

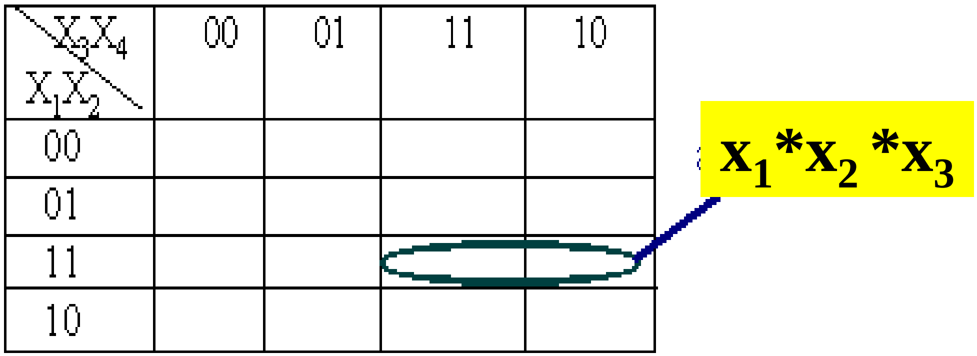




- B= ab
- A= bc′
- Set theory :
  B = ab = abx = $a^{\{1\}}b^{\{1\}}c^{\{0,1\}}$
- A= bc′= xbc′= $a^{\{0,1\}}b^{\{1\}}c^{\{0\}}$
- B ∩ A=$a^{\{1\}\cap\{0,1\}}b^{\{1\}\cap\{1\}}c^{\{0,1\}\cap\{0\}}$ = abc′

Kmap representation of Intersection

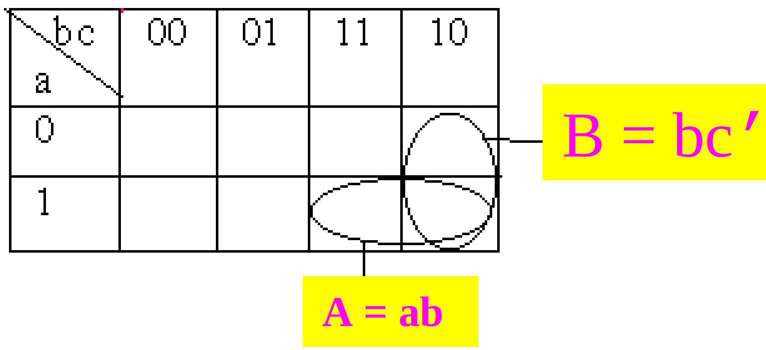


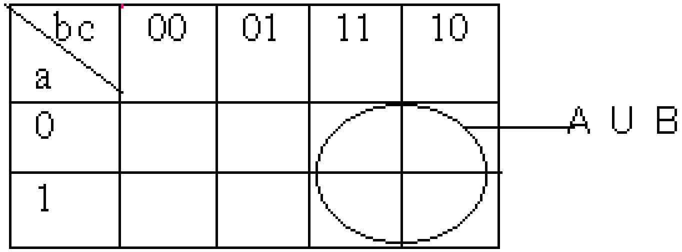


- Fig 1: Input Cubes A and B to be Intersected.
- Here
  - $A = x_2*x_3$
  - $B = x_1*x_2$
- Fig 2:Resultant Cube $C = A \cap B =$

  $x_1*x_2*x_3$

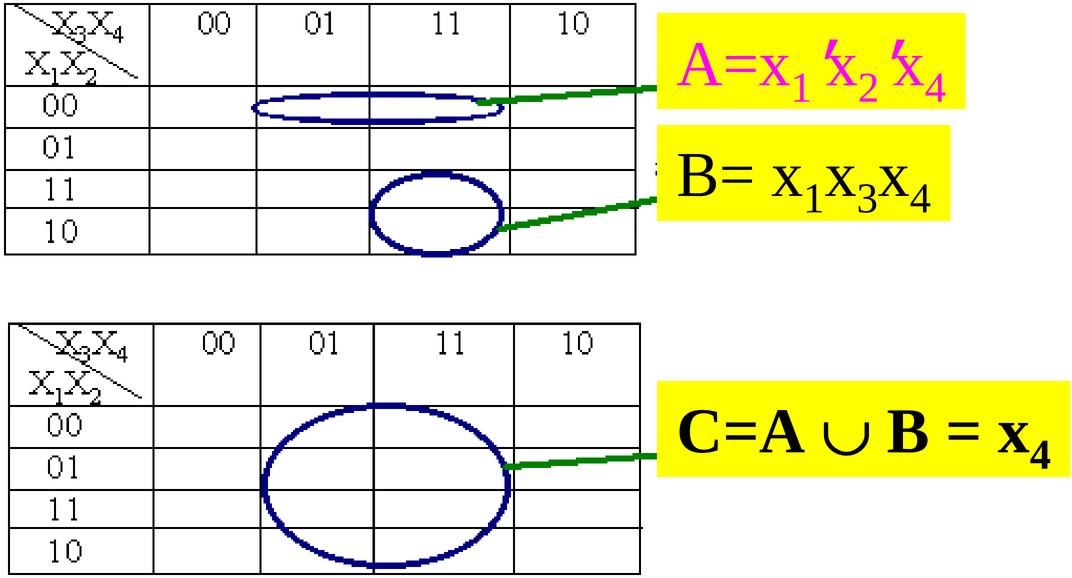
Kmap representation of Supercube





- A and B has 3 variables.
- B = bc′
- In set theory A= abx B=xbc′
- $A \cup B = b$

K-map representation of Super cube



- Input Cubes A and B to be supercubed.

$A = x_1' x_2' x_4$

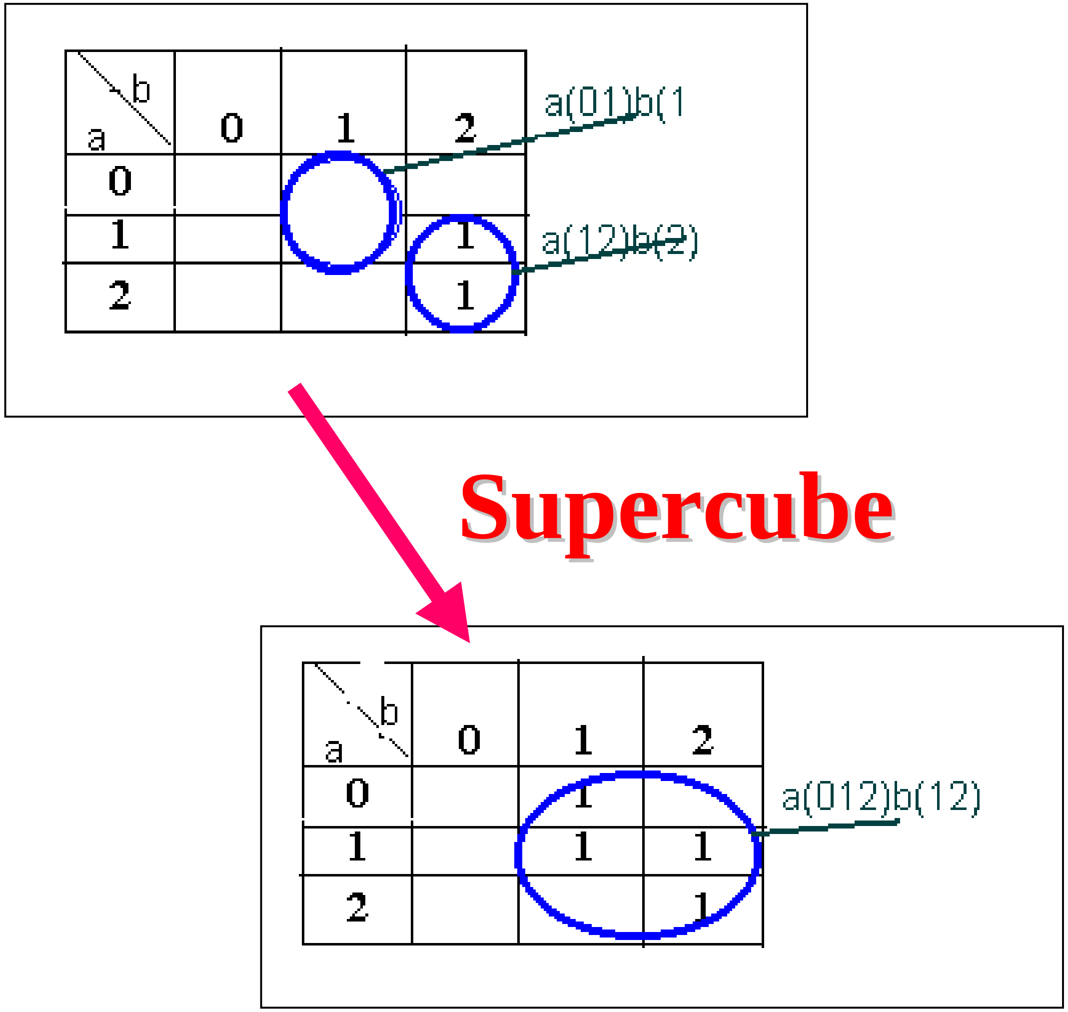
$B = x_1 x_3 x_4$

- Resultant Cube

  $C = A \cup B = x_4$

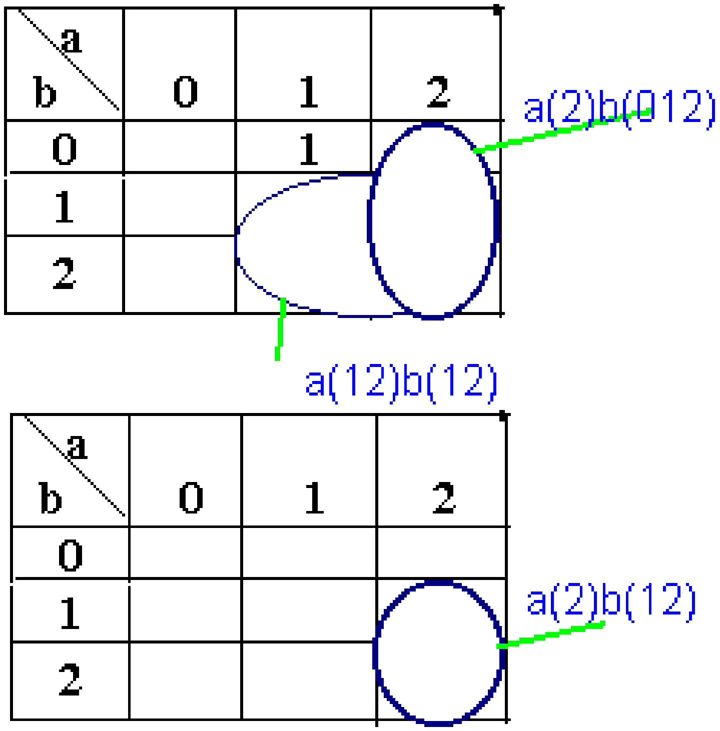
Examples Of Multi-Valued Functions

K-map representation of Supercube



- For 4 valued input logic.
- Input Cubes A and B to be supercubed. $A=a^{01}b^{1}$. $B=a^{12}b^{2}$
- Resultant Cube $C=A \cup B = a^{012}b^{12}$

K-map representation of Intersection



- For 4 valued input logic.
- Input Cubes A and B for Intersection. $A=a^{12}b^{12}$. $B=a^{2}b^{012}$
- Resultant Cube $C=A \cap B = a^{2}b^{12}$

Definitions of Simple Combinational Cube Operations

Intersection of two cubes :

Is the largest cube that is included in both A and B.

Supercube of two cubes :

Is the smallest cube that includes both cubes.

Simple Combinational Operation

- Intersection operation mathematically is defined as

$$A \cap B = \begin{cases} x_{1\,1}^{s^A \cap s_1^B} \ldots\ldots x_{n\,n}^{s^A \cap s_n^B} & \text{if } s_i^A \cap s_i^B \neq \emptyset \\ = \emptyset & \text{otherwise.} \end{cases}$$

- Union operation mathematically is defined as

$$A \cup B = x_{1\,1}^{s^A \cup s_1^B} \ldots\ldots x_{n\,n}^{s^A \cup s_n^B}$$

Complex Combinational Cube Operations

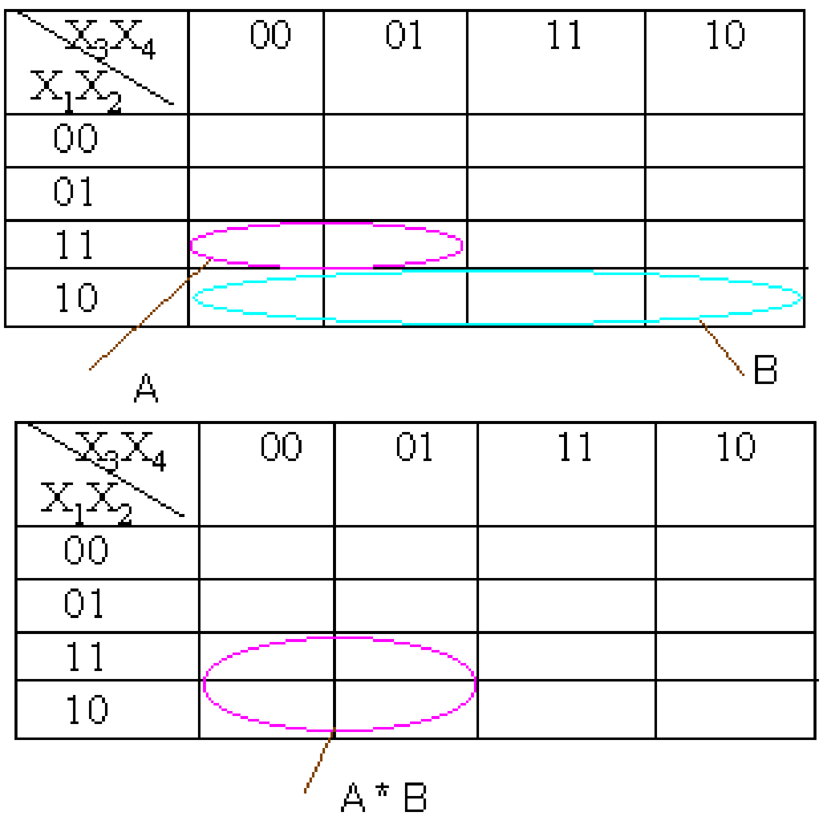
Complex combinational cube operations

- They have *two* set operations and *one* set relation.
- All variables whose pair of true sets satisfy a relation are said to be *Special Variables*.
- Two set operations are called before(*bef*) and active(*act*),the active set operation is applied on true sets of special variables and before set operation to others
- All Combinational cube operations (Complex and Simple) produce one resultant cube, all special variables taken at a time.
- The examples are *prime* operation, *cofactor* operation, *consensus* operation.

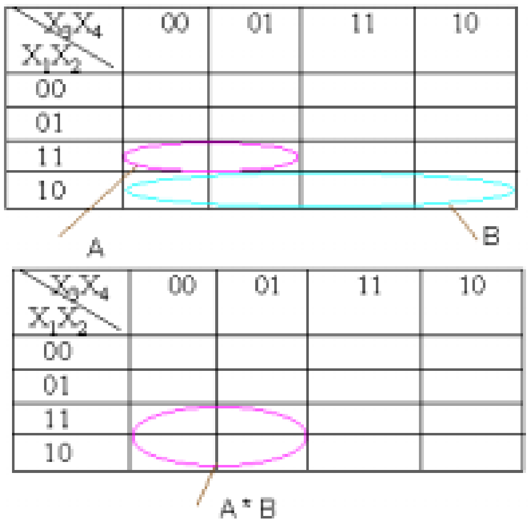
Examples of Complex Combinational Cube Operations

Example Of Binary Function

Example of consensus operation



- A & B have 4 binary variables
  - $A = x_1x_2x_3'$
  - $B = x_1x_2'$
- Steps: find the set relation $si^A \cap si^B = \varnothing$.
- If satisfied then it is a special variable.
- Active operation ($si^A \cup si^B$) applied to <u>special variables</u>.
- And to remaining variables (before variables) the set operator $si^A \cap si^B$ is applied



Explanation of consensus operation

- In set theory

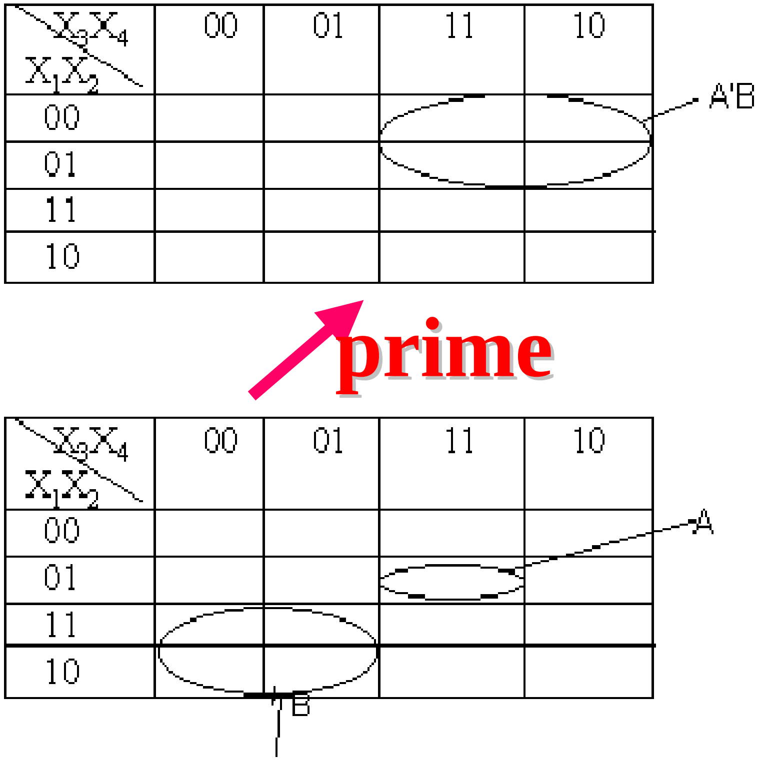
$$A = x_1^{\{1\}} x_2^{\{1\}} x_3^{\{0\}} x_4^{\{0,1\}}$$

$$B = x_1^{\{1\}} x_2^{\{0\}} x_3^{\{0,1\}} x_4^{\{0,1\}}$$

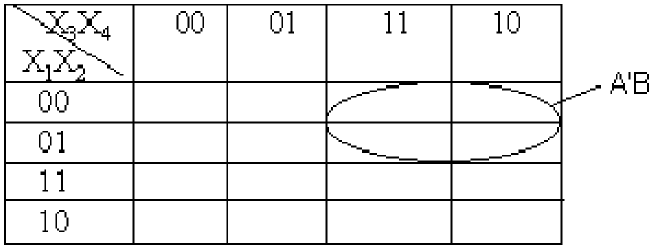
- Applying the set relation $si^A \cap si^B = \emptyset$ on all the true set of variables $\Rightarrow x_1$ is a special variable and to x_1 active operation $(si^A \cup si^B)$ is applied
- and to x_2 ,x_3 ,x_4 before set operator $(si^A \cap si^B)$ is applied

$$\Rightarrow A * B = x_1\{1\}\ x_3\{0\} = x_1 x_3'$$

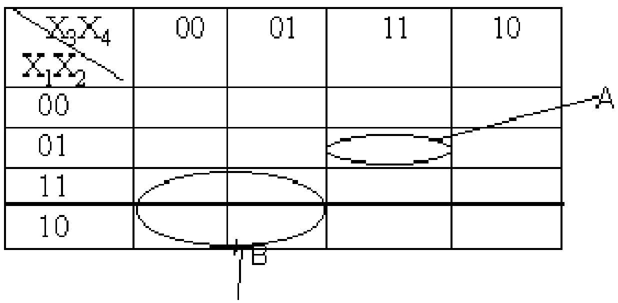
Example of prime operation



- A&B has 4binary variables
- $A = x_1' x_2 x_3 x_4$
- $B = x_1 x_3'$
- **Step 1**: Find intersection operation(set relation) on all true sets($s_i^A \cap s_i^B \neq \emptyset$)
- **Step 2**: If the set relation is satisfied apply active set operation($s_i^A \cup s_i^B$) else before set operation(s_i^A).



Explanation of the Example



- Using set theory

$B = x_1^{\{1\}}\ x_2^{\{0,1\}} x_3^{\{0\}} x_4^{\{0,1\}}$

$A = x_1^{\{0\}} x_2^{\{1\}} x_3^{\{1\}} x_4^{\{1\}}$

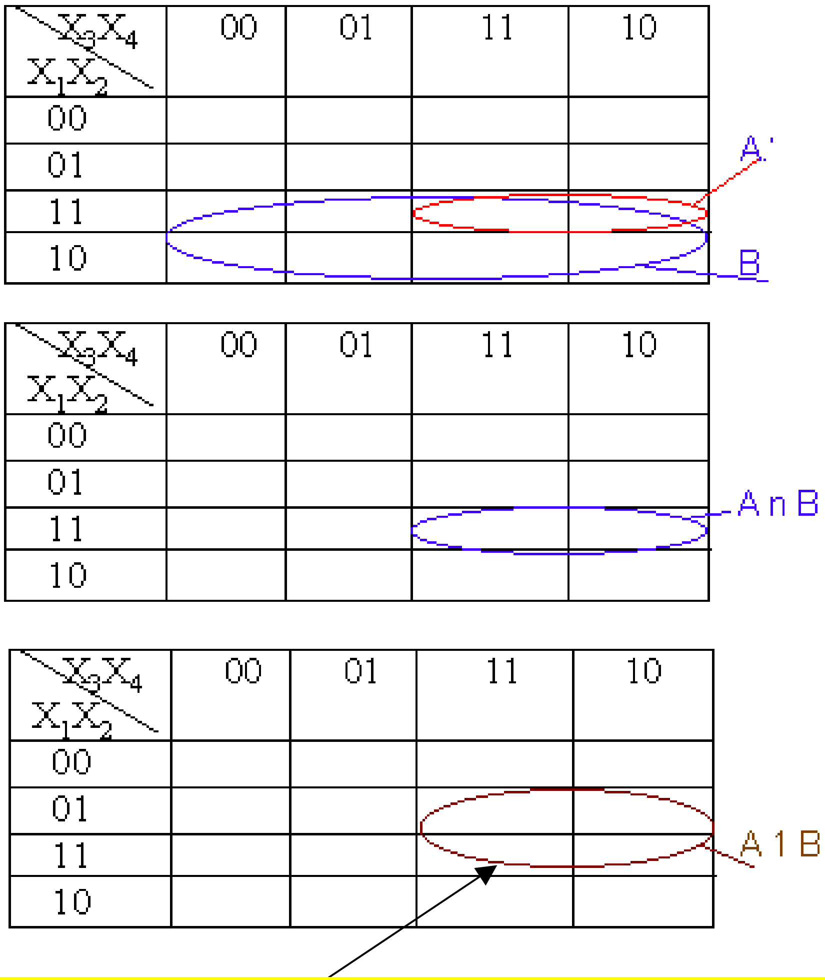
Step 1. Find if set relation ($s_i^A \cap s_i^B \neq \emptyset$) is satisfied or not

. $\Rightarrow x_2$ and x_4 are special variables.

Step 2. As the given set relation is satisfied.for the variables x_2 and x_4 to these variables active set operation ($s_i^A \cup s_i^B$) is applied and to the others (x_1 , x_3) before set operation (s_i^A) is applied

$\Rightarrow\ A'B = x_1^{\{0\}} x_3^{\{1\}} = x_1' x_3$

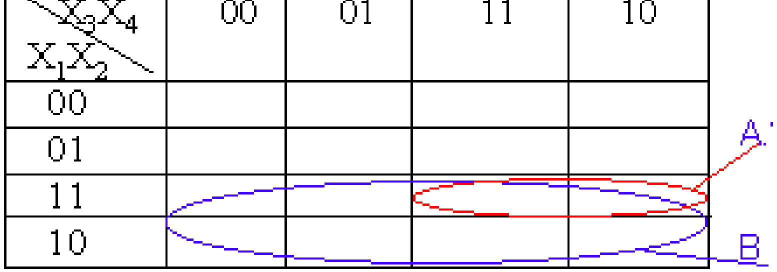
Example of cofactor operation

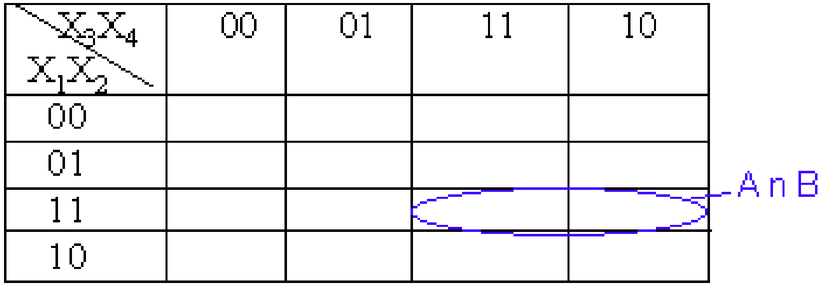


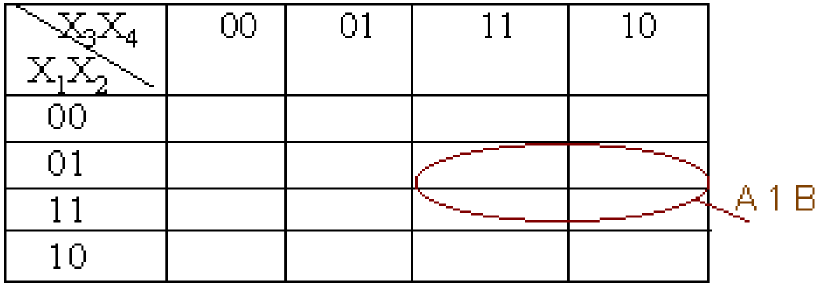
This is cofactor of cube A with respect to cube B (variable x_1)

- A and B have 4 binary variables.
  - $A = x_1x_2x_3$
  - $B = x_1$
- Steps: Find $si^A \supseteq si^B$ (If satisfied $\Rightarrow$ special variable)
- Apply U(Universal set(0,1)) for special variables and $si^A \cap si^B$ for others

Explanation







Set theory:

$A = x_1^{\{1\}}x_2^{\{1\}}x_3^{\{1\}}x_4^{\{0,1\}}$

$B =$

$x_1^{\{1\}}x_2^{\{0,1\}}x_3^{\{0,1\}}x_4^{\{0,1\}}$

- Testing the relation $si^A \supseteq si^B$ for the variables. $\Rightarrow$ x_1 and x_4 are special variables. $\Rightarrow$ Universal set operator applied to these variables.
- To the rest of the variables the intersection operation is applied. $\Rightarrow$ The result is x_2 x_3.
- In K-map apply the intersection operator and remove the special variables.

Example Of Multi-Valued Function

K-map representation of Consensus

A

B

X{012}Y{23}

X{01234} Y{01}

Y \ X	0	1	2	3	4	5
0	1	1	1	1		
1	1	1	1	1		
2	1	1	1	1		
3	1	1				
4	1	1				
5						

X{012}Y{0123}

Y \ X	0	1	2	3	4	5
0	1	1	1	1		
1	1	1	1	1		
2	1	1	1	1		
3						
4						
5						

- A= $X^{01234}\ Y^{01}$
- B = $X^{012}\ Y^{23}$
- A * B =

$X^{01234}Y^{01} * X^{012}Y^{23}$

$= X^{012}\ Y^{0123}$

Definitions of Complex Combinational Cube Operations

General Description of Consensus Cube Operation

Complex combinational cube operation

- The consensus operation on cubes A and B is defined as

$$A * B = \begin{cases} A \cap B \text{ when distance } (A,B) = 0 \\ \varnothing \quad \text{when distance } (A,B) > 1 \\ A *_{basic} B \text{ when distance } (A,B) = 1 \end{cases}$$

- where $A *_{basic} B =$

$$x_1^{s1^A \cap s1^B} \ldots . x_{k-1}^{sk-1^A \cap sk-1^B} \; x_k^{sk^A \cup sk^B} \; x_{k+1}^{sk+1^A \cap sk+1^B} \ldots \ldots x_n^{sn^A \cap sn^B}.$$

- Set relation: $\mathbf{si^A \cap si^B = \varnothing}$, if satisfied, a special variable
- Active set operation : $\mathbf{si^A \cup si^B}$
- Before set operation : $\mathbf{si^A \cap si^B}$

Complex combinational cube operation

- Applications of consensus operator:
  - For finding prime implicants (Used for two-level logic minimization),
  - three level,
  - multilevel minimization,
  - and machine learning.

General Description of Cofactor Cube Operation

Complex combinational cube operation

Cofactor operation of two cubes A and B is

$$A \mid B = \begin{cases} A \mid_{basic} B \text{ when } A \cap B \neq \varnothing \\ = \varnothing \quad \text{otherwise} \end{cases}$$

- Set relation for cofactor operation = $\mathbf{si}^{\mathbf{A}} \supseteq \mathbf{si}^{\mathbf{B}}$
  Active set operator = U (Universal set(0,1))
  Before set operator = $\mathbf{si}^{\mathbf{A}} \cap \mathbf{si}^{\mathbf{B}}$
- Application: Used in functional decomposition.

review

Applications of Crosslink

Assume the we have a function that is expressed in SOP form and we need to have it in the ESOP form, then we need to perform a sequential operation on this function which is the so called Crosslink operation.

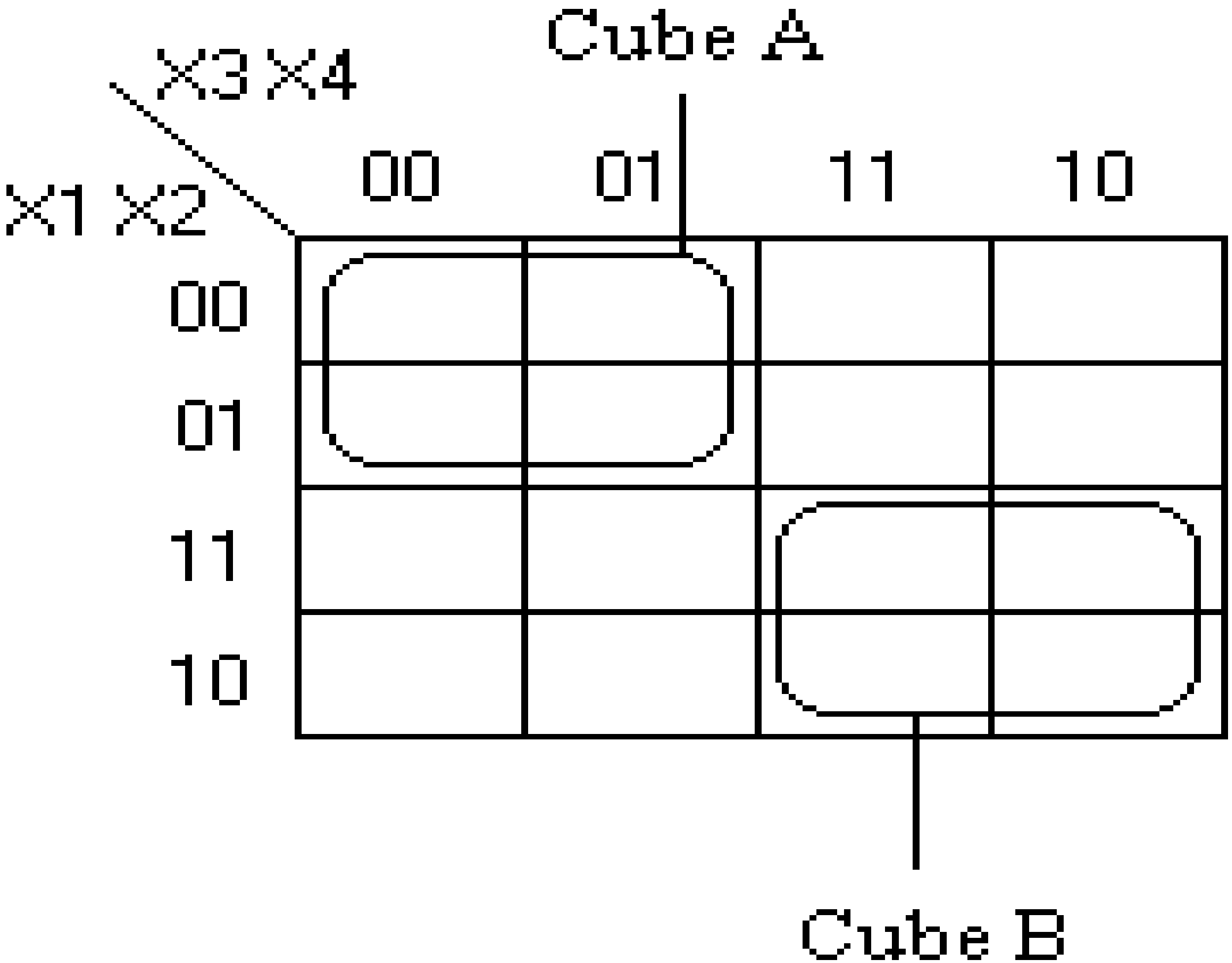
Suppose it is a function with two cubes:

$$f(X1,X2,X3,X4)= \overline{X1}.\overline{X3}+X1X3$$

first cube (cube A for reference) is $\overline{X1}.\overline{X3}$

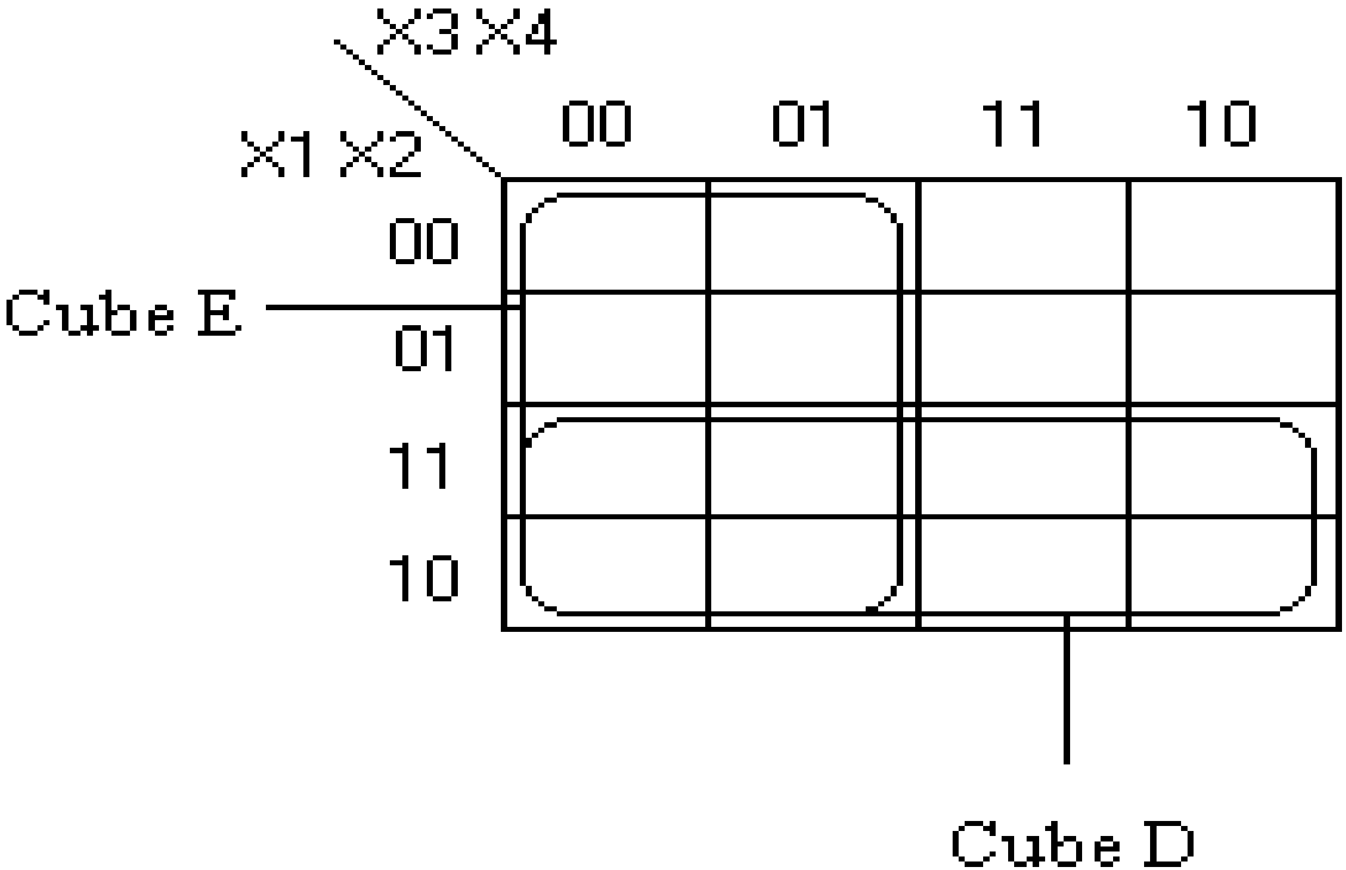
second cube (cube B for reference) is $X1.X3$

Let us see these two cubes on K-map


Cube A
X3 X4
X1 X2
00
01
11
10
00
01
11
10
Cube B

And we all know that the answer is that we are looking for two cubes as shown down here,

so that our function is to be expressed in the ESOP form as follows: $f = E \odot D$



General Description of Prime Operation

Complex combinational cube operations

- Prime operation of two cubes A and B is defined as

$$A \,'\, B = x_1^{s1^A} \ldots x_{k-1}^{sk-1^A} x_k^{sk^A \cup sk^B} x_{k+1}^{sk+1^A} \ldots . x_n^{snA}$$

where set relation : $s_i^A \cap s_i^B \neq \emptyset$ is applied to true sets and if satisfied active set operation is applied

else before set operation is applied.

- Active set operation(act): $s_i^A \cup s_i^B$
- Before set operation(bef): s_i^A
- Application: Used in ESOP minimization .

Sequential Cube Operations

Sequential cube operations

- **They have *three* set operations and *one* set relation.**
- **All variables whose pair of true sets satisfy a relation are said to be *Special Variables*.**
- **Three set operations are called before(*bef*),active(*act*) and after(*aft*)**
- **The <u>active</u> set operation is applied on true sets of special variable**
- **The <u>before</u> set operation to variables before the special variable**
- **The <u>after</u> set operation to variables after the special variable.**
- **Every special variable is taken once at a time.**

Sequential cube operations

- All *sequential cube operations* produce *'n' resultant cubes*, where *n* is the number of special variables.
- The examples are:
  - *sharp* operation,
  - *cross link* operation.

Examples of Sequential Cube Operations

Example Of Binary Function

Let us consider the following example, where the relation operation is $S^A \cap S^B = \varphi$

Cube $A = a^{\{0,1\}} b^{\{0\}} c^{\{0\}}$

Cube $B = a^{\{0,1\}} b^{\{1\}} c^{\{0\}}$

$a^{\{0,1\}} \quad b^{\{0\}} \quad c^{\{0\}}$

$\updownarrow$

$a^{\{0,1\}} \quad b^{\{1\}} \quad c^{\{0\}}$

a is not a special variable.

Cubes A and B are of 4 variables

Let us consider the following two cubes,

Cube A = x1 x2 x3′=

$$x_1^{\{1\}} * x_2^{\{1\}} * x_3^{\{0\}} * x_4^{\{0,1\}}$$

Cube B = x1 x2′=

$$x_1^{\{1\}} * x_2^{\{0\}} * x_3^{\{0,1\}} * x_4^{\{0,1\}}$$

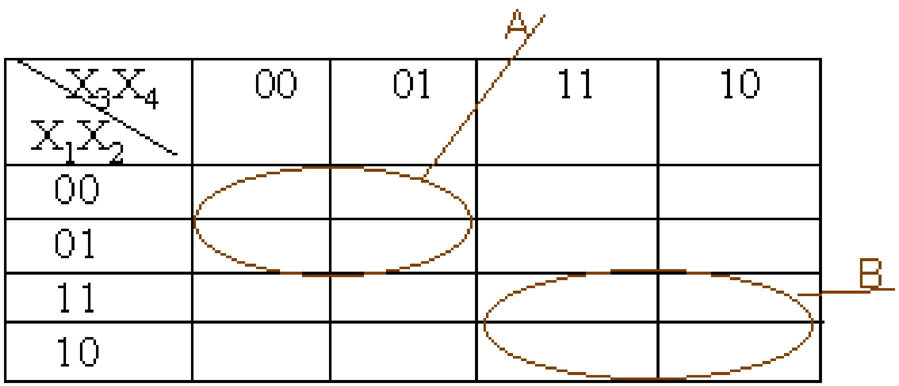
These two variables (x_1 and x_3) are the so called *special variables* , and we find them out by checking the relation between every literal in both cubes for a relation which is for the *crosslink* (is the intersection between these two variables is empty?).

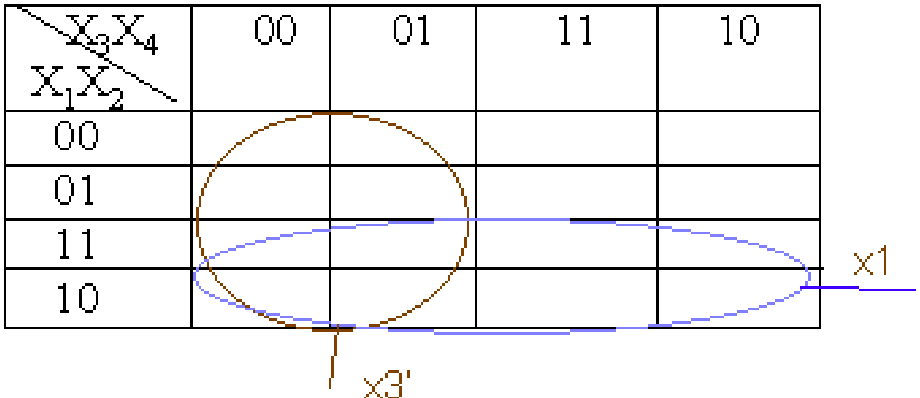
Cube A	$x1^{\{0\}}$	$x2^{\{0,1\}}$	$x3^{\{0\}}$	$x4^{\{0,1\}}$
	‖	‖	‖	‖
Cube B	$x1^{\{1\}}$	$x2^{\{0,1\}}$	$x3^{\{1\}}$	$x4^{\{0,1\}}$

We can see that the intersection is empty for only X1 and X3

therefore they are **special variables**.

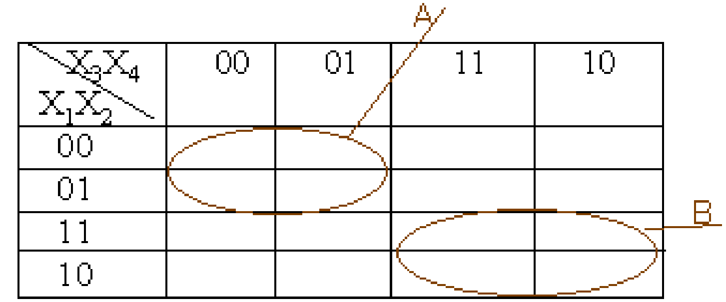
Example of crosslink operation

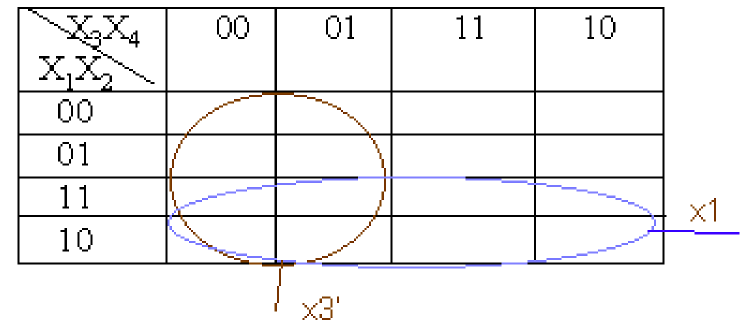




- A and B has 4 binary variables.
- $A = x_1' x_3' = x_1^{\{0\}} x_2^{\{0,1\}} x_3^{\{0\}} x_4^{\{0,1}$
- $B = x_1 x_3 = x_1^{\{1\}} x_2^{\{0,1\}} x_3^{\{1\}} x_4^{\{0,1\}}$
- $A \; c \; B = x_1' x_3' \oplus x_1 x_3 = x_3' \oplus x_1$

Example





- Cubes :

  $x_1^{\{0\}}x_2^{\{0,1\}}\ x_3^{\{0\}}x_4^{\{0,1\}}$

  $x_1^{\{1\}}x_2^{\{0,1\}}x_3^{\{1\}}\ x_4^{\{0,1\}}$

  Set Relation: $si^A \cap si^B = \varphi$

  $\Rightarrow x_1$ and x_3 are special variables..

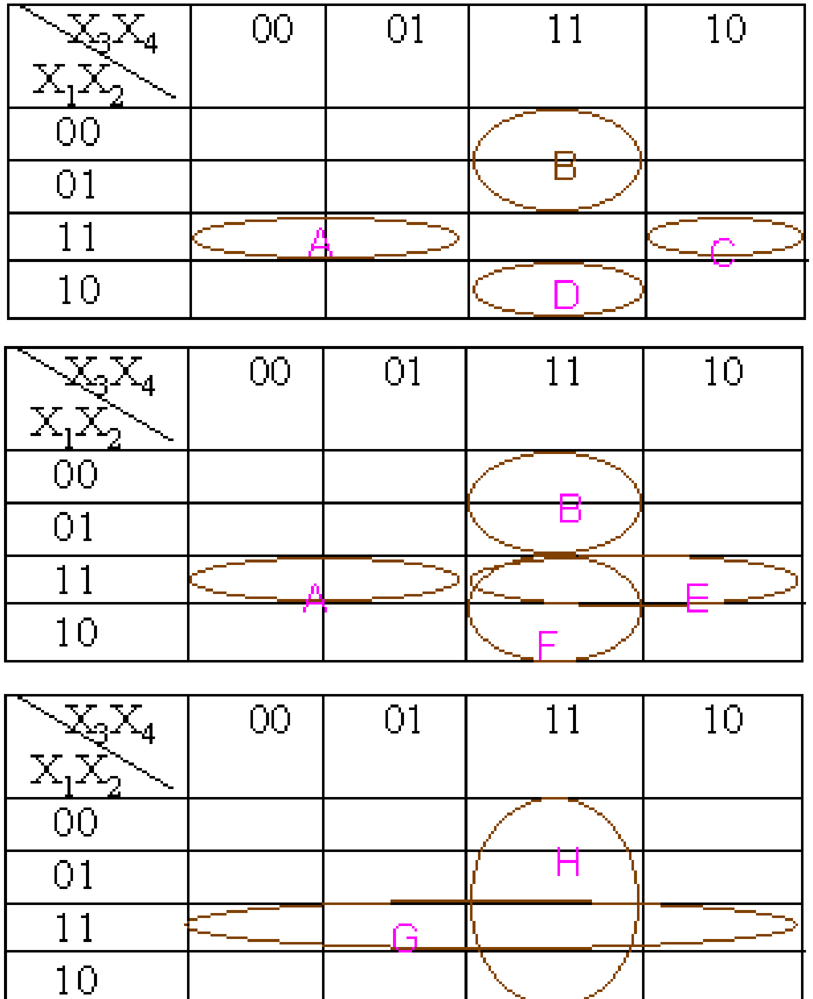
- $\Rightarrow$ **Act:** $si^A \cup si^B$, **Bef:** si^A, **Aft:** si^B

  $\Rightarrow x_1^{\{0\}} \cup {}^{\{1\}}x_2^{\{0,1\}}\ x_3^{\{0\}}x_4^{\{0,1\}}$ and

  $x_1^{\ \{1\}}x_2^{\{0,1\}}\ x_3^{\{0\}} \cup {}^{\{1\}}\ x_4^{\{0,1\}}$

- $\Rightarrow$ Two resultant cubes one for each special variable. $\Rightarrow A\ \ B = x_3' + x_1$

Complex crosslink example



- A,B,C,D are 4 cubes
- F = A⊕B⊕C⊕D
- F = A⊕B⊕E⊕F
- F = G⊕H

Example of nondisjoint sharp

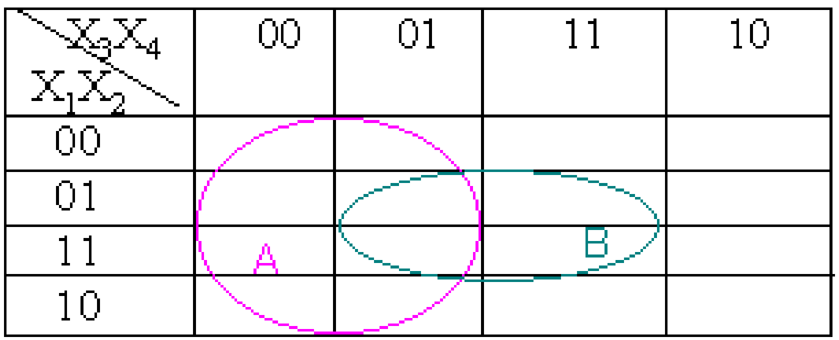
- A and B has 4 binary variables.
- $A = x_3' = x_1^{\{0,1\}} x_2^{\{0,1\}} x_3^{\{0\}} x_4^{\{0,1\}}$
- $B = x_2 x_4 \;\; x_1^{\{0,1\}} x_2^{\{1\}} x_3^{\{0,1\}} x_4^{\{1\}}$
- Set relation: $\neg(si^A \subseteq si^B)$
  - 1. $x_1^{\{0,1\}} \; x_2^{\{0,1\}n(\neg\{1\})} \; x_3^{\{0\}} x_4^{\{0,1\}}$

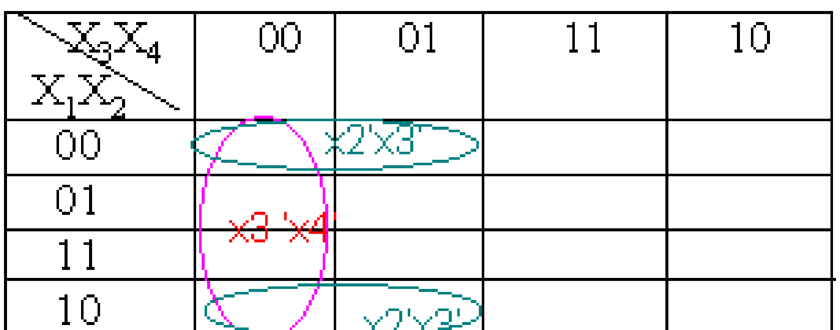
    Act: $si^A \cap si^B$, **Bef:** si^A, **Aft:** si^A
  - 2. $x_1^{\{0,1\}} x_2^{\{0,1\}} \; x_3^{\{0\}} \;\; x_4^{\{0,1\}n(\neg\{1\})}$

Act: $si^A \cap si^B$, **Bef:** si^A,

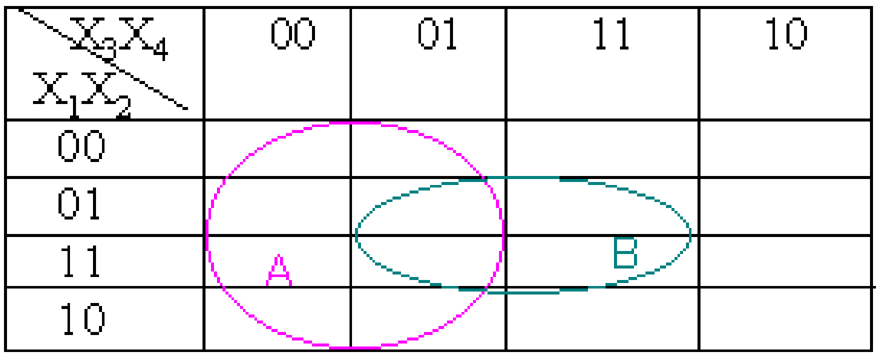
Aft: si^A

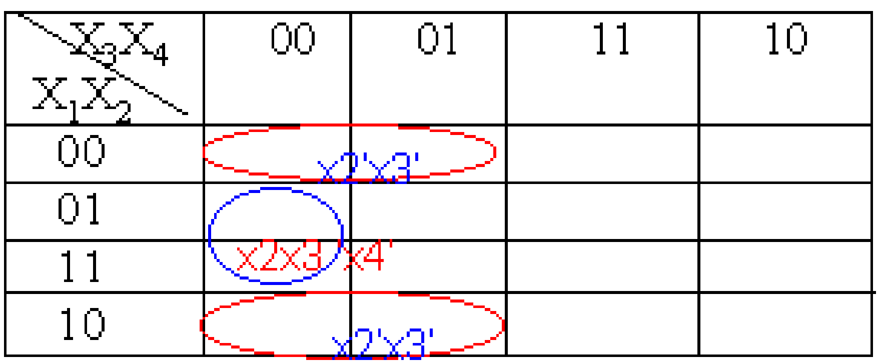
- $A \# B = x_2' x_3' + x_3' x_4'$





Example of disjoint sharp





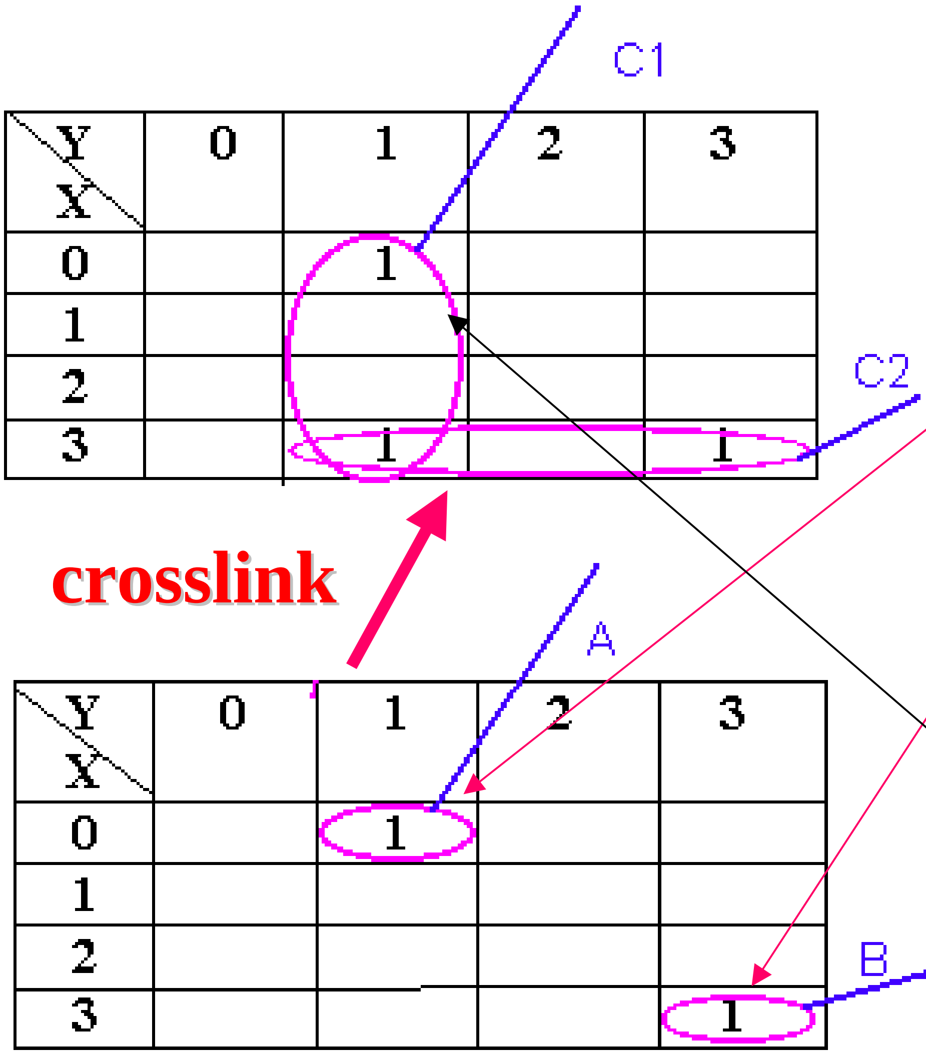
- A and B has 4 binary variables.
- $A = x_3'$

  $x_1^{\{0,1\}}x_2^{\{0,1\}}x_3^{\{0\}}x_4^{\{0,1\}}$
- $B = x_2x_4$

  $x_1^{\{0,1\}}x_2^{\{1\}}x_3^{\{0,1\}}x_4^{\{1\}}$
- Set relation: $\rceil(si^A \subseteq si^B)$
- $x_1^{\{0,1\}n\{0,1\}}x_2^{\{0,1\}n(\rceil\{1\})}$ $x_3^{\{0\}}x_4^{\{0,1\}}$ **Act:** $si^A \cap (\rceil si^B)$, **Bef**: si^A, **Aft**: $si^A \cap si^B$
- $x_1^{\{0,1\}n\{0,1\}}x_2^{\{0,1\}n\{1\}}$ $x_3^{\{0\}n\{0,1\}}$ $x_4^{\{0,1\}n(\rceil\{1\})}$
- **Act:** $si^A \cap (\rceil si^B)$, **Bef:** si^A, **Aft:** $si^A \cap si^B$
- A #d B= $x_2'x_3' + x_2x_3'x_4'$

Example Of Multi-Valued Functions

Example of crosslink operation



- Let $A = x^0 y^1$ and $B = x^3 y^3$

$\Rightarrow$ X and Y are special variables.

$\Rightarrow x^{\{0\} \cup \{3\}} \; y^1$ and

$\Rightarrow x^{\{3\}} \; y^{\{1\} \cup \{3\}}$

$\Rightarrow$ Resultant cubes: $x^{03} y^1 \oplus x^3 y^{13}$

Definitions of Sequential Cube Operations

General Description of Nondisjoint sharp Operation

Sequential cube operations.

- The nondisjoint sharp operation on cubes A and B

$$A \# B = A \quad \text{when } A \cap B \neq \varnothing$$
$$= \varnothing \quad \text{when } A \subseteq B$$
$$= A \#_{basic} B \text{ otherwise}$$

- $A \#_{basic} B = x_1^{s1^A} \ldots x_{k-1}^{sk-1^A} x_k^{sk^A \cap (\rceil sk^B)} x_{k+1}^{sk+1^A} \ldots . x_n^{sn^A}$ for such that k=1,….n for which the set relation is true.

- <u>**Set relation**</u>: $\rceil(\mathbf{si^A \subseteq si^B})$.

  Active set operation: $\mathbf{si^A \cap (\rceil si^B)}$,

  After set operation: $\mathbf{si^A}$,

  Before set operation: $\mathbf{si^A}$

- Used in **tautology** problem.

General Description of disjoint sharp operation

Sequential cube operations.

- The disjoint sharp operation on cubes A and B

  $A \# d\, B = A$ when $A \cap B \neq \emptyset$

  $= \emptyset$ when $A \subseteq B$

  $= A \#d_{basic}\, B$ otherwise

- $A \#d_{basic}\, B = x_1^{s1^A} \ldots x_{k-1}^{sk-1^A} x_k^{sk^A \cap (\rceil sk^B)} x_{k+1}^{sk+1^A} \ldots . x_n^{sn^A}$

  for such that k=1,….n for which the set relation is true.

- Set relation: $\rceil(si^A \subseteq si^B)$

  Active set operation: $si^A \cap (\rceil si^B)$,

  After set operation: $si^A \cap si^B$,

  Before set operation: si^A

- Used in tautology problem, conversions between SOP and ESOP representations

General Description of Crosslink Operation

Sequential cube operations.

- Generalized equation of Crosslink operation:

  $$A \;\; B = x_1^{s1^B} \ldots x_{k-1}^{sk\text{-}1^B} x_k^{sk^A \cup sk^B} x_{k+1}^{sk+1^A} \ldots . x_n^{sn^A}$$

  where k=1,….n for which the set relation is true.
- **Set relation**: $si^A \cap si^B = \varnothing$,
- **Active** set operation : $si^A \cup si^B$,
- **After** set operation : si^B,
- **Before** set operation : si^A.
- Used in the minimization of logic function on EXOR logic, Generalized Reed-Muller form.

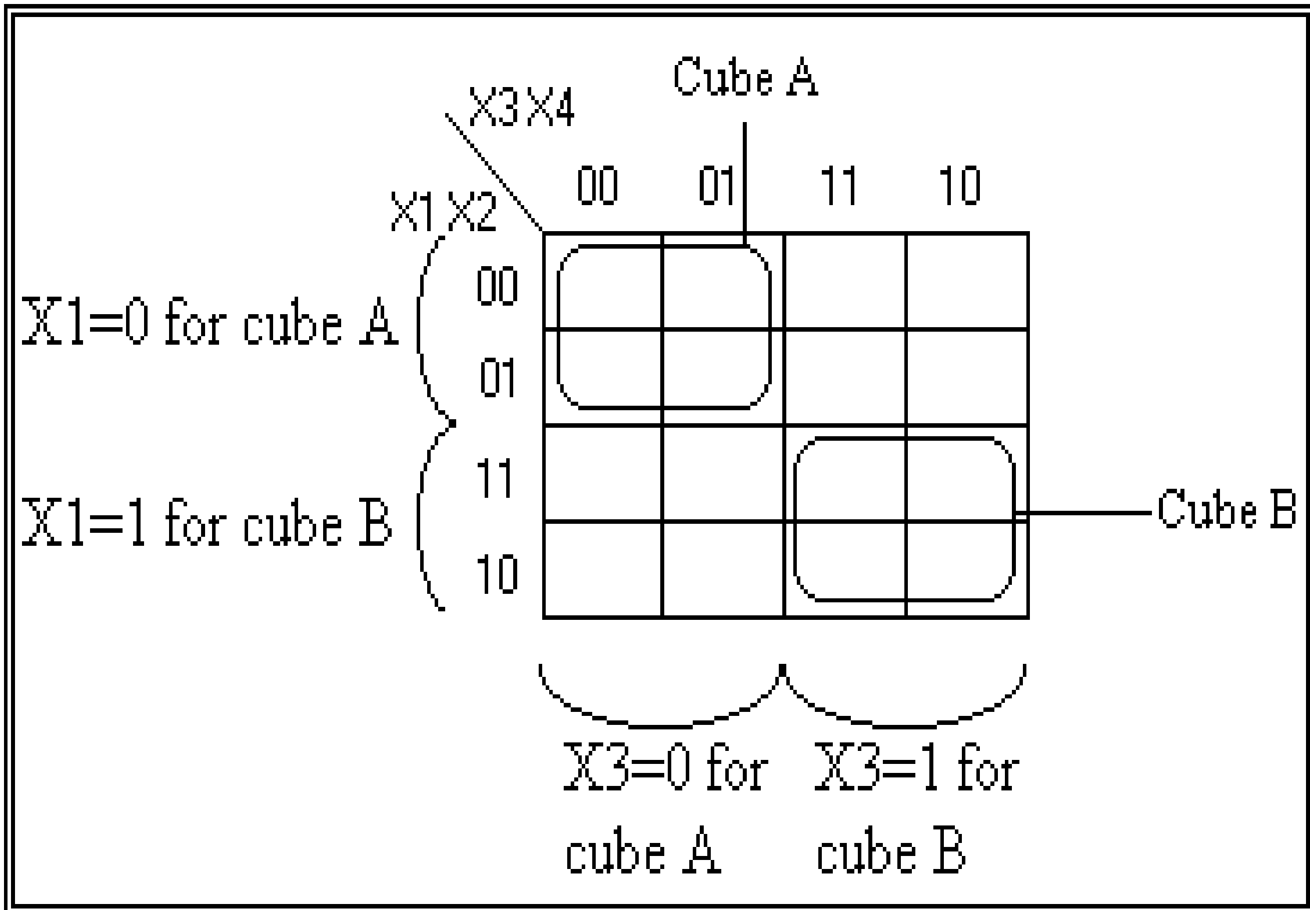
So the solution is to find these two cubes out (Cubes E, D).

Let us see how the crosslink operation does that!!!

Here is the K map for the original function again

Here if we think about it, we are looking for literals, in which the intersection of these literals in both cubes is empty, that is

X1 and X3,
Because in the first cube we have
$\overline{X1}$ and $\overline{X3}$
in the 2nd cube
we have X1 & X3



Summary of the Cube Calculus Operations

Summary of cube calculus operations

Operation	Notation	Relation	Before	Active	After
Intersection	$A \cap B$	1	$s_i^A \cap s_i^B$	-	-
Supercube	$A \cup B$	1	$s_i^A \cup s_i^B$	-	-
Prime	$A'B$	$s_i^A \cap s_i^B \neq \emptyset$	s_i^A	$s_i^A \cup s_i^B$	-
Consensus	$A *_{basic} B$	1	$s_i^A \cap s_i^B$	$s_i^A \cup s_i^B$	$s_i^A \cap s_i^B$
Cofactor	$A \rfloor_{basic} B$	$si^A \supseteq si^B$	$s_i^A \cap s_i^B$	U	-
Crosslink	$A B$	$s_i^A \cap s_i^B = \emptyset$	s_i^A	$s_i^A \cup s_i^B$	s_i^B
Sharp	$A \#_{basic} B$	$\neg(si^A \subset si^B)$	s_i^A	$s_i^A \cap (\neg s_i^B)$	s_i^A
Disjoint Sharp	$A \#_{dbasic} B$	$\neg(si^A \subseteq si^B)$	s_i^A	$s_i^A \cap (\neg s_i^B)$	$s_i^A \cap s_i^B$

Positional Notation

Positional notation

- Cube operations were broken into several set relations and set operations.$\Rightarrow$ Easy to carry out by hand.
- To Process set operations efficiently by computers, we use positional notation.
- Positional notation : Possible value of a variable is 0 or 1.
- P valued variable $\Rightarrow$ a string of p bit
- Positional notation of binary literals:$x' \Rightarrow x^{\{0\}} \Rightarrow x^{10} \Rightarrow$ 10 $x \Rightarrow x^{\{1\}} \Rightarrow x^{01} \Rightarrow$ 01, don't care $x \Rightarrow x^{\{0,1\}} \Rightarrow x^{11} \Rightarrow$ 11, contradiction $\in \Rightarrow x^{\{\varnothing\}} \Rightarrow x^{00} \Rightarrow$ 00

Set operations in positional notation

- Three basic set operations are executed using bit wise operations in positional notation.
- Set intersection $\Rightarrow$ bitwise AND
- Example A = ab B=bc$'$A $\cap$ B = abc$'$
- In positional notation: A = 01-01-11 B = 11-01-10

  A $\cap$ B = (01/11)-(01/01)-(11/10) $\Rightarrow$ 01-01-10(abc$'$)
- Set union $\Rightarrow$ bitwise OR
- Example A = ab B=bc$'$A $\cup$ B = b
- In positional notation: A = 01-01-11 B = 11-01-10

  A $\cup$ B = (01/11)-(01/01)-(11/10) $\Rightarrow$ 11-01-11(b)

Positional notation

- Set complement operation $\Rightarrow$ bitwise NOT
- Example: A= ab then A'= $(ab)'$
- In positional notation: A = 01-01 $\Rightarrow$ A'= 10-10
- Example of multi valued variable:A= x{0,1,2} B=x{0,2,3} $A \cap B$ = {0,2}, $A \cup B$ = {0,1,2,3}
- In positional notation: A=1110, B=1011 $A \cap B$ =1110/1011=1010{0,2},$A \cup B$ =1110/1011=1111{0,1,2,3} A'=0001{3}, B'=0100{1}

Set relations in positional notation

- $1 \Rightarrow$ True and $0 \Rightarrow$ False
- Set relation cannot be done by bit wise operation as it is a function of all bits of operands.
- Set relation is broken into Partial relation and Relation type.
- The partial relation determine whether or not the two literals satisfy the relation locally.
- The relation type determines the method of combining partial relations.
- Relation(A,B) = $(a_0'+b_0').(a_1'+b_1')\ldots\ldots\ldots\ldots(a_{n-1}'+b_{n-1}')$ for crosslink operation $\Rightarrow$ Partial relation $a_i'+b_i'$ and relation type is AND.

Table of partial relation

a_i	b_i	$A_i \subseteq B_i$	$\neg A_i \subseteq B_i$	$a_i b_i'$	$A_i \supseteq B_i$	$a_i + b_i'$	$A_i \cap B_i = \emptyset$	$a_i' + b_i'$	$A_i \cap B_i \neq \emptyset$	$a_i b_i$
0	0	1	0	0	1	1	1	1	0	0
0	1	1	0	0	0	0	1	1	0	0
1	0	0	1	1	1	1	1	1	0	0
1	1	1	0	0	0	0	0	0	1	1

Summary of cube operations in positional notation

Operation	Notation	Relation Relation(Type)	Before	Active	After
Intersection	$A \cap B$	-	$a_i b_i$	-	-
Supercube	$A \cup B$	-	$a_i + b_i$	-	-
Prime	$A'B$	$a_i b_i$(Or)	a_i	$a_i + b_i$	-
Consensus	$A *_{basic} B$	1	$a_i b_i$	$a_i + b_i$	$a_i b_i$
Cofactor	$A \|_{basic} B$	$a_i + b_i'$(And)	$a_i b_i$	1	-
Crosslink	$A B$	$a_i' + b_i'$(And)	a_i	$a_i + b_i$	b_i
Sharp	$A \#_{basic} B$	$a_i b_i'$ (Or)	a_i	$a_i b_i'$	a_i
Disjoint Sharp	$A \#_{dbasic} B$	$a_i b_i'$(Or)	a_i	$a_i b_i'$	$a_i b_i$

Why We Need a Cube Calculus Machine?

Why Using Cube Calculus Machine?

- The cube calculus Operations can be implemented on general-purpose computers,
- But in general-purpose computers, the control is located in the program that is stored in the memory.
- This results in a considerable ***control overhead***.
- Since the instructions have to be fetched from the memory, if an algorithm contains loops, the same instruction will be read many times.

Why Using Cube Calculus Machine?

That makes the memory interface the bottleneck of these architectures.

The cube calculus operations involve nested loops, it leads to poor performance on these general-purpose computers.

Sources

Nouraddin Alhagi
Syeda Mohsina
Qihong Chen