Contents

Preface xxix

1 Overview 1

1.0 Introduction 1
1.1 Laying the Foundation 1
1.2 Design Decisions 2
  1.2.1 Number of Dimensions 2
  1.2.2 Type of Processing 2
  1.2.3 Arithmetic Format 2
  1.2.4 Weighting Functions 3
  1.2.5 Transform Length 3
  1.2.6 Algorithm Building Blocks 3
  1.2.7 Algorithm Construction 3
  1.2.8 DSP Chips 3
  1.2.9 Architectures 3
  1.2.10 Mapping Algorithms onto Architectures 4
  1.2.11 Board Decisions and Selection 4
  1.2.12 Test Signals and Procedures 4
1.3 Types of Examples 4
  1.3.1 Eight-Point DFT to FFT Example 5
  1.3.2 Algorithm Steps and Memory Maps 5
  1.3.3 Fifteen-Point or 16-Point FFT Algorithm Examples 5
  1.3.4 Sixteen-Point Radix-4 FFT Algorithm Examples 5
  1.3.5 Four-Point FFT and 16-Point Radix-4 FFT Algorithm Examples 5
1.4 Design Examples 6
1.4.1 Doppler Radar 6
1.4.2 Power Spectrum Estimator 6
1.4.3 Speech Recognition 6
1.4.4 Image Deblurring 6
1.5 Conclusions 7

2 The Discrete Fourier Transform 9
2.0 Introduction 9
2.1 Common Uses of the DFT 10
2.2 Equation and Block Diagram 10
2.3 Properties 10
2.3.1 Frequency Limits 10
2.3.2 DFT Filter Spacing/Nulls 12
2.3.3 Linearity 12
2.3.4 Symmetry 12
2.3.5 Inverse DFT 12
2.3.6 Ease of IDFT Computation 12
2.3.7 Time and Frequency Scaling 13
2.3.8 Time and Frequency Shifting 13
2.3.9 Parseval’s Theorem 14
2.3.10 Zero Padding 14
2.3.11 Resolution 15
2.3.12 Periodicity 16
2.3.13 Summary of Properties 16
2.4 Real Input Signals 16
2.4.1 Two-Signal Algorithm 17
2.4.2 Double-Length Algorithm 18
2.5 Strengths 20
2.5.1 Periodic Signals 20
2.5.2 Real or Complex Input Data 21
2.5.3 Sets of Data 21
2.5.4 Coherent Integration Gain 22
2.6 Weaknesses 22
2.6.1 Computational Load 22
2.6.2 Quantization Noise Error 23
2.6.3 High Sidelobes 23
2.6.4 Frequency Straddle Loss 23
2.6.5 Transient Signals 23
2.7 Conclusions 24
3 The Fast Fourier Transform 27

3.0 Introduction 27
3.1 Improvements to the DFT 27
3.1.1 Computational Load 28
3.1.2 Quantization Noise 28
3.2 FFT-Specific Weakness 28
3.3 Eight-Point DFT to FFT Example 28
3.3.1 Eight-Point DFT Equations in Matrix Form 29
3.3.2 180° Redundant Computations 30
3.3.3 90° Redundant Computations 30
3.3.4 45° Redundant Computations 31
3.4 Building-Block Construction of FFT Algorithms 32
3.5 Conclusions 34

4 Weighting Functions 35

4.0 Introduction 35
4.1 Six Performance Measures 35
4.1.1 Highest Sidelobe Level 36
4.1.2 Sidelobe Fall-off Ratio 36
4.1.3 Frequency Straddle Loss 36
4.1.4 Coherent Integration Gain 36
4.1.5 Equivalent Noise Bandwidth 36
4.1.6 Three dB Main-Lobe Bandwidth 37
4.2 Weighting Function Equations and Their FFTs 37
4.2.1 Rectangular 37
4.2.2 Triangular 38
4.2.3 Sine Lobe 39
4.2.4 Hanning 40
4.2.5 Sine Cubed 40
4.2.6 Sine to the Fourth 41
4.2.7 Hamming 42
4.2.8 Blackman 43
4.2.9 Three-Sample Blackman-Harris 43
4.2.10 Four-Sample Blackman-Harris 45
4.2.11 Kaiser-Bessel 46
4.2.12 Gaussian 48
4.2.13 Dolph-Chebyshev 49
4.2.14 Finite Impulse Response Filter Design Techniques 52
4.3 Weighting Function Comparison Matrix 52
4.4 Conclusions 53
5 Frequency Analysis  55
5.0 Introduction  55
5.1 Five Performance Measures  55
  5.1.1 Input Sample Overlap  55
  5.1.2 Sidelobe Level  56
  5.1.3 Frequency Straddle Loss  56
  5.1.4 Frequency Resolution  56
  5.1.5 Coherent Integration Gain  57
5.2 Computational Techniques  57
  5.2.1 Nonoverlapped  57
  5.2.2 Overlapped  58
  5.2.3 Weighting Functions  58
5.3 Conclusions  59

6 Linear Filtering and Pattern Matching  61
6.0 Introduction  61
6.1 Equations  61
6.2 Three Performance Measures  62
  6.2.1 Number of Computations per Data Point  62
  6.2.2 Number of Data Memory Locations  62
  6.2.3 Computational Latency  63
6.3 Direct Method  63
  6.3.1 Complex Input Signal  63
  6.3.2 Real Input Signal  63
6.4 Single-Step Frequency Domain Method  64
  6.4.1 Complex Input Signal  64
  6.4.2 Real Input Signal  64
6.5 Multiple-Step Frequency Domain Method  65
6.6 Overlap-and-Add Frequency Domain Algorithm  65
  6.6.1 Introduction  65
  6.6.2 Complex Input Signals  65
  6.6.3 Real Input Signals  67
6.7 Overlap-and-Save Frequency Domain Algorithm  68
  6.7.1 Introduction  68
  6.7.2 Complex Input Signals  69
  6.7.3 Real Input Signals  70
6.8 Linear Filtering and Pattern Matching
  Comparison Matrix  70
6.9 Conclusions  71
7 Multidimensional Processing 73

7.0 Introduction 73
7.1 Frequency Analysis 74
  7.1.1 Two Dimensions 74
  7.1.2 Three or More Dimensions 75
7.2 Linear Filtering 75
  7.2.1 Separable Two-Dimensional Filter 76
  7.2.2 Frequency Domain Approach 76
  7.2.3 Three and More Dimensions 77
7.3 Pattern Matching 78
  7.3.1 Separable Two-Dimensional Pattern Matching 78
  7.3.2 Frequency Domain Approach 79
  7.3.3 Three and More Dimensions 80
7.4 Conclusions 80

8 Building-Block Algorithms 81

8.0 Introduction 81
8.1 Four Performance Measures 81
  8.1.1 Number of Adds 82
  8.1.2 Number of Multiplies 82
  8.1.3 Number of Memory Locations for Multiplier Constants 82
  8.1.4 Number of Data Memory Locations 83
8.2 Ten Building-Block Algorithm Constraints 83
8.3 Two-Point FFT 84
8.4 Three-Point FFT 85
  8.4.1 Winograd 3-Point FFT 85
  8.4.2 Singleton 3-Point FFT 86
8.5 Four-Point FFT 87
8.6 Five-Point FFT 88
  8.6.1 Winograd 5-Point FFT 89
  8.6.2 Singleton 5-Point FFT 91
  8.6.3 Rader 5-Point FFT 93
8.7 Seven-Point FFT 96
  8.7.1 Winograd 7-Point FFT 97
  8.7.2 Singleton 7-Point FFT 101
8.8 Eight-Point FFT 103
  8.8.1 Winograd 8-Point FFT 104
  8.8.2 Eight-Point Radix-4 and -2 Algorithm 107
  8.8.3 Eight-Point Radix-2 Algorithm 110
  8.8.4 PTL 8-Point FFT 113
9 Algorithm Construction 145

9.0 Introduction 145
9.1 Four Performance Measures 145
  9.1.1 Number of Adds 146
  9.1.2 Number of Multiplies 146
  9.1.3 Number of Memory Locations
      for Multiplier Constants 146
  9.1.4 Number of Data Memory Locations 146
9.2 Nine Algorithm Constraints 146
9.3 Three Construction Approaches 147
9.4 Algorithm Data Mapping Relabeling 148
  9.4.1 General Address Relabeling 148
  9.4.2 Four-Point FFT Address Relabeling Example 148
9.5 Convolution Approach 149
  9.5.1 Bluestein Algorithm Introduction 149
  9.5.2 Number of Bluestein Algorithm
      Adds and Multiplies 151
  9.5.3 Number of Bluestein Algorithm
      Memory Locations 151
  9.5.4 General Bluestein Algorithm 152
  9.5.5 Fifteen-Point Bluestein Example 158
  9.5.6 Winograd Algorithm Introduction 167
  9.5.7 Number of Winograd Algorithm
      Adds and Multiplies 169
  9.5.8 General Winograd Algorithm 169
  9.5.9 Fifteen-Point Winograd Algorithm Example 173
9.6 Prime Factor Approach 185
  9.6.1 Prime Factor Algorithm Introduction 185
  9.6.2 Number of Prime Factor Algorithm
      Adds and Multiplies 187
9.6.3 General Prime Factor Algorithm for Two Factors 187
9.6.4 Fifteen-Point Kolba-Parks FFT Example 191
9.6.5 Fifteen-Point SWIFT Example 199

9.7 Mixed-Radix Approach 207
9.7.1 Mixed-Radix Algorithm Introduction 207
9.7.2 Number of Mixed-Radix Algorithm Adds and Multiplies 210
9.7.3 Categories of the Mixed-Radix Algorithm 211
9.7.4 General Mixed-Radix Algorithm for Two Factors 211
9.7.5 Sixteen-Point Radix-4 Primes-to-a-Power FFT Example 213
9.7.6 Sixteen-Point Radix-8 and -2, Mixed Power-of-Primes Example 222
9.7.7 Fifteen-Point Singleton Mixed-Radix FFT Example 230

9.8 Comparison Matrices 242
9.9 Conclusions 243

10 Arithmetic Building Blocks for Architectures 245
10.0 Introduction 245
10.1 Five Performance Measures 246
10.1.1 Input Data Organization 246
10.1.2 Output Data Organization 246
10.1.3 Internal Data Bus Loading 246
10.1.4 Throughput from Computations 246
10.1.5 Latency from Computations 247
10.2 Bit-Slice Arithmetic 247
10.2.1 Multiplier 248
10.2.2 Multiplier-Accumulator 250
10.3 Integrated Arithmetic 250
10.3.1 Multiplier 250
10.3.2 Multiplier-Accumulator 250
10.4 Special Purpose 251
10.4.1 FFT Data Separation Patterns 251
10.4.2 Decimation-in-Time Building Block 253
10.4.3 Decimation-in-Frequency Building Block 253
10.5 Conclusions 254

11 Multiprocessor Architectures 255
11.0 Introduction 255
11.1 Two Single Processors 255
11.1.1 Von Neumann Architecture 256
11.1.2 Harvard Architecture 257
11.2 Three Linear Arrays 258
  11.2.1 Pipeline 258
  11.2.2 Linear Bus 259
  11.2.3 Ring Bus 260
11.3 Three Parallel Arrays 262
  11.3.1 Crossbar 262
  11.3.2 Massively Parallel 264
  11.3.3 Star 267
11.4 Three Multidimensional Arrays 268
  11.4.1 Hypercube 269
  11.4.2 Massively Parallel 270
  11.4.3 Hybrids 270
11.5 Conclusions 272

12 Algorithm and Data Mappings 273
12.0 Introduction 273
12.1 Five Performance Measures 273
  12.1.1 Input Data Overhead 274
  12.1.2 Intermediate Results Reorganization Overhead 274
  12.1.3 Output Data Overhead 274
  12.1.4 Computational Throughput 274
  12.1.5 Processing Latency 274
12.2 Mappings 274
12.3 Single Processor 275
  12.3.1 Data I/O Requirements 276
  12.3.2 Memory Requirements 276
  12.3.3 Arithmetic Unit Requirements 277
  12.3.4 Von Neumann Architecture 277
  12.3.5 Harvard Architecture 278
  12.3.6 Harvard 16-Point Radix-4 FFT Example 279
12.4 Three Linear Arrays 279
  12.4.1 Pipeline 279
  12.4.2 Linear Bus 283
  12.4.3 Ring Bus 283
  12.4.4 Pipeline 16-Point Radix-4 Example 284
  12.4.5 Linear and Ring Bus 16-Point Radix-4 FFT Examples 286
12.5 Three Parallel Arrays 287
  12.5.1 Crossbar 16-Point Radix-4 FFT Examples 288
  12.5.2 Massively Parallel 16-Point Radix-4 FFT Examples 293
  12.5.3 Star 16-Point Radix-4 FFT Examples 300
12.6 Three Multidimensional Arrays 304
12.6.1 Hypercube 16-Point Radix-4 FFT Examples 305
12.6.2 Massively Parallel 16-Point Radix-4 FFT Examples 312
12.6.3 Hybrid 16-Point Radix-4 FFT Examples 313
12.7 Algorithm Mapping Examples
   Comparison Matrix 313
12.8 Conclusions 313

13 Arithmetic Formats 315
13.0 Introduction 315
13.1 Three Performance Measures 315
   13.1.1 Dynamic Range 316
   13.1.2 Arithmetic Accuracy 316
   13.1.3 Quantization Noise Escalation 316
13.2 Three Arithmetic Formats 316
   13.2.1 Fixed-Point 317
   13.2.2 Floating-Point 318
   13.2.3 Block-Floating-Point 320
13.3 Arithmetic Format Comparison Matrix 321
13.4 Conclusions 322

14 Chips 323
14.0 Introduction 323
14.1 Five FFT Performance Measures 324
   14.1.1 1024-Point Complex FFT 324
   14.1.2 Data I/O Ports 324
   14.1.3 On-Chip Data Memory Words 325
   14.1.4 On-Chip Program Memory Words 325
   14.1.5 Number of Address Generators 325
14.2 Generic Programmable DSP Chip 325
   14.2.1 Block Diagram 326
   14.2.2 On-Chip Data Memory 326
   14.2.3 On-Chip Program Memory 327
   14.2.4 On-Chip Data Buses 327
   14.2.5 Off-Chip Data Bus 327
   14.2.6 On-Chip Address Buses 328
   14.2.7 Off-Chip Address Bus 328
   14.2.8 Address Generators 328
   14.2.9 Serial I/O Ports 329
   14.2.10 Program Control 332
14.2.11 Multiplier-Accumulator and Arithmetic Logic Unit 332
14.2.12 Estimating FFT Performance 334

14.3 Programmable Fixed-Point Chip Families 335
14.3.1 Analog Devices ADSP-21xx Family 336
14.3.2 AT&T DSP16 Family 338
14.3.3 AT&T DSP161x Family 339
14.3.4 Motorola DSP56001 Family 341
14.3.5 Motorola DSP561xx Family 343
14.3.6 NEC μPD77xxx Family 344
14.3.7 NEC μPD7701x Family 346
14.3.8 NEC μPD77220 Family 347
14.3.9 Texas Instruments TMS320C1x Family 348
14.3.10 Texas Instruments TMS320C2x Family 350
14.3.11 Texas Instruments TMS320C5x Family 351
14.3.12 Zilog Z89Cxx Family 353
14.3.13 Zoran ZR38000 Family 354

14.4 Programmable Fixed-Point Chips
   Comparison Matrix 355

14.5 Programmable Floating-Point Chips 357
14.5.1 Analog Devices 21020 Family 357
14.5.2 Analog Devices ADSP-21060 Family 358
14.5.3 AT&T DSP32C Family 359
14.5.4 Intel i860 Family 361
14.5.5 Motorola DSP96002 Family 363
14.5.6 NEC μPD77240/230A Family 364
14.5.7 Texas Instruments TMS320C3x Family 365
14.5.8 Texas Instruments TMS320C40 Family 367

14.6 Programmable Floating-Point Chips
   Comparison Matrix 369

14.7 FFT-Specific Chips and Chip Sets 369
14.7.1 Array Microsystems a66110/66210 Chip Set 370
14.7.2 Sharp LH9124/LH9320 Chip Set 372
14.7.3 Raytheon TMC2310 Chip 373
14.7.4 Plessey Semiconductor PDSP16510 Chip 374

14.8 FFT-Specific Chip and Chip Set
   Comparison Matrix 375

14.9 Application-Specific Integrated Circuits 376
14.9.1 DSP Semiconductor Pine/Oak Core Family 376

14.10 ASIC Programmable DSP Chip
   Cores Comparison Matrix 377
16.6 Test Signal Error Patterns 406
  16.6.1 Unit Pulse 407
  16.6.2 Constants 408
  16.6.3 Single Sine Waves 408
  16.6.4 Pair of Sine Waves 409
16.7 Isolating Errors: A 16-Point Example 409
  16.7.1 Assumptions 409
  16.7.2 Test Signal Strategy 410
  16.7.3 Error Isolation 410
16.8 Conclusions 412

17 Design Examples 413
17.0 Introduction 413
17.1 Example 1: Doppler Radar Processor 414
  17.1.1 Definition of the Product 414
  17.1.2 Specification 414
  17.1.3 Description 415
  17.1.4 Design Decisions 416
  17.1.5 Board Selection Process 422
  17.1.6 Test Signals 423
  17.1.7 Design Decisions Summary 423
17.2 Example 2: Power Spectrum Estimator 424
  17.2.1 Definition of the Product 424
  17.2.2 Specification 424
  17.2.3 Description 425
  17.2.4 Design Decisions 427
  17.2.5 Board Selection Process 430
  17.2.6 Test Signals 430
  17.2.7 Design Decision Summary 431
17.3 Example 3: Speech Analyzer 431
  17.3.1 Definition of the Product 432
  17.3.2 Specification 432
  17.3.3 Description 432
  17.3.4 Design Decisions 435
  17.3.5 Board Selection Process 438
  17.3.6 Test Signals 439
  17.3.7 Design Decision Summary 439
17.4 Example 4: Image Deblurring 440
  17.4.1 Definition of the Product 440
  17.4.2 Specification 441
  17.4.3 Description 441
  17.4.4 Design Decisions 443
CONTENTS

17.4.5 Board Selection Process 447
17.4.6 Test Signals 447
17.4.7 Design Decision Summary 447
17.5 Conclusions 448

Glossary 449

Appendix: Table of Comparison Matrices 455

Index 457