LECTURE 2

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1 Code Converter

SYNTHESIS EXAMPLE

binary natural code

truth table

binary code a	b	c	d	Gray code A	B	C	D
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	0	1	1	1	1
1	0	1	1	1	1	1	0
1	1	0	0	1	0	1	0
1	1	0	1	1	0	1	1
1	1	1	0	1	0	0	1
1	1	1	1	1	0	0	0

We are designing code converter circuit from binary natural code to Gray code.

This is an example of reversible circuit. (permutation of input/output vectors)

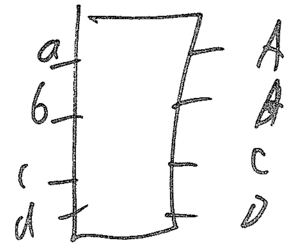
2

ab \ cd	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

A

ab \ cd	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	1	1	1	1

B



ab \ cd	00	01	11	10
00	0	0	1	1
01	1	1	0	0
11	1	1	0	0
10	0	0	1	1

C

ab \ cd	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	0	1	0	1
10	0	1	0	1

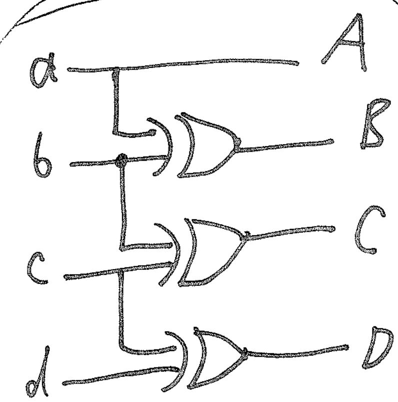
D

$D = c \oplus d$

$C = c \oplus b$

$B = a \oplus b$

$A = a$



binary to Gray converter

to be used in K-map drawing for project

also good example of what is reversible circuit in practice

(2) Project 1 for Matlab group

Given is a single-output function in array format

Draw Kmap of this function

a	b	c	X
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$\longrightarrow$

ab \ c	0	1
00	1	1
01	0	1
11	0	1
10	1	0

essential prime

essential prime

essential vertex

End of project

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	1	1	0	1
10	0	0	1	0

ac

DSOP

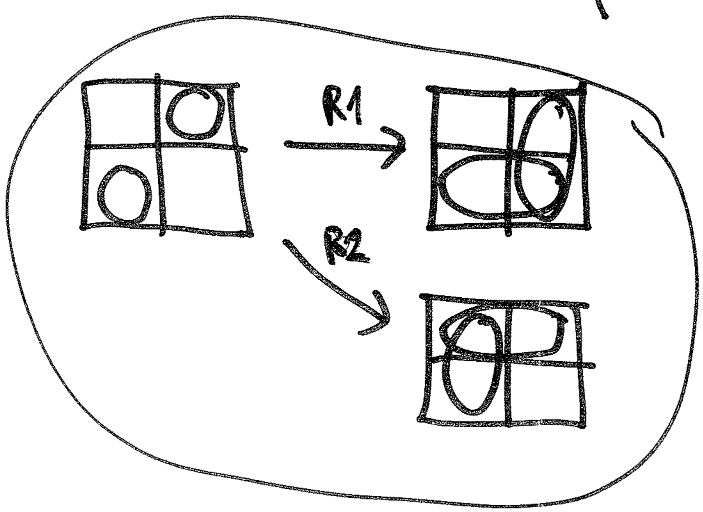
$a\bar{b}cd$ $\quad$ $abc\bar{d}$

1. Find arbitrary DSOP
2. Apply rewriting transformations on cubes to minimize the cost = total # of literals in products

RULE-BASED ESOP MINIMIZER

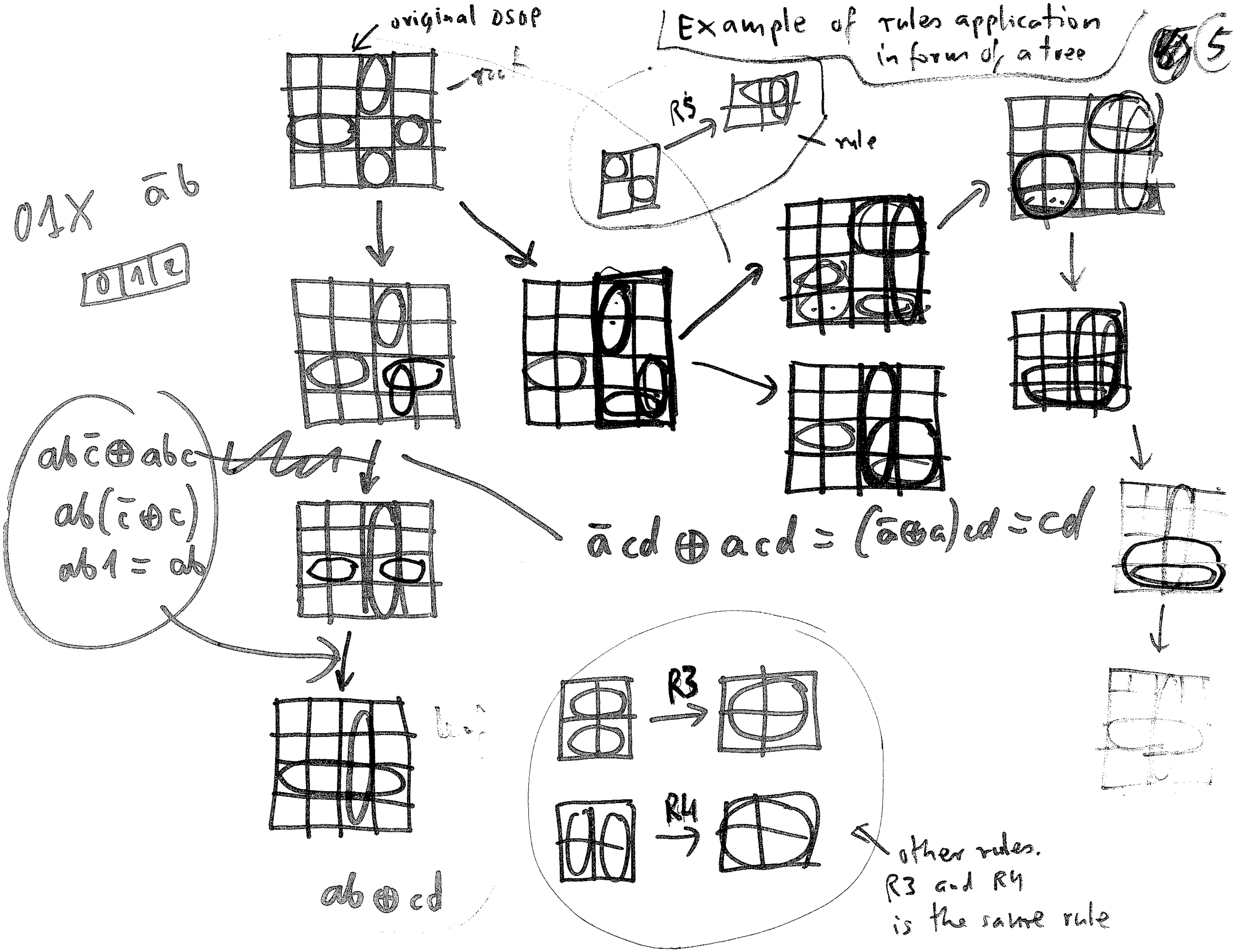
most ESOP minimizers are like this

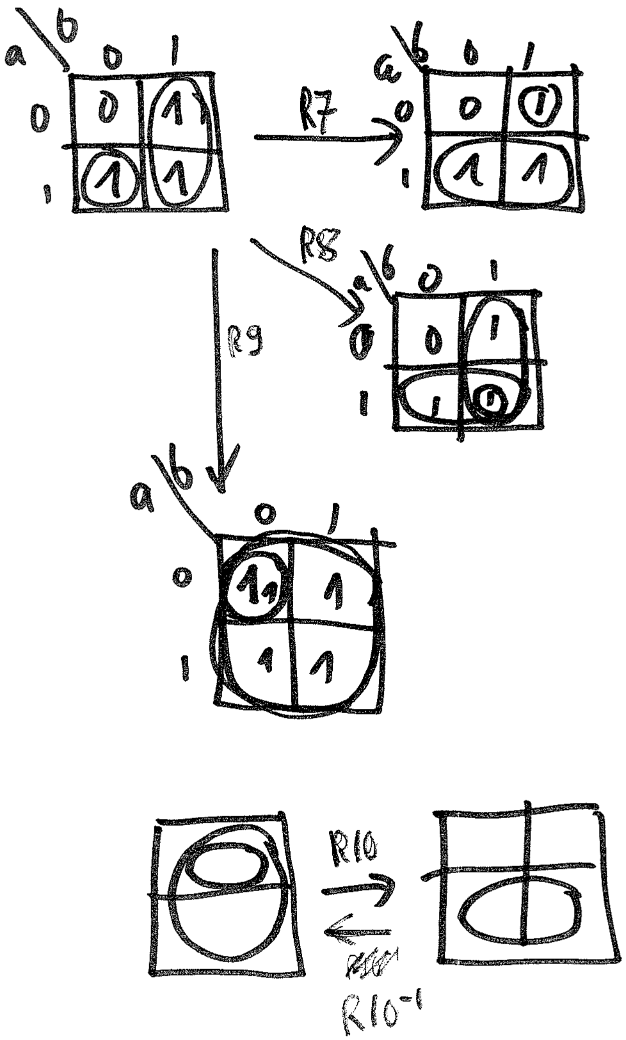
$$a\bar{b}cd + abc\bar{d} = a\bar{b}cd \oplus abc\bar{d}$$
$$= ac(\bar{b}d \oplus b\bar{d}) = ac(b \oplus d)$$



Transformation rules drawn in symbolic way to separate general patterns from Exor algebra formulas

Example of rules application in form of a tree





$$
\begin{aligned}
a + b &= a \oplus b \oplus ab \\
&= a \oplus b(1 \oplus a) \\
&= a \oplus b\bar{a}
\end{aligned}
$$

important formulas used in ESOP rewriting rules

more examples of rules

Sequential (Branching) ESOP minimizer based on group removal

		1	
		1	
1	1		1
		1	

selected group g

$\oplus$

		1	
		1	
		1	
		1	

g

=

		0	
		0	
1	1	1	1
		0	

result of exoring group g

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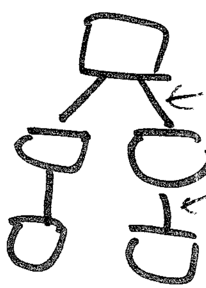
$$f \oplus 0 = f$$

$$f \oplus 1 = \bar{f}$$

$\boxed{\text{inside group g you have to negate values of f}}$

SEQUENTIAL BRANCHING MINIMIZER FOR ESOP BASED ON EXORING Products of Literals



we create tree when we exor various groups

(8)

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	1	1	0	1
10	0	0	1	0

g

f

3 ones, 1 zero so I select group cd

select cd and exor it : all ones changed to zeros and zeros to ones inside the exored group

$f \oplus g$

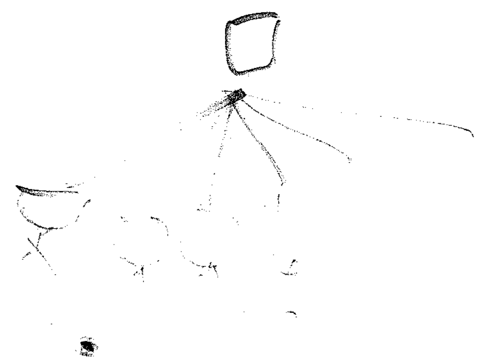
0	0	0	0
0	0	0	0
1	1	1	1
0	0	0	0

h

$f = ab \oplus cd$

select ab and exor it

We get the Kmap of all zeros so this branch of tree is terminated with solution $f = g \oplus h = cd \oplus ab$



generally we need a lot of branching

Exorlog style heuristically selected cubes

1. Select a cube that has as many ones and as few zeros covered by it. # of 1's > # 0's
2. Exor it from function
3. Repeat until function is 0
4. Do for all (or many) branches.
5. Select the best solution (cost = literals or cost = terms)

QUANTUM REVERSIBLE

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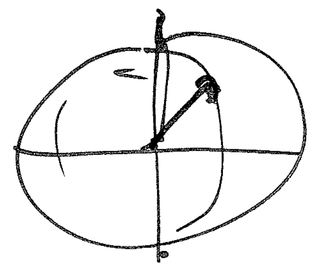
Reversible and quantum logic

$0, 1 \rightarrow |0\rangle, |1\rangle$

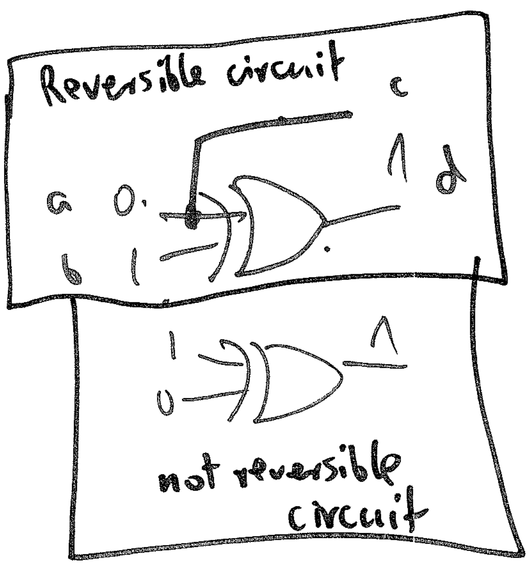


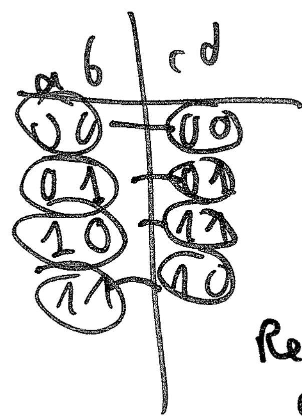
all quantum values
are on the sphere

$\text{qubit} = \alpha|0\rangle + \beta|1\rangle$

α, β = complex numbers
$|0\rangle, |1\rangle$ basic values

qubit is a superposition





Reversible
circuit
is a one-to-one
mapping
in binary values

linear and affine functions

0

a

b

$a \oplus b$

all linear functions of 2 variables

$1 \oplus 0 = 1$

$1 \oplus a = \overline{a}$

$1 \oplus b = \overline{b}$

$1 \oplus a \oplus b = \overline{a \oplus b}$

all affine functions of 2 variables

Question

How many affine functions of 3 variables exist?

$2^3 \times 2$ — affine functions

$ab \backslash c$	0	1
00	1	1
01	0	0
11	1	1
10	0	0

f

Is this an affine function?

$\bar{a}\bar{b} + ab = a \odot b = \overline{a \oplus b}$

yes it is

"Nearly affine" functions and decomposition with remainder

1		1	
	0	1	1
1	1	0	
	1		1

=

1		1	
	1		1
1		1	
	1		1

$\oplus$

	1	1	
	1	1	

f "nearly" affine function

affine function

remainder function.

Observation

The "more ones than zeros" algorithm should be applied after extracting the linear part from the circuit.

NEARLY AFFINE FUNCTIONS AND EXOR DECOMPOSITION WITH REMAINDER

Unate functions

ab \ cd	00	01	11	10
00			1	
01			1	
11		1	1	1
10		1	1	

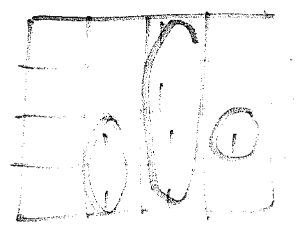
We already know reversible and affine functions.
Now new classification

product of not negated variables

abc

cd

ad



$f = cd + a\bar{c}d + abc\bar{d}$

$f = abc + cd + ad$

Def
the function that can be represented as sum of positive product terms is called positive unate function

"positive polarity"

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	1	1	1
11	1	1	1	1
10	0	1	1	1

bd cd bc ac ab ad

g

is function g positive unate?

Yes – find all primes and write the expression

This has all positive polarities of variables

Polarity of a boolean function with n variables is a binary vector of length n

0 → negative polarity

1 → positive polarity

Ab

a, b

$\bar{a}\bar{b} = 00$ negative polarity

$\bar{a}b = 01$

$a\bar{b} = 10$

$ab = 11$ positive polarity

Def

Unate function with all its variables of positive polarity is called positive unate function.

$$\boxed{2^n}$$

Binary vector $[a_1\ a_2\ a_3 \dots a_n]$ is stronger than binary vector $[b_1\ b_2 \dots b_n]$ if for every i either $a_i = b_i$ or $a_i > b_i$ $\boxed{1 > 0}$

$$11101 > 11001$$

$$10 \ngtr 01$$
$$\nless$$

We can also write

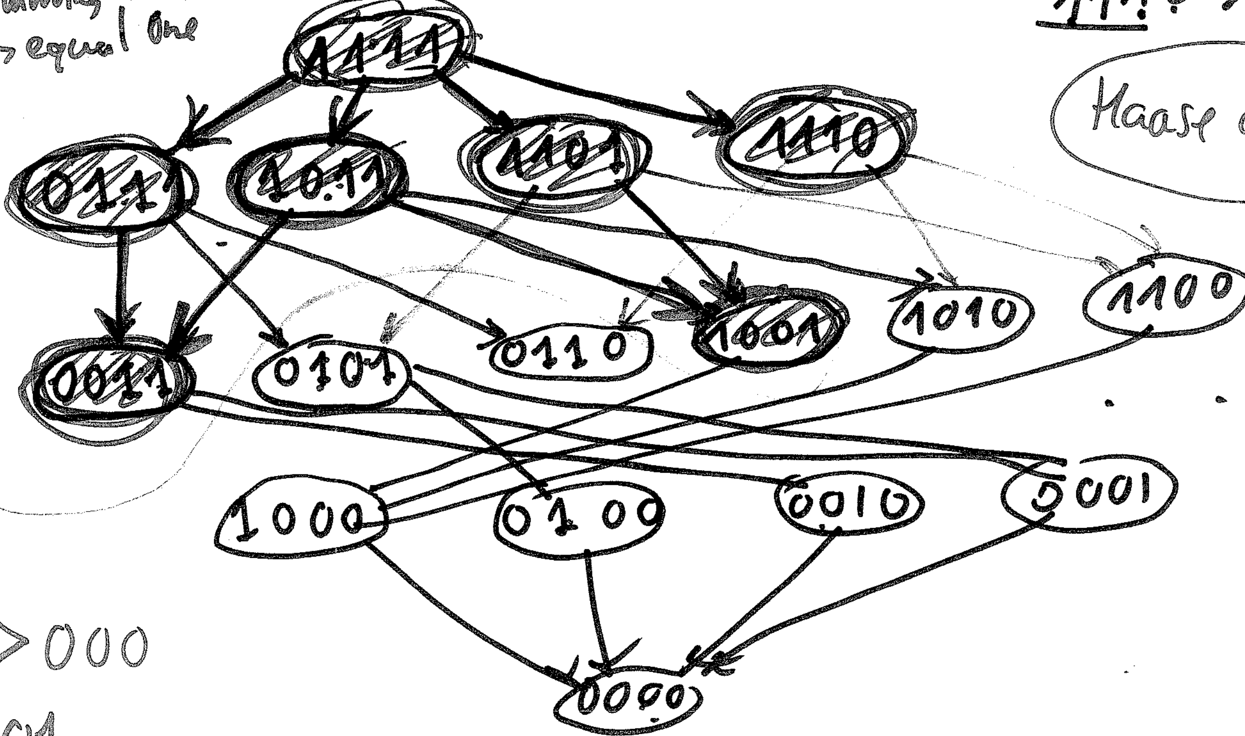
$$a_i \geq b_i \ \forall i$$

Kmap → truth table → hypercube → ? lattice

$\underline{111}0 > \underline{11}00$

Haase diagram

Hamming distance is equal one



$111 > 000$

$10 \not> 01$

If on every path up from 0000 the values of function are monotonously increasing then function is positive unate.

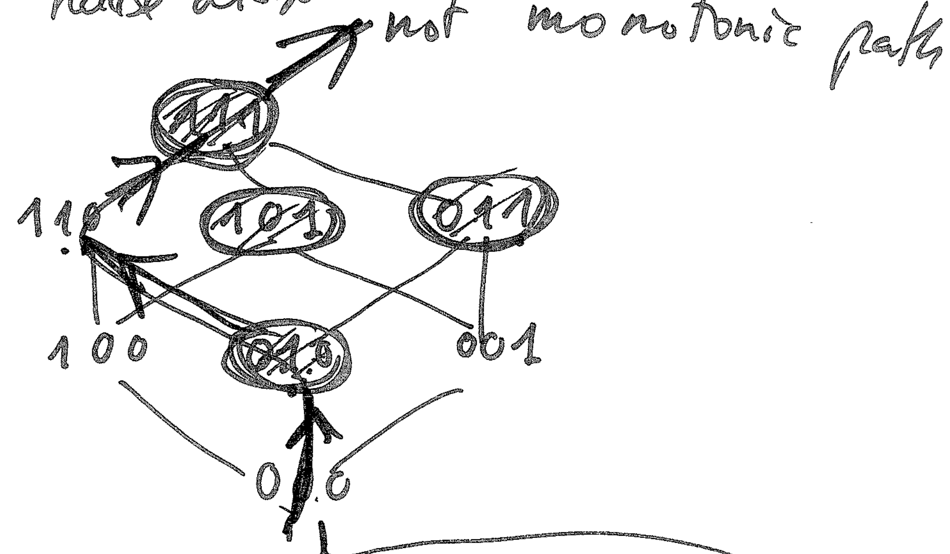
Here we use Haase diagram to verify and illustrate positive unate fun

ab \ c	0	1
00	0	0
01	1	1
11	0	1
10	0	1

Problem 2

Is this function positive unate?

Hasse diagr



answer.
It is not because there exists a non-monotonic path

Problem 3

(20)

ab \ cd	00	01	11	10
00	1	1	1	1
01	1	1		1
11	1			1
10	1			1

$$f = \overline{\bar{a} b} + \overline{\bar{a} c} + \bar{d}$$

I represent f as a sum of products where all variables are negated

Problem 4

ab \ cd	00	01	11	10
00			1	
01		1	1	1
11			1	
10				

g

$\bar{a}cd$

$\bar{a}bc$

$\bar{a}.bd$

bcd

is function g unate?
if yes, of which polarity

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Polarity
$$\bar{a}bcd = 0111$$

Polarity is "center of my flower"