Soft Computing and Fuzzy Theory

Outline

- ◆be introduced to the topics of:
  - fuzzy sets,
  - fuzzy operators,
  - fuzzy logic
  - and come to terms with the technology
- ◆learn how to represent concepts using fuzzy logic
- ◆understand how fuzzy logic is used to make deductions
- ◆familiarise yourself with the `fuzzy' terminology

What is fuzziness

- The concept of fuzzy logic was introduced in a 1965 paper by Lotfi Zadeh.
- Professor Zadeh was motivated by his realization of the fact that people base their decisions on imprecise, **non-numerical information.**
- Fuzzification should not be regarded as a single theory but as a methodology
  - It generalizes any specific theory from **a discrete** to a **continuous** form.
- For instance:
  - from Boolean logic to fuzzy logic,
  - from calculus to fuzzy calculus,
  - from differential equations to fuzzy differential equations,
  - and so on.

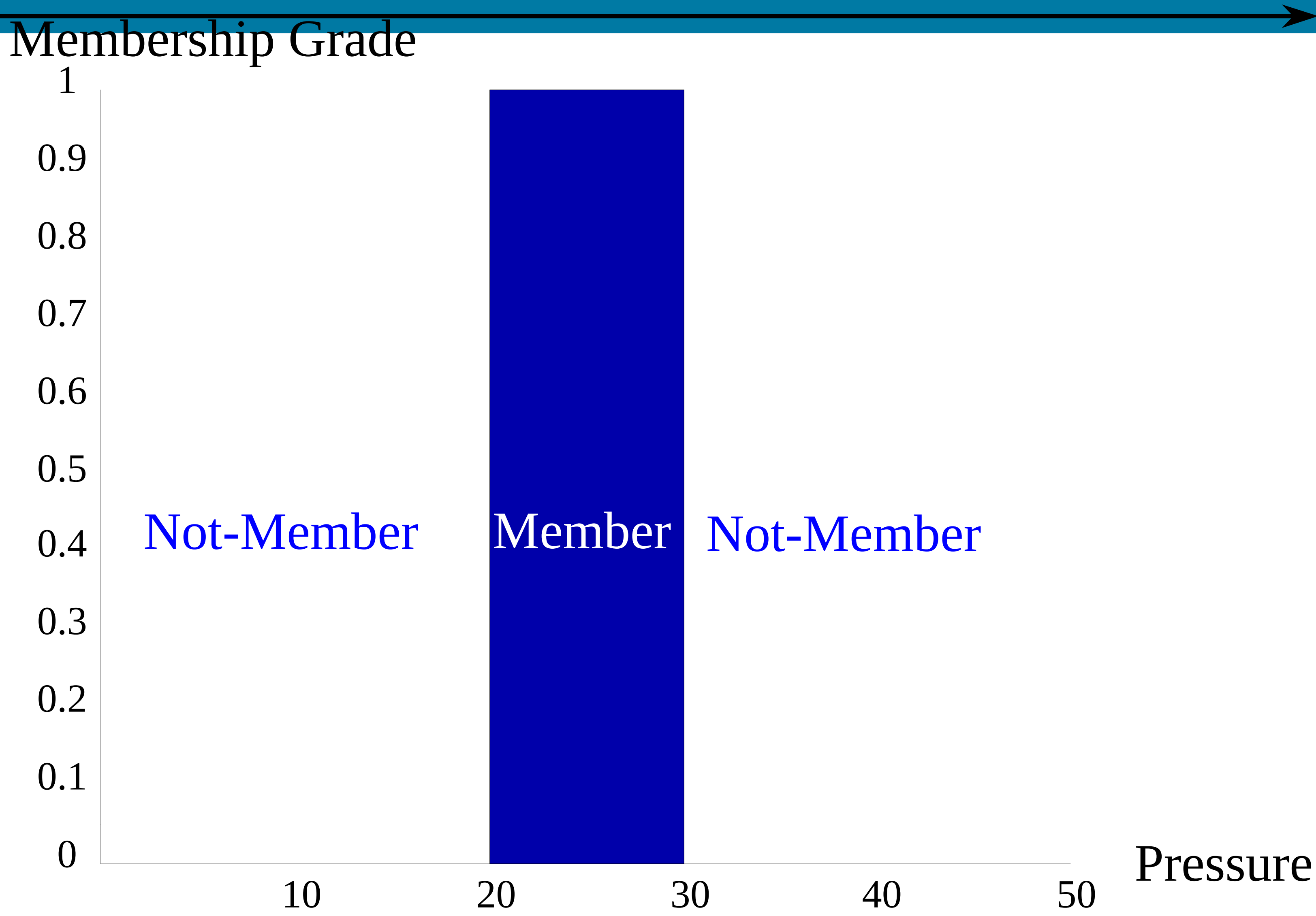
What is fuzziness

- Fuzzy logic is then a **superset** of conventional Boolean logic.
- In ***Boolean logic*** propositions take a value of either completely true or completely false
- **Fuzzy logic** handles the concept of partial truth, i.e., values between the two extremes.

What is fuzziness

- For example, if pressure takes values between 0 and 50 *(**the universe of discourse**)* one might label the range 20 to 30 as medium pressure *(the subset)*.
- Medium is known as a ***linguistic*** variable.
- Therefore, with Boolean logic 15.0 (or even 19.99) is not a member of the medium pressure range.
- As soon as the pressure equals 20, then it becomes a member.

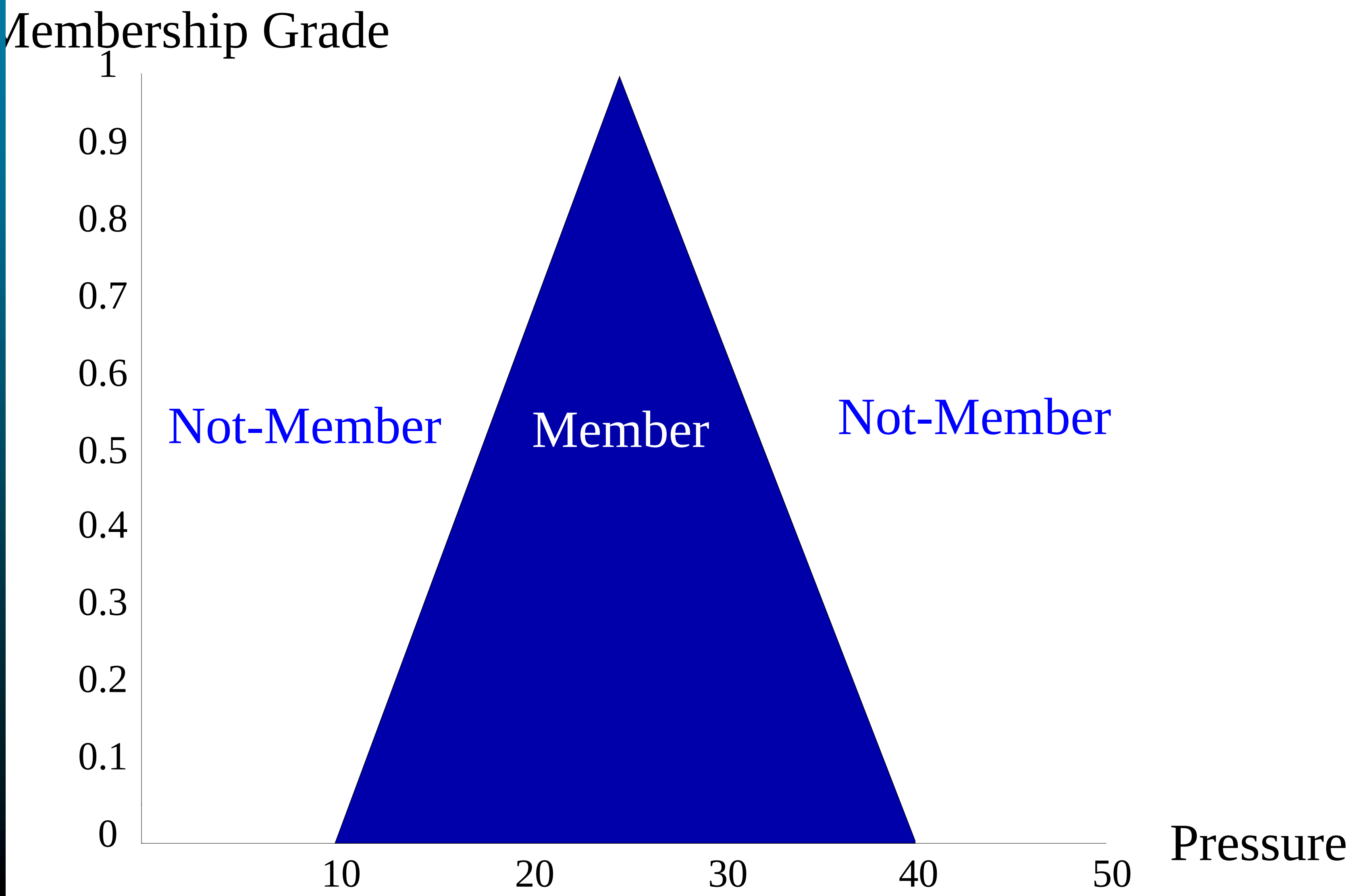
Boolean Medium Pressure



What is fuzziness

- Contrast with the Figure of the next page which shows the membership function using fuzzy logic.
- Here, a value of 15 is a member of the medium pressure range with a membership grade of about 0.3.
- Measurements of 20, 25, 30, 40 have grade of memberships of 0.5, 1.0, 0.8, and 0.0 respectively.
- Therefore, a membership grade progresses from non-membership to full membership and again to non-membership.

Fuzzy Medium Pressure



Fuzziness is not *Vague*

- we shall have a look at some propositions.

- **Dimitris is six feet tall**
  - The first proposition (traditional) has a crisp truth value of either TRUE or FALSE.

- **He is tall**
  - The second proposition is vague.
  - It does not provide sufficient information for us to make a decision, either fuzzy or crisp.
  - We do not know the value of the pronoun.
  - Is it Dimitris, John or someone else?

Fuzziness is not *Vague*

- **Dimitris is tall**
  - The third proposition is a fuzzy proposition.
  - It is true to some degree depending in the context, i.e., the universe of discourse.
  - It might be ***SomeWhat True*** if we are referring to basketball players or it might be ***Very True*** if we are referring to horse-jockeys.

Fuzziness is not ***Multi-valued logic***

- The limitations of two-valued logic were recognised very early.
- A number of different logic theories based on multiple values of truth have been formulated through the years.
- For example, in three-valued logic three truth values have been employed.
- These are TRUTH, FALSE, and UNKNOWN represented by 1, 0 and 0.5 respectively.

- In 1921 the first N-valued logic was introduced.
- The set of truth values T_n were assumed to be evenly divided over the closed interval [0,1].
- Fuzzy logic may be considered as an extension of multi-valued logic but they are somewhat different.
- Multi-valued logic is still based on exact reasoning whereas fuzzy logic is approximate reasoning.

Fuzziness is not ***Probability***

- This is better explained using an example.
- Let X be the set of all liquids (i.e., the universe of discourse) .
- Let L be a subset of X which includes all suitable for drinking liquids.
- Suppose now that you find two bottles, A and B.
- The labels do not provide any clues about the contents.
- Bottle A label is marked as membership of L is 0.9.
- The label of bottle B is marked as probability of L is 0.9.
- Given that you have to drink from the one you choose, the problem is of how to interpret the labels.

Fuzziness is not ***Probability***

- Well, membership of 0.9 means that the contents of A are ***fairly similar*** to perfectly potable liquids.
- If, for example, a perfectly liquid is pure water then bottle A might contain, say, tonic water.
- Probability of 0.9 means something completely different.
- You have a 90% chance that the contents are potable and 10% chance that the contents will be unsavoury, some kind of acid maybe.
- Hence, with bottle A you might drink something that is not pure but with bottle B you might drink something deadly. So choose bottle A.

Fuzziness is not *Probability*

- Opening both bottles you observe beer (bottle A) and hydrochloric acid (bottle B).
- The outcome of this observation is that the membership stays the same whereas the probability drops to zero.
- All in all:
  - probability measures the likelihood that a future event will occur,
  - fuzzy logic measures the **ambiguity of events that have already occurred.**
- In fact, fuzzy sets and probability exist as parts of a greater Generalized Information Theory.
- This theory also includes:
  - Demster-Shafer evidence theory,
  - possibility theory,
  - and so on.

Where is fuzzy logic used?

- Fuzzy logic is a powerful problem-solving methodology.
  - **It is used directly and indirectly in a number of applications.**
- Fuzzy logic is now being applied all over Japan, Europe and more recently in the United States of America.
- It is true though that all we ever hear about is Japanese fuzzy logic.
- Products such as:
  - **the Panasonic rice cooker,**
  - **Hitachi's vacuum cleaner,**
  - **Minolta's cameras,**
  - **Sony's PalmTop computer,**
  - **and so on.**
- This is not unexpected since Japan adopted the technology first.

Where is fuzzy logic used?

- whereas in the West companies might keep their fuzzy development secret because of the implication of the word `fuzzy',
- or because companies want to preserve competitive advantage,
- or because fuzzy logic is embedded in products without advertisement.
- Most applications of fuzzy logic use it as the concealed logic system for expert systems.

Where is fuzzy logic used?

◆ The areas of potential fuzzy implementation are numerous and not just for control:

- **Speech recognition,**
- **fault analysis,**
- **decision making,**
- **image analysis,**
- **scheduling**
- and many more are areas where fuzzy thinking can help.

◆ Hence, fuzzy logic is not just control but can be utilized for other problems.

Where is fuzzy logic used?

◆One business problem, namely that of **fraud detection**, was recently addressed using fuzzy logic.

◆The system detects probable fraudulent behavior:

- by evaluating all the characteristics of a provider's claim data in parallel,
- against the normal behaviour of a small in demographic terms community.

◆An all-American success story is the use of fuzziness on keeping a **commercial refrigerator** thermally controlled (0.1 C).

- The excitement comes due to the fact that this refrigerator has flown on several space shuttle flights.

Where is fuzzy logic used?

◆Another interesting application has been reported by Aptronix Inc., where f

◆ Fuzzy logic was used as a means of determining correct focus distance for cameras with *automatic focusing system*.

◆ Traditionally, such a camera focuses at the middle of the view finder.

◆ This can be inaccurate though when the object of interest is not at the center.

◆ Using fuzzy logic, three distances are measured from the view finder; left, center and right.

◆ For each measurement a plausibility value is calculated and the measurement with the highest plausibility is deemed as the place where the object of interest is located.

When to use Fuzzy Logic?

- If the system to be modelled is a linear system which can be represented by a mathematical equation or by a series of rules then straightforward techniques should be used.
- Alternatively, if the system is complex, fuzzy logic may be the technique to follow.

When to use Fuzzy Logic?

- We define a **complex system** :
  - when it is nonlinear, time-variant, ill-defined;
  - when variables are continuous;
  - when a mathematical model is either too difficult to encode or does not exist or is too complicated and expensive to be evaluated;
  - when noisy inputs;
  - and when an expert is available who can specify the rules underlying the system behaviour.

Fuzzy sets, logic, inference, control

- This is the appropriate place to **clarify not what the terms mean but their relationship.**
- This is necessary because different authors and researchers use the same term either for the same thing or for different things.
- The following have become widely accepted:
  - **Fuzzy logic system**
    - anything that uses fuzzy set theory
  - **Fuzzy control**
    - any control system that employs fuzzy logic
  - **Fuzzy associative memory**
    - any system that evaluates a set of fuzzy *if-then* rules uses fuzzy inference. Also known as **fuzzy rule base** or **fuzzy expert system**
  - Fuzzy inference control
    - a system that uses fuzzy control and fuzzy inference

Traditional sets

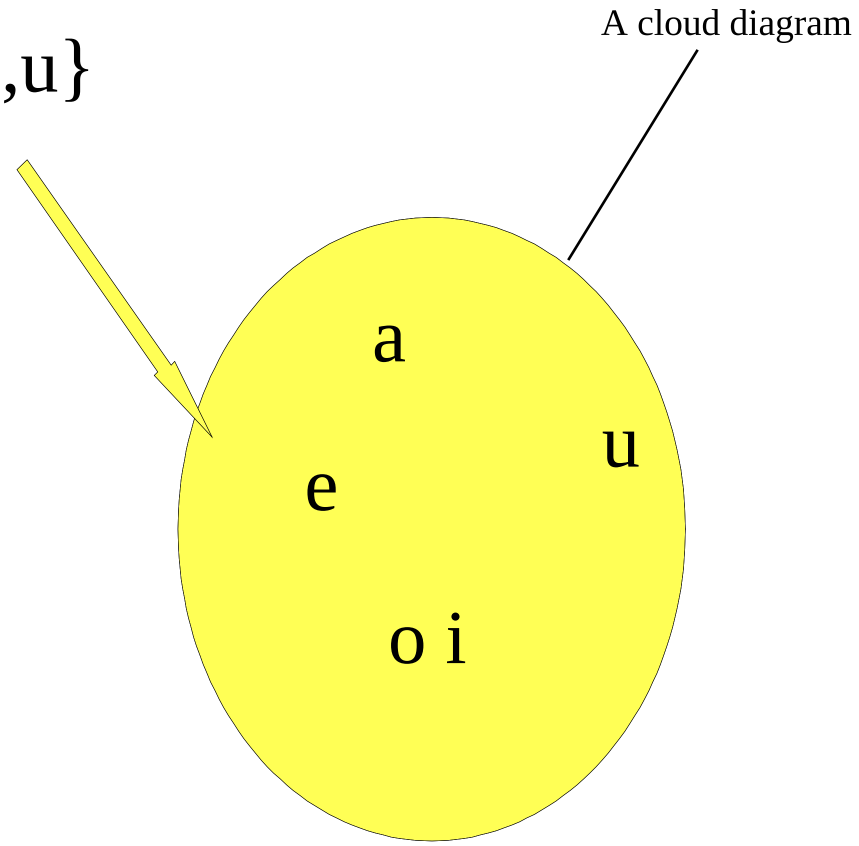
- In order to represent a set we use curly brackets { }.
- Within the curly brackets we enclose the names of the items, separating them from each other by commas.
- The items within the curly brackets are referred to as the elements of the set.
  - **Example:** Set of vowels in the English alphabet = {a,e,i,o,u}
- When dealing with numerical elements we may replace any number of elements using 3 dots.
  - **Example:** Set of numbers from 1 to 100 = {1,2,3,...,100}
  - Set of numbers from 23 to infinity = {23,24,25,...}

Traditional sets

- Rather than writing the description of a set all the time we can give names to the set.
- The general convention is to give sets names in capital letters.
  - **Example:**
    - **V = set of vowels in the English alphabet.**
    - **Hence any time we encounter V implies the set {a, e, i, o, u}.**
  - For finite size sets a diagrammatic representation can be employed which can be used to assist in their understanding.
    - **These are called the cloud diagrams**

Cloud Diagrams

$V = \{a,e,i,o,u\}$



Set order

- The order in which the elements are written down is not important.
  - **Example:** V = {a,e,i,o,u} = {u,o,i,e,a} = {a,o,e,u,i}
- The names of the elements in a set must be unique.
  - **Example:**
    - **V = {a,a,e,i,o,u}**
    - **If two elements are the same then there is no point writing them down twice (waste of effort)**
    - **but if different then we must introduce a way to tell them apart.**

Set membership

- Given any set, we can test if a certain thing is an element of the set or not.
- The Greek symbol, $\in$, indicates an element is a member of a set.
- For example, $x \in A$ means that x is an element of the set A.
- If an element is not a member of a set, the symbol $\notin$ is used, as in $\notin A$.

Set equality & subsets

- Two sets A and B are equal, (A= B) if every element of A is an element of B and every element of B is an element of A.

- A set A is a subset of set B, ($A \subseteq B$) if every element of A is an element of B.

- A set A is a proper subset of set B, ($A \subset B$) if A is a subset of B and the two sets are not equal.

Set equality & subsets

◆Two sets A and B are disjoint, (A ∩ b) if and only if their intersection is the empty set.

◆There are a number of special sets. For instance:

- Boolean B={True, False}
- Natural numbers N={0,1,2,3,...}
- Integer numbers Z={...,-3,-2,-1,0,1,2,3,...}
- Real numbers R
- Characters Char
- Empty set ∅ or {}
- The empty set is not to be confused with {0} which is a set which contains the number zero as its only element.

Set operations

◆ We have a number of possible operators acting on sets.

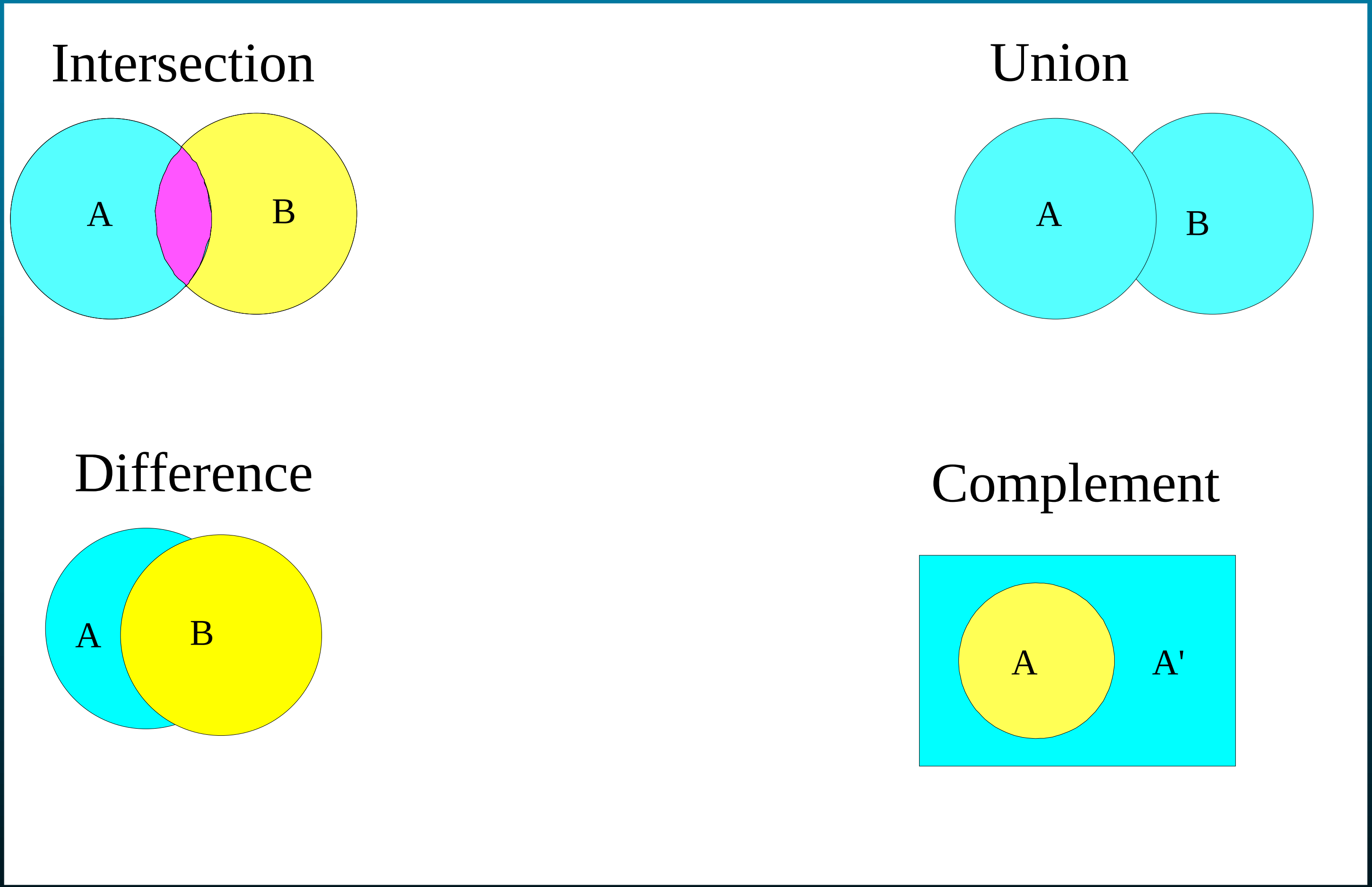
◆ The intersection (∩), the union (∪), the difference (/), the complement (').

- Intersection results in a set with the common elements of two sets.
- Union results in a set which contains the elements of both sets.
- The difference results in a set which contains all the elements of the first set which do not appear in the second set.
- The complement of a set is the set of all element not in that set.

Set operations example

- Using as an example the two following sets A and B the mathematical representation of the operations will be given.
  - A={cat, dog, ferret, monkey, stoat}
  - B={dog, elephant, weasel, monkey}
- C=A ∩ B={x e u | (x e A)∧ (x e B)}={dog, monkey}
- C=A ∪ B={x e u | (x e A)∨ (x e B)}={cat, ferret, stoat, dog, elephant, weasel, monkey}
- C=A / B={x e u | (x e A) ∧ ~(x e B)}={cat, ferret, stoat}
- C=A'={x e u | ~(x e A)}

Set operations example using Venn diagrams



Fuzzy sets

- A traditional set can be considered as a special case of fuzzy sets.
- A fuzzy set has 3 principal properties:
  - **the range of values over which the set is mapped**
  - **the degree of membership axis that measures a domain value's membership in the set**
  - **the surface of the fuzzy set - the points that connect the degree of membership with the underlying domain**

Fuzzy sets

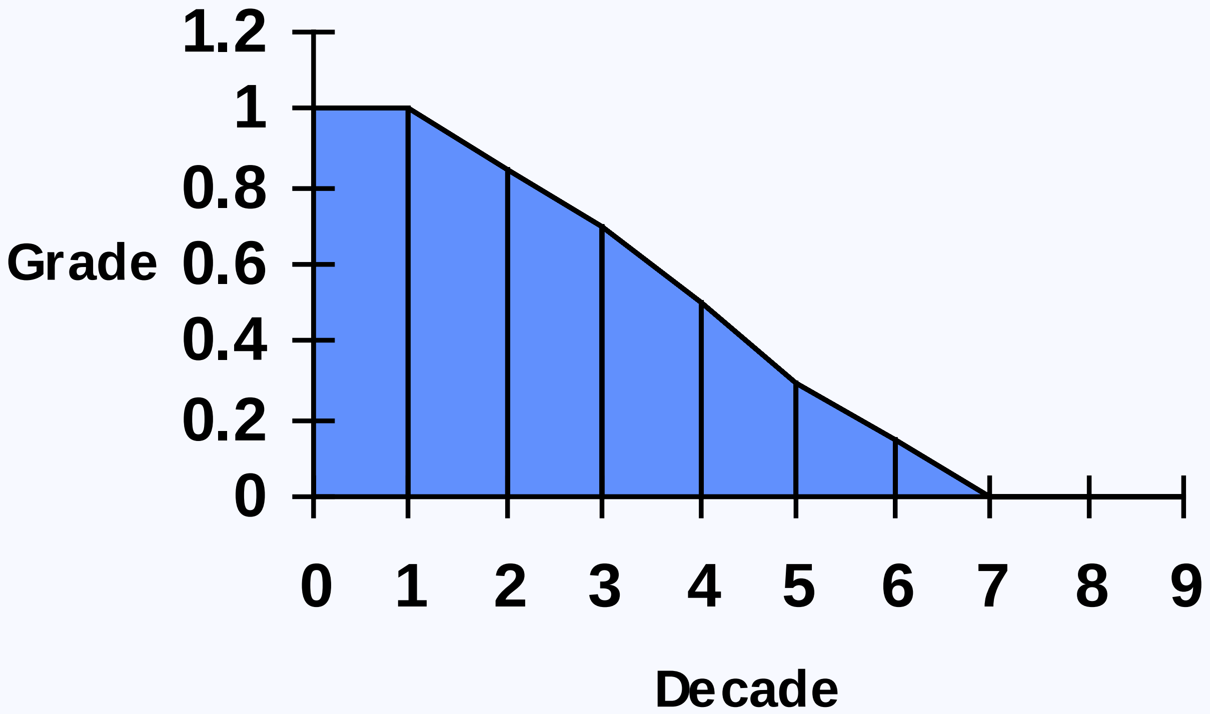
◆Therefore, a fuzzy set in a universe of discourse **U** is characterised by the membership function μ_x, which takes values in the interval **[0,1]** namely $\mu_x: U \rightarrow [0,1]$.

- A fuzzy set **X** in **U** may be represented as a set of ordered pairs of a generic element **u** and its grade of membership μ_x as $X=\{u,\mu_X(u)/u \_ U\}$,
  - **i.e., the fuzzy variables u take on fuzzy values $\mu_x(u)$.**
- When a fuzzy set, say **X**, is discrete and finite it may expressed as

$$X=\mu_x(u_1)/u_1+...+\mu_x(u_n)/u_n$$

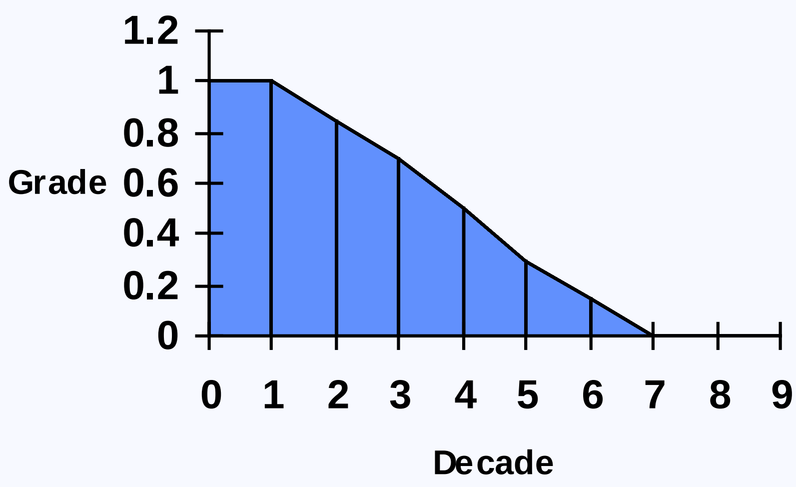
  - **where `+' is not the summation symbol but the union operator, the `/' does not denote division but a particular membership function to a value on the universe of discourse.**

Fuzzy set



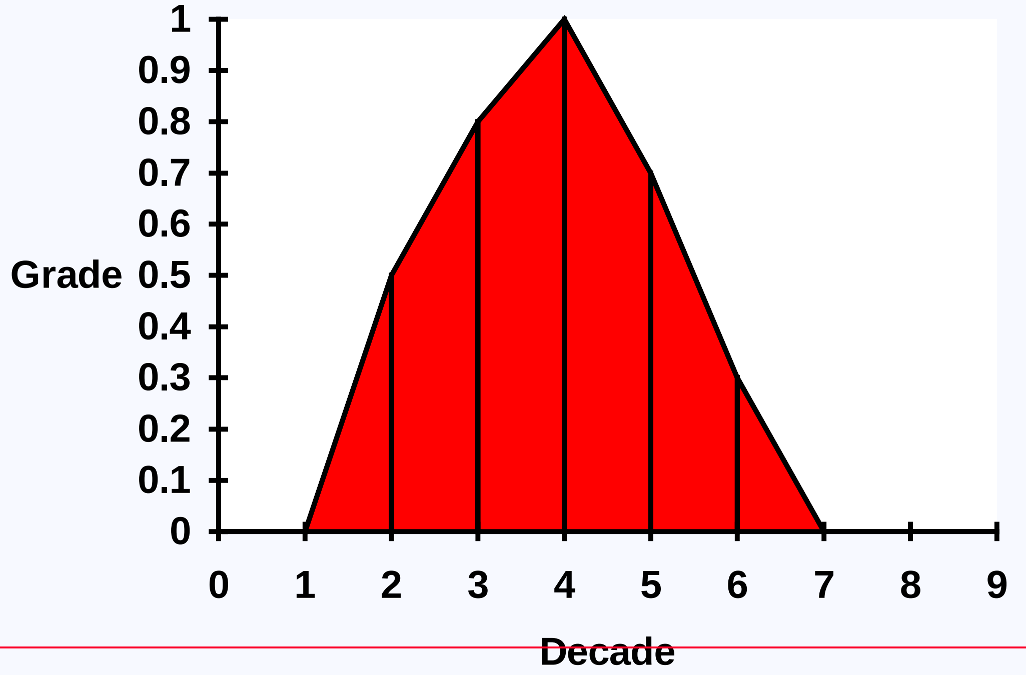
Fuzzy sets

- As an example consider:
  - the universe of discourse U={0,1,2,...,9}
  - and a fuzzy set X_1, `young generation decade'.
- A possible presentation now follows:
  - X_1={1.0/0+1.0/1+0.85/2+0.7/3+0.5/4+0.3/5+0.15/6+0.0/7+0.0/8+0.0/9.
  - The set is also shown in a **graphical form** below.



Fuzzy set

◆Another set, X_2 might be `mid-age generation decade. In discrete form this can be depicted as
X_2={0.0/0+0.0/1+0.5/2+0.8/3+1.0/4+0.7/5+0.3/6+0.0/7 +0.0/8+0.0/9.



Support, Crossover, Singleton

- **Support of a fuzzy set:**
  - The support of a fuzzy set is the set of all elements of the universe of discourse that their grade of memebrship is greater than zero.
  - For X_2 the support is {2,3,4,5,6}.
  - Additionally, a fuzzy set has ***compact support*** if its support is finite.
- **Crossover point:**
  - The element of a fuzzy set that has a grade of membership equal to 0.5 is known as the crossover point.
  - For X_2 the crossover point is 2.

Support, Crossover, Singleton

- **Fuzzy singleton:**
  - The fuzzy set whose support is a single point in the universe of discourse with grade of membership equal to one is known as the fuzzy singleton.
- **-Level sets:**
  - The fuzzy set that contains the elements which have a grade of membership greater than the -level set is known as the -Level set.
  - For X_2 the -Level set when =0.6 is {3,4,5}.
  - Whereas for X_2 the -Level set when =0.4 is {2,3,4,5}.

Fuzzy operators

- What follows is a summary of some fuzzy set operators in a domain X.
- For illustration purposes we shall use the following membership sets:
  - $\mu_A = 0.8/2 + 0.6/3 + 0.2/4$, and $\mu_B = 0.8/3 + 0.2/5$
  - as well as X_1 and X_2 from above.
- **Set equality:**
  - A=B if $\mu_A(x)=\mu_B(x)$ for all $x \in X$
- **Set complement:**
  - A' $\mu_{A'}(x)=1-\mu_A(x)$ for all $x \in X$.
  - This corresponds to the logic `NOT' function.
  - $\mu_{A'}(x) = 0.2/2 + 0.4/3 + 0.8/4$

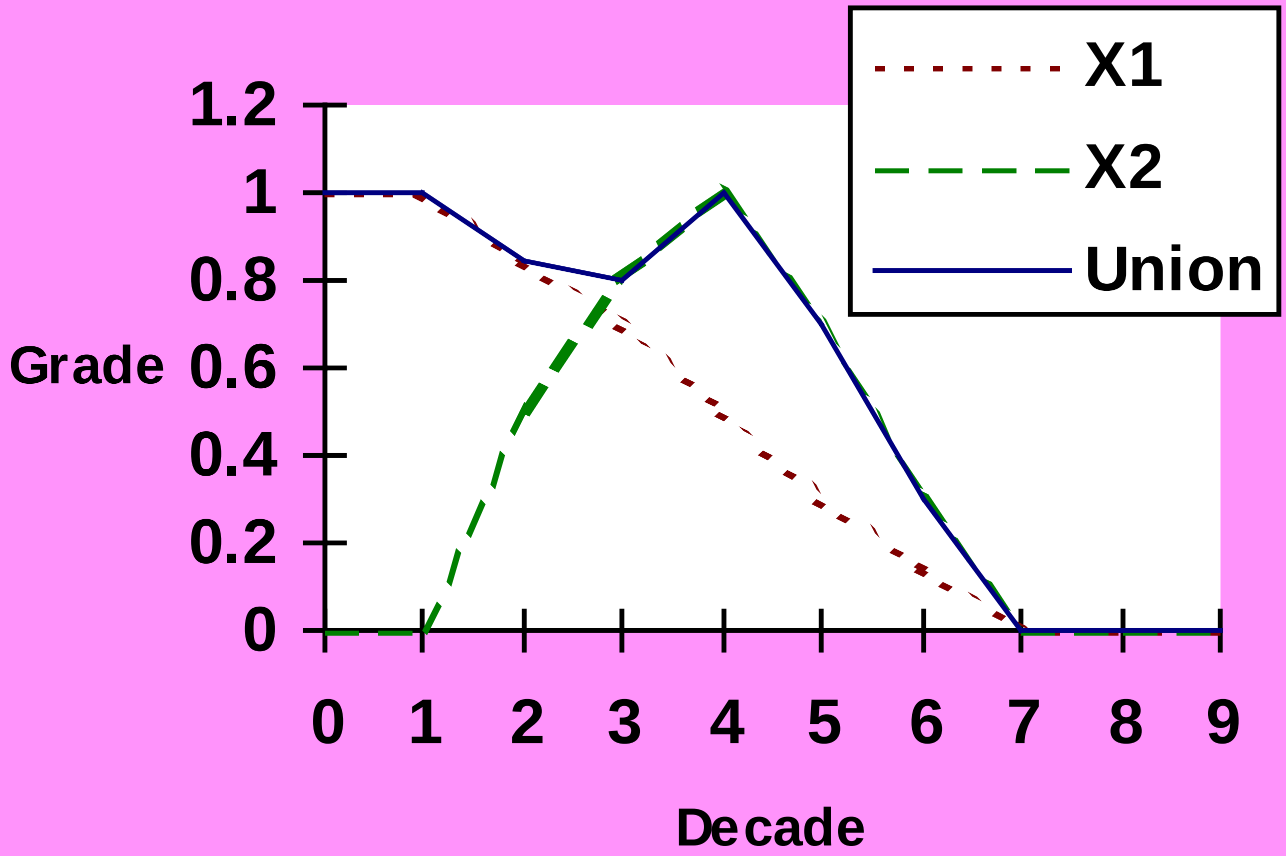
Fuzzy operators

- **Subset:** $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$

- **Proper Subset:**
  - $A \subset B$ if $\mu_A(x) \leq \mu_B(x)$ and $\mu_A(x) \quad \mu_B(x)$ for at least one $x \in X$

- **Set Union:**
  - $A \cup B$ $\mu_{A \cup B}(x) = \vee(\mu_A(x), \mu_B(x))$ for all $x \in X$ where $\vee$ is the *join operator* and means the maximum of the arguments.
    - **This corresponds to the logic `OR' function.**
  - $\mu_{A \cup B}(x) = 0.8/2 + 0.8/3 + 0.2/4 + 0.2/5$

Fuzzy Union Diagram



Fuzzy operators

Set Intersection:

- $A \cap B$ $\quad {}_{A\cap B}(x)=\wedge(\quad {}_{A}(x), \quad {}_{B}(x))$ for all $x \in X$ where $\wedge$ is the *meet operator* and means the minimum of the arguments.
  - **This corresponds to the logic `AND' function.**
- $\quad {}_{A}\cap_{B}(x) = 0.6/3$

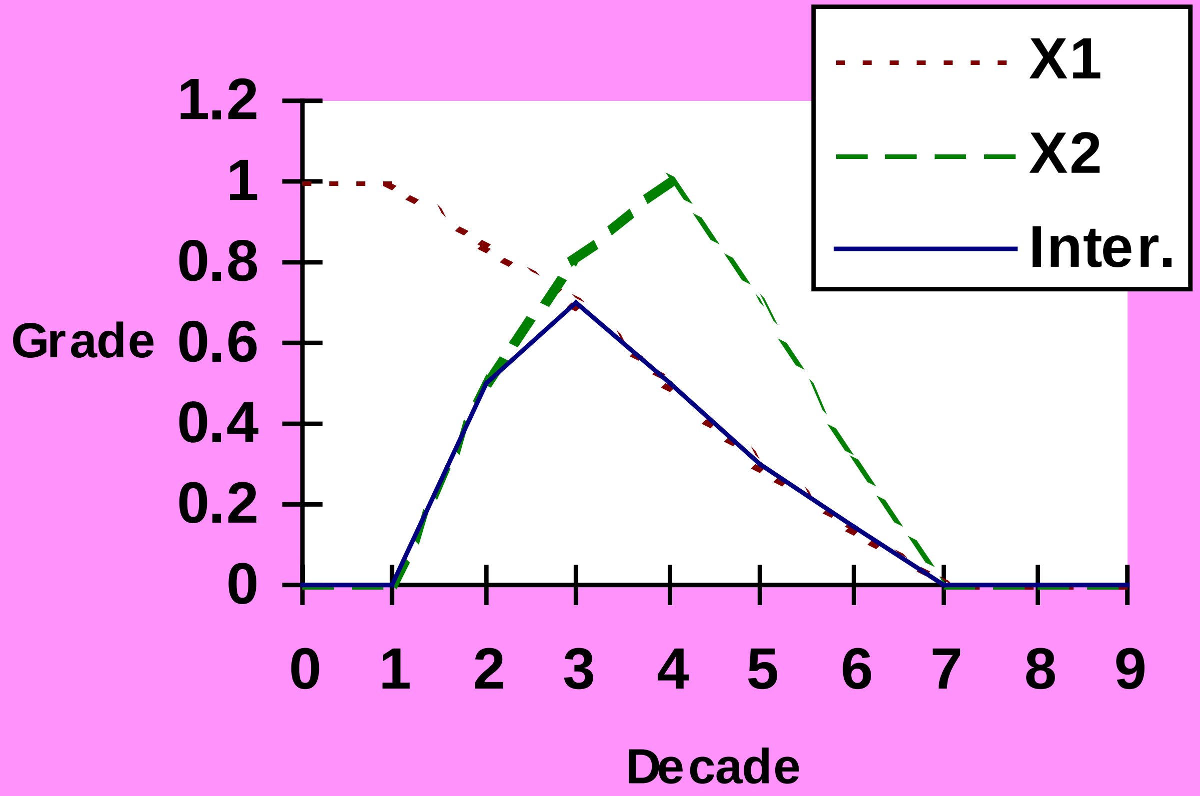
Set product:

- AB $\quad \mu_{AB}(x)=\mu_{A}(x)\mu_{B}(x)$

Power of a set:

- A^{N} $\quad \mu_{A}{}^{N}(x)=(\mu_{A}(x))^{N}$

Fuzzy Intersection diagram



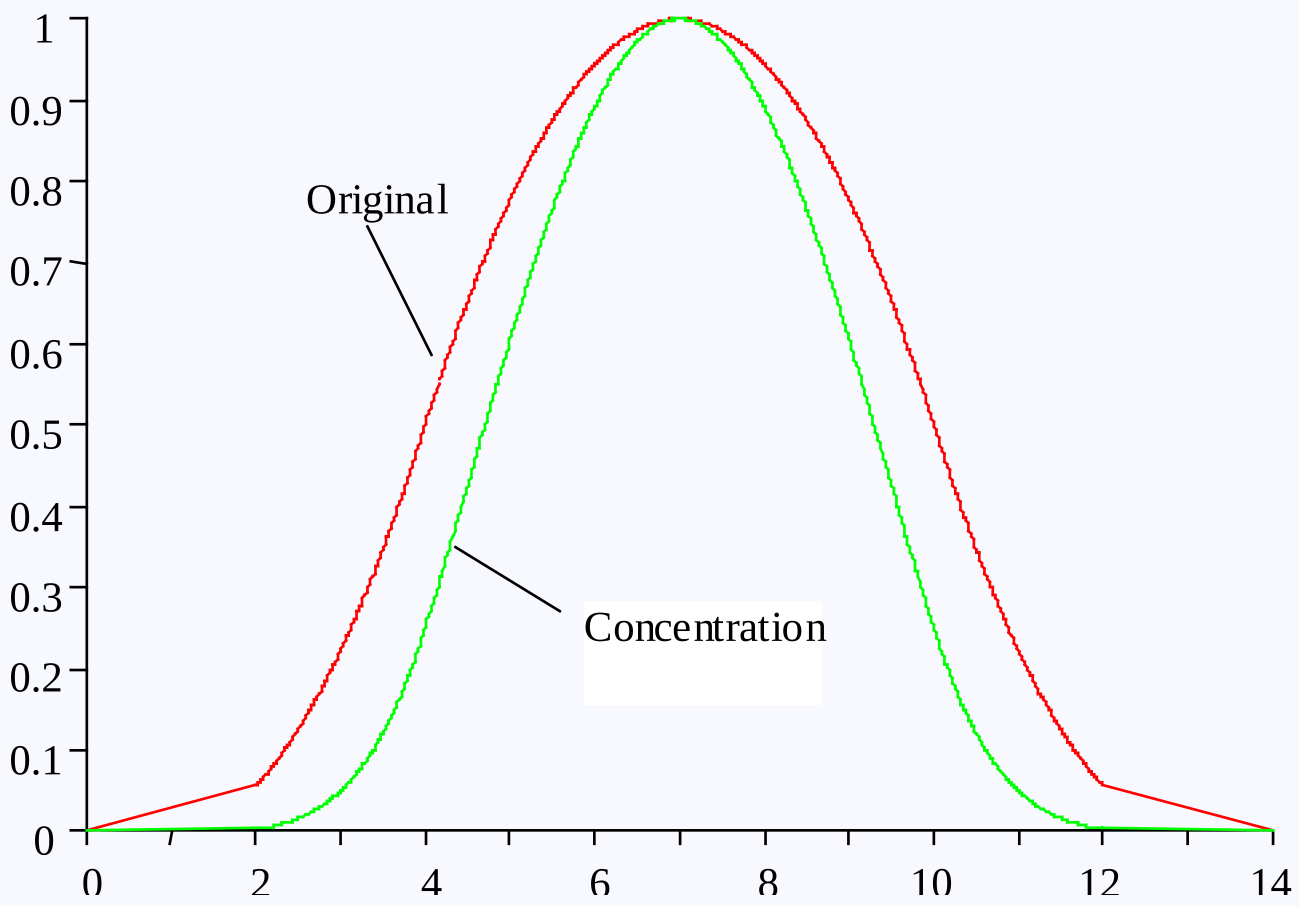
Fuzzy operators

- **Bounded sum or bold union:** $A \oplus B$
  - $\mu_{A \oplus B}(x) = \wedge(1, (\mu_A(x) + \mu_B(x)))$ where $\wedge$ is minimum and + is the arithmetic add operator.
- **Bounded product or bold intersection:** $A \otimes B$
  - $\mu_{A \otimes B}(x) = \vee(0, (\mu_A(x) + \mu_B(x) - 1))$ where $\vee$ is maximum and + is the arithmetic add operator.
- **Bounded difference:** $A| - |B$
  - $\mu_{A|-|B}(x) = \vee(0, (\mu_A(x) - \mu_B(x)))$
    - **where $\vee$ is maximum and - is the arithmetic minus operator.**
  - This operation represents those elements that are more in A than B.

Concentration set operator

- **CON(A)** $\mu_{CON(A)}=(\mu A(x))^2$
  - This operation reduces the membership grade of elements that have small membership grades.
- If TALL=-.125/5+0.5/6+0.875/6.5+1/7+1/7.5+1/8 then
- VERY TALL = 0.0165/5+0.25/6+0.76/6.5+1/7+1/7.5+1/8 since VERY TALL=TALL^2.

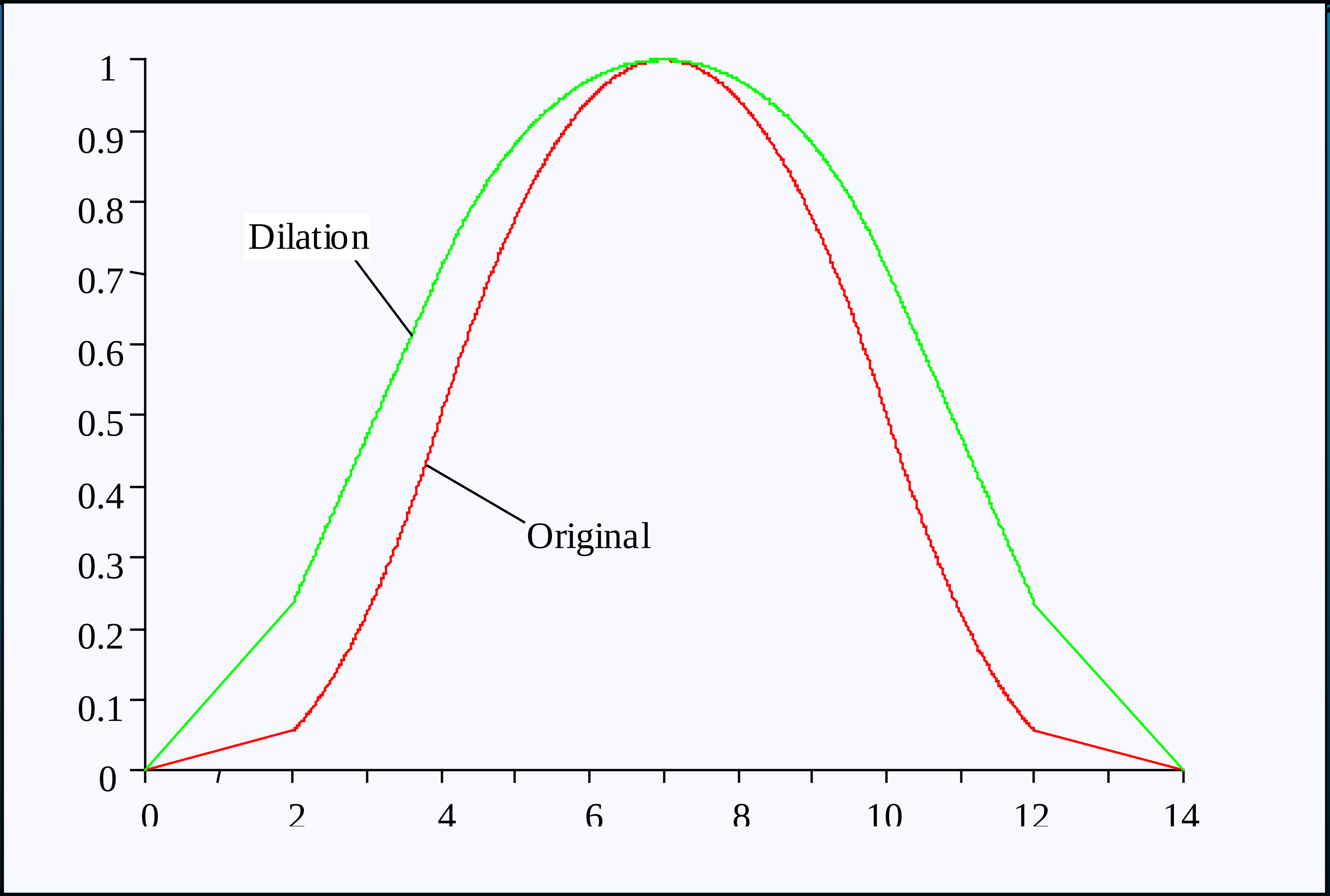
Concentration set operator



Dilation set operator

- **DIL(A)** $\mu_{DIL(A)}=(\mu_A(x))^{0.5}$
  - This operation increases the membership grade of elements that have small membership grades.
  - It is the inverse of the concentration operation.
- If TALL=-.125/5+0.5/6+0.875/6.5+1/7+1/7.5+1/8 then
- MORE or LESS TALL = 0.354/5+0.707/6+0.935/6.5+1/7+1/7.5+1/8 since MORE or LESS TALL=$TALL^{0.5}$.

Dilation set operator



Intensification set operator

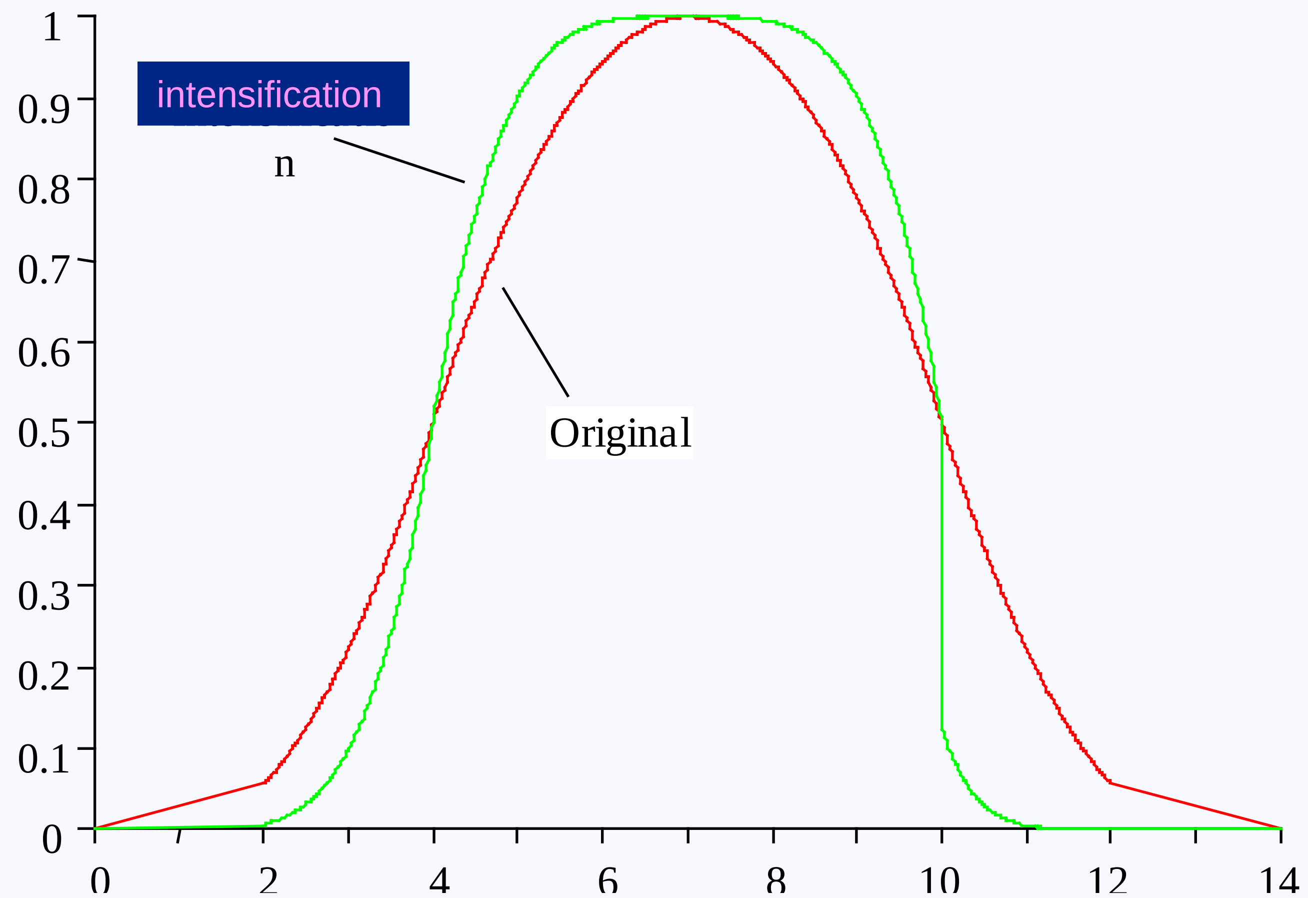
◆This operation raises the membership grade of those elements within the 0.5 points and reduces the membership grade of those elements outside the crossover (0.5) point.

◆ Hence, intensification amplifies the signal within the bandwidth while reducing the `noise'.

- If TALL=-.125/5+0.5/6+0.875/6.5+1/7+1/7.5+1/8 then
- INT(TALL) = 0.031/5+0.5/6+0.969/6.5+1/7+1/7.5+1/8.

$$\mu_{INT(A)}(x) = \begin{cases} 2(\mu_A(x))^2 \text{ for } 0 \le \mu_A(x) \le 0.5 \\ 1-2(1-\mu_A(x))^2 \text{ for } 0.5 < \mu_A(x) \le 1 \end{cases}$$

Intensification set operator



Normalisation set operator

◆ $\mu_{NORM(A)}(x)=\mu_A(x)/\max\{\mu_A(x)\}$ where the max function returns the maximum membership grade for all elements of x.

- If the maximum grade is <1, then all membership grades will be increased.
- If the maximum is 1, then the membership grades remain unchanged.

◆ NORM(TALL)=TALL since the maximum is 1

Sources

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