Introduction to Logic

Aristotle

Aristotle

(3000 years ago) was a Greek philosopher, scientist, and mathematician.

He wanted to create a system for explaining reality - and he called this system “Logos” or “Logic”

Aristotle's system

- Was very popular for the ancient Greeks.
- The system of logic became the basis for Western thought and Western writing.

It's like Mathematics

If A, then B

If B, then C

Therefore……

If A, then C

Anatomy of an argument

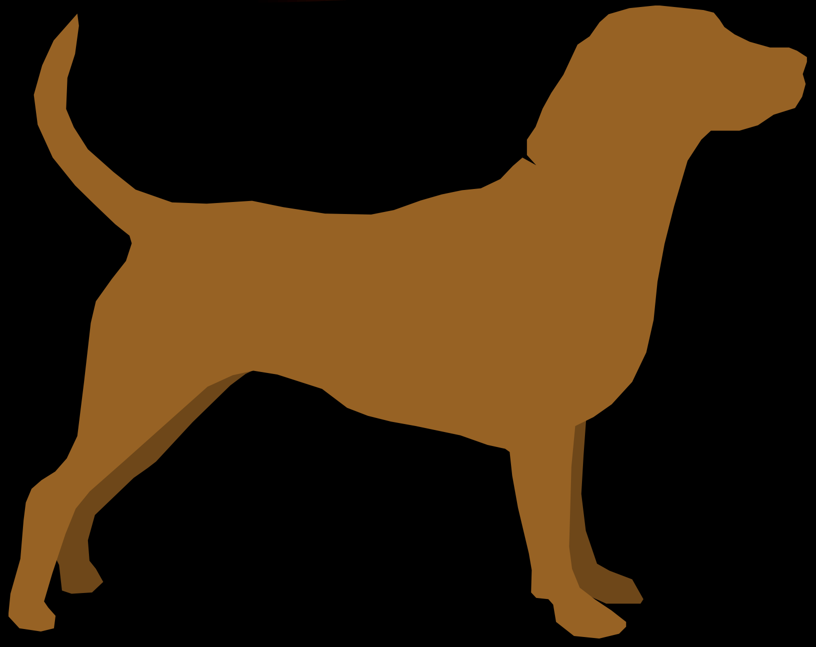
- The bottom statement after the "therefore" is called the "**conclusion**" of the argument
- *Conclusions are often called "thesis statements"*
- The top two statements are called "**premises**"
- *Premises are often called "supporting statements"*

Why do we argue???

To make people believe the same thing we do

Meet Fred the Dog

- If Fred is a healthy dog, then he has four legs
- If he has four legs, then he can run quickly
- Therefore, if Fred is a healthy dog, he can run quickly.



See the flow?

- If Fred is a healthy dog, then he has four legs.
- If Fred Has four legs, then he can run quickly
- If Fred is a healthy dog, then he can run quickly

- If A then B
- If B then C
- If A then C

We See a "Z"

- If Fred is a healthy dog , then he has four legs
- If Fred has four legs, then he can run fast
- If Fred is a healthy dog, then he can run fast

- If A then B
- If A then C
- If B then C

Let's try it with another form

- If Fred is a healthy dog , then he has four legs
- If Fred is a healthy dog, then he can run fast
- If fred has four legs, then he can run fast

- If A then B
- If A then C
- If B then C

It Doesn't Work

- If Fred is a healthy dog , then he has four legs
- If Fred is a healthy dog, then he can run fast
- If Fred has four legs, then he can run fast

- If A then B
- If A then C
- If B then C

Connecting the Ideas

- When the Ideas flow together in a "Z" chain, then we say the argument has a good "form"
- There are many kinds of forms that work
- We will learn two more forms

An argument is VALID when

- The argument has a proper form that chains together without interruption
- The premises are true.

IF THESE RULES ARE MET, THEN THE ARGUMENT IS VALID, AND THE CONCLUSION ***<u>MUST</u>*** BE TRUE!!!!

Argument form #1 - Hypothetical Syllogism (the Fred argument)

If A, then B

If B, then C

<u>If C, then D</u>

If A, then D

Something a little more real..

- If the United Nations help Poochnia develop it’s economy, then Poochnia can make enough food for it’s people.
- If Poochnia can make enough food, then it won’t attack it’s neighbors for food.
- If the United Nations help Poochnia develop it’s economy, it won’t attack it’s neighbors for food.

Argument form #2 - Modus Ponens

If A, then B

If B, then C

A

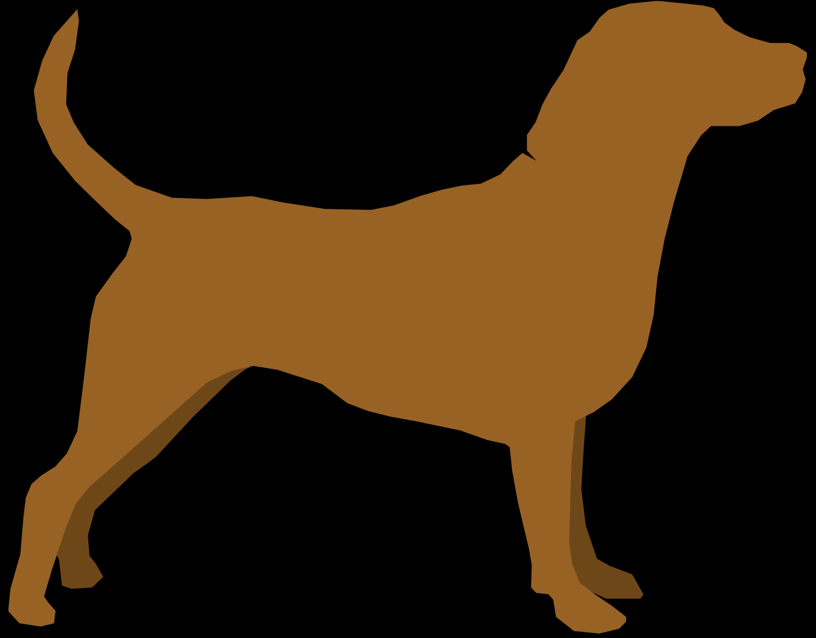
C

Example

- If you throw a ball at Fred, he will try to catch it.
- You throw a ball to Fred.

Therefore

- He will try to catch it.



A little more real

- If we help Poochnia develop it's economy, it will be able to make enough food.
- If Poochnia can make enough food, then it won't attack it's neighbors for food.
- If Poochnia doesn't attack it's neighbors, then there will be peace in that region.
- We will help Poochnia develop it's economy.

Therefore

- There will be peace in the region.

Argument form #3 - Transitive Syllogism

A = B

B = C

<u>C = D</u>

A = D

Meet Fred's friend -- Fluffy

- Fluffy is a cat
- cats are meat eaters
- meat eaters are killers

therefore

- Fluffy is a killer

Another trip to Poochnia

- Poochnia is a poor country
- Poor countries have bad economies
- Bad economies are the result of bad political systems

therefore

- Poochnia has a bad political system

Argument Grammar

- Infinitives - *'to protect our society'*
- Prediction Modals - *'may/will/would/shall'*
- Suasive verbs - *'agree, demand, prove recommend, suggest, indicate, show'*
- Conditional subordination - *'if - then', 'either - or', 'unless'*
- Necessity modals - *'will/would/shall '*
- Possibility modals - *'can/may/might/could'*

Symbolic Logic

The Statement

The fundamental unit in symbolic logic is the ***statement*** which is a declarative statement that may be assigned a value of being either False (0) or True (1)

The following are examples of statements:

p: The sky is blue.

q: The sky is black

r: The cat is fat.

s: The cat ate the rat.

t: The cat is black.

u: Today is Thursday.

Not a Statement

What time is it?

(Interrogative not declarative.)

Not a Statement

Stop making that noise.

(Imperative not declarative.)

Not a Statement

Pow!

Holy Cow, Batman!

(Exclamatory not declarative)

The Statement

At different times the same statement may take on different truth values.

For example tonight statement t is True (1) but tomorrow it will be False (0).

A statement is ***simple*** if it has a single subject and predicate and if it doesn't use the words we define as connectives.

The connectives are:

not, and, or, if…then, if and only if

A statement that uses connectives is ***compound***.

Simple & Compound Statements

Simple Statements:

Today is Thursday.   Tomorrow is Friday.   The cat is black.

Compound Statements:

Today is not Thursday.

The cat is fat and black.

If the cat ate the rat then the cat is fat.

We have a class today, if and only if today is Thursday.

Wiggles & Squiggles

Symbol	Connective	Key Word
$\neg$	negation	not
$\wedge$	conjunction	and
$\vee$	disjunction	or
$\rightarrow$	conditional	If…then
$\leftrightarrow$	biconditional	If and only if

Translating

p: The cat is black. q: The cat ate the rat r: The cat is fat.

¬p: The cat is not fat. Or It is not the case that, the cat is fat.

p∧r: The cat is black and fat. Or The cat is black and the cat is fat.

p∨q: The cat is black or the cat ate the rat.

q→r: If the cat ate the rat then the cat is fat.

r↔q: The cat is fat if and only if the cat ate the rat.

When are Things True of False?

The truth value of a compound statement depends of the truth values of simple statements that make it up,

and how their truth values are combined.

It is the ***structure*** that determines the truth value not the words.

Truth Tables

- We use a Truth Table to tell when compound statements are true or false.
- There is a basic truth table for each one of the connectives we use as logical operations.
- A summary of truth tables for the connectives is in Russell/Norvig text!

Truth Table for Negations

p	¬p
0	1
1	0

A statement and its negation always have opposite truth values

Truth Table for Conjunction

p	q	p∧q
0	0	0
0	1	0
1	0	0
1	1	1

The only time a conjunction is true is when the simple statements are all true.

Truth Table for Disjunction

p	q	p∨q
0	0	0
0	1	1
1	0	1
1	1	1

The only time a disjunction is false is when all the simple statements are false.

Truth Table for Implication (Conditional)

p	q	p→q
0	0	1
0	1	1
1	0	0
1	1	1

The only time a conditional is false is when you have 1→0

Conditional = implication

Truth Table for Equivalence (Biconditional)

p	q	p↔q
0	0	1
0	1	0
1	0	0
1	1	1

Biconditional is true when the simple statements match and false when the don’t match.

Order of Operations

Inside ( ) or [ ] first.

Negations

Conjunctions or Disjunctions

Conditional

Biconditional (equivalence of language - <-->

Try Some on Your Own

$(p \lor \neg p)$

$(p \land \neg p)$

$\neg(p \lor q)$

$\neg p \land \neg q$

$(p \lor q) \land r$

Equivalence of metalanguage (<=>)

Two statements are equivalent if their truth values match in all possible cases.

p q	$p \rightarrow q$	$\neg p \vee q$
0 0	1	1
0 1	1	1
1 0	0	0
1 1	1	1

So we write $p \rightarrow q \Leftrightarrow \neg p \vee q$. Equivalent statements have the same meaning.

Equivalencies

$\neg\neg p \Leftrightarrow p$	**Double Negation**
$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	**DeMorgan's Laws**
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	
$p \vee q \Leftrightarrow q \vee p$	**Commutative Laws**
$p \wedge q \Leftrightarrow q \wedge p$	
$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$	**Associative Laws**
$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$	

More Equivalencies

$$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

Distributive Laws

$$p \vee p \Leftrightarrow p$$

$$p \wedge p \Leftrightarrow p$$

Idempotent Laws

$$p \vee (p \wedge q) \Leftrightarrow p$$

$$p \wedge (p \vee q) \Leftrightarrow p$$

Absorption Laws

Conditionals and Equivalencies

Which of the following (if any) have the same meaning (are equivalent):

I. If the cat ate the rat then the cat is fat.

II. If the cat didn’t eat the rat then the cat isn’t fat.

III. If the cat is fat then the cat ate the rat.

IV. If the cat isn’t fat then the cat didn’t eat the rat.

Translating the Conditionals

p:The cat ate the rat.   q: The cat is fat.

I. $p \rightarrow q$ (conditional)

II. $\neg p \rightarrow \neg q$ (inverse)

III. $q \rightarrow p$ (converse)

IV. $\neg q \rightarrow \neg p$ (contrapositive)

The Truth Tables

p	q	$p \to q$	$\neg p \to \neg q$	$q \to p$	$\neg q \to \neg p$
0	0	1	1	1	1
0	1	1	0	0	1
1	0	0	1	1	0
1	1	1	1	1	1

From the truth table above we see any conditional statement is equivalent to its contapositive (flip and negate).

Arguments

The construction shown stands for the conditional statement:

$$(p_1 \wedge p_2 \wedge p_3 \wedge \ldots \wedge p_n) \to c$$

is called an argument. If the conditional statement shown is a tautology then the argument is valid, otherwise it is invalid.

Uses of Arguments

• Arguments form the basis of the methods we use for mathematical proofs.

• Once one accepts the premises in a valid argument then the conclusion follows logically from the premises.

•We say the premises imply ($\Rightarrow$) the conclusion.

•Several standard forms of arguments are recognized as valid or invalid.

Valid Forms

Modus Ponens

$$p \to q$$
$$\underline{p}$$
$$\therefore q$$

Modus Tollens

$$p \to q$$
$$\underline{\neg q}$$
$$\therefore \neg p$$

Proof by Contradiction

$$\underline{\neg p \to F_0}$$
$$\therefore p$$

Law of Syllogism

$$p \to q$$
$$\underline{q \to r}$$
$$\therefore p \to r$$

Law of Disjunctive Syllogism

$$p \vee q$$
$$\underline{\neg p}$$
$$\therefore q$$

Invalid Forms

Fallacy of the Inverse

$$p \to q$$
$$\underline{\neg p}$$
$$\therefore \neg q$$

Fallacy of the Converse

$$p \to q$$
$$\underline{\neg q}$$
$$\therefore \neg p$$

Quantifiers and Negations

$\forall =$ all

$\exists =$ some

The use of quantifiers sometimes causes problems for people.

This is especially with regards to forming negations when using DeMorgan’s Laws

or in forming the contrapositive of a conditional statement.

Examples of Negations

Given the statement: All of the dogs have fleas.

Which if any of the following is the negation.

None of the dogs have fleas.

Some of the dogs have fleas.

All of the dogs don’t have fleas.

Some of the dogs don’t have fleas.

Conditions on Negations

To correctly determine which of the previous statements is the negation of the given statement we rely on the fact that a statement and its negation must ***always*** have opposite truth values.

Consider evaluating the statements in the under the following conditions.

I) Every dog in the world is simultaneously given a flea bath.

II) Some dogs actually have fleas while others do not.

III) Every dog in the world is simultaneously given fleas.

The Truths of the Matter

	I	II	III
All dogs have fleas	F	F	T
No dogs have fleas	T	F	T
Some dogs have fleas	F	T	T
All dogs don’t have fleas	T	F	F
Some dogs don’t have fleas.	T	T	F

Negating Quantifiers

From the previous example we see that the correct way to negate:

all do, all have, all are $\leftrightarrow$ some don't, some haven't, some aren't

you can also check and see that the correct way to negate:

none do, none have, none are $\leftrightarrow$ some do, some have, some are

Check on Negations

Statements:	Negations:
No pigs have wings.	Some pigs have wings.
Some numbers are odd.	No numbers are odd.
All fish have gills	Some fish don’t have gills

sources

- Prof. Mark Duston
  - John Hazen White School of Arts & Sciences
  - Johnson & Wales University

Mercer University

English 100