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Application of modified sign Haar transform in logic functions

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Abstract: Modified sign Haar transform with sign Walsh-like structure is introduced in this article. This nonlinear transform converts binary/ternary vectors into digital spectral domain and is invertible. Recursive definitions for the calculation of this transform have been developed. The properties of logic functions and variables in the spectral domain of the modified sign Haar transform are presented.

Keywords: sign transform, ternary transform, Haar functions, logic functions

Classification: Science and engineering for electronics

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1 Introduction

In many applications of computer engineering and science, where logic functions need to be analyzed or synthesized, it is useful to transform such functions to the corresponding spectral domain that provides various new insights

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into solving some important problems [1, 2, 3]. A family of invertible non-linear transforms, which uniquely map ternary logic functions into ternary transform space, are sign transforms. The first transform under the name of 'sign transform' was based on Walsh functions and is known as sign Walsh transform [3]. The sign Haar transform [4] and sign Hadamard-Haar transform [5] have also been developed later.

The modified sign Haar transform [6] is related to both standard sign Haar and sign Walsh transforms, so it is also a non-linear, unique and invertible quantized transform. An important property of all quantized transforms is that the computer memory required to store functional and spectral data is exactly the same since all of them operate on ternary values. The basic definitions for this transform have been given here. In the new recursive sign transform, the butterfly structure is similar to the one of sign Walsh transform. The sign Walsh transform has been used for logic design [3]. The properties of logic functions based on modified sign Haar transform are shown for two different codings (S-coding and R-coding) of incompletely specified functions in this article.

2 Modified sign Haar transform

The following symbols are used: Let $\overrightarrow{x}_n = \{x_n, x_{n-1}, \ldots, x_p, \ldots, x_2, x_1\}$ and $\overrightarrow{\omega}_n = \{\omega_n, \omega_{n-1}, \ldots, \omega_p, \ldots, \omega_2, \omega_1\}$ be n-tuples over GF(2). The symbol x_p stands for a data variable, ω_p represents a modified sign Haar transform variable, p is an integer and $1 \leq p \leq n$. Let $\overrightarrow{F} = [F_0, F_1, \ldots, F_j, \ldots, F_{N-2}, F_{N-1}]$ be a ternary vector. Let $\overrightarrow{H}_F = [h_0, h_1, \ldots, h_j, \ldots, h_{N-2}, h_{N-1}]$ be the vector corresponding to modified sign Haar spectrum of $\overrightarrow{F}$. The value of h_j $(0 \leq j < N)$ is given by $\overrightarrow{H}_F(\omega)$ when $\sum_{p=1}^{n} \omega_p 2^{p-1} = j$. Let the symbol $\oplus$ represent logical EXOR.

2.1 Definitions of modified sign Haar transform

Definition 1 An invertible modified sign Haar transform h and its inverse transform h^{-1} are the mappings $h:\ \{+1,0,-1\}^N \rightarrow \{+1,0,-1\}_h^N$ and $h^{-1}:$ $\{+1,0,-1\}_h^N \rightarrow \{+1,0,-1\}^N$, where $N{=}2^n$. In the above equations, $\{+1,0,-1\}_h^N$ represents a set with the elements from $\{+1,0,-1\}^N$ permuted by the mapping h of all the elements of the set $\{+1,0,-1\}^N$. The cardinality of the original data set $\{+1,0,-1\}^N$ and its transformed spectrum is equal to 3^N.

Definition 2 An invertible forward modified sign Haar transform h is:

$$h(\overrightarrow{\omega}_n) = \mathrm{sign}\{\sum_{x_n=0}^{1}(-1)^{\omega_n x_n[1-\mathrm{sign}(\sum_{p=0}^{n-1}\omega_p)]}[1-(x_n\oplus\omega_n)\mathrm{sign}(\sum_{p=0}^{n-1}\omega_p)]\ldots$$
$$\mathrm{sign}\{\sum_{x_i=0}^{1}(-1)^{\omega_i x_i[1-\mathrm{sign}(\sum_{p=0}^{i-1}\omega_p)]}[1-(x_i\oplus\omega_i)\mathrm{sign}(\sum_{p=0}^{i-1}\omega_p)]\ldots$$
$$\mathrm{sign}\{\sum_{x_2=0}^{1}(-1)^{\omega_2 x_2(1-\omega_1)}[1-(x_2\oplus\omega_2)\omega_1]\mathrm{sign}[\sum_{x_1=0}^{1}(-1)^{x_1\omega_1}f(\overrightarrow{x}_n)]\}\ldots\}\ldots\} \quad (1)$$

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The inverse modified sign Haar transform is:

$$f(\overrightarrow{x}_n)=\text{sign}\{\sum_{\omega_1=0}^{1}(-1)^{x_1\omega_1}\text{sign}\{\sum_{\omega_2=0}^{1}(-1)^{\omega_2 x_2(1-x_1)}[1-(x_2\oplus\omega_2)x_1]\ldots$$
$$\text{sign}\{\sum_{\omega_i=0}^{1}(-1)^{\omega_i x_i[1-\text{sign}(\sum_{p=0}^{i-1}x_p)]}[1-(x_i\oplus\omega_i)\text{sign}(\sum_{p=0}^{i-1}x_p)]\ldots$$
$$\text{sign}\{\sum_{\omega_n=0}^{1}(-1)^{\omega_n x_n[1-\text{sign}(\sum_{p=0}^{n-1}x_p)]}[1-(x_n\oplus\omega_n)\text{sign}(\sum_{p=0}^{n-1}x_p)]h(\overrightarrow{\omega}_n)\}\ldots\}\ldots\}\} \quad (2)$$

In Eqs. (1) and (2), $1 \le p \le n$, $1 \le i \le n$ and $\text{sign}\, z = \begin{cases} -1 & \text{if } z < 0, \\ 0 & \text{if } z = 0, \\ 1 & \text{if } z > 0. \end{cases}$

2.2 Analysis of butterfly structure

The modified sign Haar transform is related to sign Haar transform, but its butterfly structure is derived from sign Walsh transform. Fig. 1 shows the butterfly diagrams for forward and inverse 16-point (n=4) modified sign Haar transform that is related to in-place fast Haar transform [7]. Comparing Fig. 1 and corresponding butterfly diagrams of sign Walsh transform [3], it can be seen that the modified sign Haar transform has similar butterfly structure with sign Walsh transform. Therefore, the modified sign Haar transform has also the advantages of symmetric butterfly structure. Using such a structure, the modified sign Haar transform can be implemented by hardware and extended more conveniently than standard sign Haar transform.

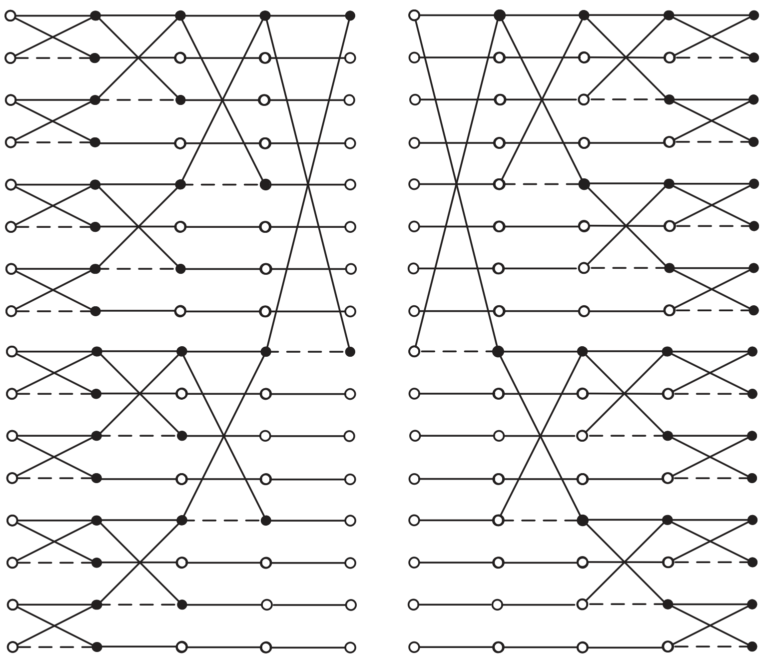

Fig. 1. Fast butterfly diagrams for (a) forward and (b) inverse 16-point modified sign Haar transforms.

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Table I. Relationship between 16-point modified and standard sign Haar spectra

Standard sign Haar spectrum	Modified sign Haar spectrum
$(0,0,0,0)$	$(0,0,0,0)$
$(0,0,0,1)$	$(1,0,0,0)$
$(0,0,1,0)$	$(0,1,0,0)$
$(0,0,1,1)$	$(1,1,0,0)$
$(0,1,0,0)$	$(0,0,1,0)$
$(0,1,0,1)$	$(0,1,1,0)$
$(0,1,1,0)$	$(1,0,1,0)$
$(0,1,1,1)$	$(1,1,1,0)$
$(1,0,0,0)$	$(0,0,0,1)$
$(1,0,0,1)$	$(0,0,1,1)$
$(1,0,1,0)$	$(0,1,0,1)$
$(1,0,1,1)$	$(0,1,1,1)$
$(1,1,0,0)$	$(1,0,0,1)$
$(1,1,0,1)$	$(1,0,1,1)$
$(1,1,1,0)$	$(1,1,0,1)$
$(1,1,1,1)$	$(1,1,1,1)$

The modified sign Haar transform can be considered as another version of standard sign Haar transform [4], since there are some relationships between the outputs of modified and standard sign Haar transforms. Comparing Fig. 1 and corresponding butterfly diagrams of standard sign Haar transform [4], the relationship between 16-point modified and standard sign Haar transforms can be found. The modified and standard sign Haar spectra can be derived from reordering each other for the same input function. Table I shows the corresponding relationship between 16-point modified and standard sign Haar transforms.

3 Application of modified sign Haar transform in logic functions

The properties of modified sign Haar spectra for logic functions are shown in this section. In the following properties, let function f and h be defined as in the previous section. There are two types of coding which have been widely used in logic design.

Definition 3 S-coding and R-coding are used to represent Boolean functions when different spectra of such functions are calculated. The truth vector for S-coding is represented in the following way: the true minterms (minterms for which Boolean functions have logical values 1) are denoted by -1, false minterms (minterms for which Boolean functions have logical values 0) by +1 , and do not care minterms (minterms for which Boolean functions can have arbitrary logical values 0 or 1) by 0. The truth vector for R-coding is represented in the following way: the true minterms are denoted by 1, false minterms by 0, and do not care minterms by 0.5.

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Property 1 Let function $f(\overrightarrow{x}_n)$ be a constant, such that its ternary vector $\overrightarrow{F}$ has all the coefficients equal and $F_j(0 \le j < 2^n)$; $F_j \in \{-1, 0, 1\}$. Then,

$$f(\overrightarrow{x}_n) = 0 \Longleftrightarrow h(\overrightarrow{\omega}_n) = 0, \tag{3}$$

$$f(\overrightarrow{x}_n) = \pm 1 \Longleftrightarrow h(\overrightarrow{\omega}_n) = \pm 1 \times (\overset{n}{\underset{j=1}{\wedge}} \overline{\omega}_j) \tag{4}$$

where $\omega_k \in \{0, 1\}$ and the logic AND operations in brackets ( ) will yield value 1 or 0.

Example 1 For a 3-variable function $f_1(\overrightarrow{x}_3) = -1 = (-, -, -, -, -, -, -, -)$, its modified sign Haar spectrum can be evaluated using (4) as follows:

$$h(\overrightarrow{\omega}_3) = -1 \times (\overset{3}{\underset{j=1}{\wedge}} \overline{\omega}_j) = -\overline{\omega}_3\overline{\omega}_2\overline{\omega}_1 = (-, 0, 0, 0, 0, 0, 0, 0).$$

Property 2 When S-coded n-variable function is functionally dependent on a single Boolean variable in affirmation, i.e.,

$$f(\overrightarrow{x}_n) = x_j, \; j \in \{1, 2, \ldots, n\}, x_j \in \{+1, -1\}.$$

Then its corresponding modified sign Haar spectrum is:

$$h(\overrightarrow{\omega}_n) = +1 \times (\omega_j \wedge \{\overset{j-1}{\underset{k=1}{\wedge}} \overline{\omega}_k\}) \tag{5}$$

where $\omega_k \in \{0, 1\}$ and the expression $\overset{j-1}{\underset{k=1}{\wedge}} \overline{\omega}_k = 1$ if $j = 1$. The restriction of the value of $\overset{j-1}{\underset{k=1}{\wedge}} \overline{\omega}_k$ when $j = 1$ is the same as above also for the following properties.

Example 2 For a 3-variable function $f_2(\overrightarrow{x}_3) = x_2 = (+, +, -, -, +, +, -, -)$, its modified sign Haar spectrum can be evaluated using (5) as follows:

$$h(\overrightarrow{\omega}_3) = +1 \times (\omega_2 \wedge \{\overset{2-1}{\underset{k=1}{\wedge}} \overline{\omega}_k\}) = \omega_2\overline{\omega}_1 = (0, 0, +, 0, 0, 0, +, 0).$$

Property 3 When S-coded n-variable function is functionally dependent on a single Boolean variable in negation, i.e.,

$$f(\overrightarrow{x}_n) = \overline{x}_j, \; j \in \{1, 2, \ldots, n\}, x_j \in \{+1, -1\}.$$

Then its corresponding modified sign Haar spectrum is:

$$h(\overrightarrow{\omega}_n) = -1 \times (\omega_j \wedge \{\overset{j-1}{\underset{k=1}{\wedge}} \overline{\omega}_k\}). \tag{6}$$

Property 4 For S-coded n-variable Boolean function $f(\overrightarrow{x}_n)$ whose modified sign Haar spectrum is $h(\overrightarrow{\omega}_n)$, the spectrum of the negated function is derived simply by inverting all the signs of the original spectra, i.e.,

$$f(\overrightarrow{x}_n) \Longleftrightarrow h(\overrightarrow{\omega}_n), \quad \text{then } \overline{f(\overrightarrow{x}_n)} \Longleftrightarrow -h(\overrightarrow{\omega}_n). \tag{7}$$

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Property 5 When R-coded n-variable function is functionally dependent on a single Boolean variable in affirmation, i.e.,

$$f(\overrightarrow{x}_n) = x_j,\ j \in \{1, 2, \ldots, n\}, x_j \in \{0, 1\}.$$

Then its corresponding modified sign Haar spectrum is:

$$h(\overrightarrow{\omega}_n) = \overset{n}{\underset{k=1}{\wedge}} \overline{\omega}_k - (\omega_j \wedge \{\overset{j-1}{\underset{k=1}{\wedge}} \overline{\omega}_k\}). \tag{8}$$

Property 6 When R-coded n-variable function is functionally dependent on a single Boolean variable in negation, i.e.,

$$f(\overrightarrow{x}_n) = \overline{x}_j,\ j \in \{1, 2, \ldots, n\}, x_j \in \{0, 1\}.$$

Then its corresponding modified sign Haar spectrum is:

$$h(\overrightarrow{\omega}_n) = \overset{n}{\underset{k=1}{\wedge}} \overline{\omega}_k + (\omega_j \wedge \{\overset{j-1}{\underset{k=1}{\wedge}} \overline{\omega}_k\}). \tag{9}$$

Example 3 For a 3-variable function $f_3(\overrightarrow{x}_3) = \overline{x}_3 = (+, +, +, +, 0, 0, 0, 0)$, its modified sign Haar spectrum can be evaluated using (9) as follows:

$$\begin{aligned} h(\overrightarrow{\omega}_3) &= \overset{3}{\underset{k=1}{\wedge}} \overline{\omega}_k + (\omega_3 \wedge \{\overset{3-1}{\underset{k=1}{\wedge}} \overline{\omega}_k\}) = \overline{\omega}_3\overline{\omega}_2\overline{\omega}_1 + \omega_3\overline{\omega}_2\overline{\omega}_1 \\ &= (+, 0, 0, 0, +, 0, 0, 0). \end{aligned}$$

4 Conclusion

The modified sign Haar transform is a new non-linear transform. The recursive definitions and fast butterfly diagram have been introduced to facilitate the calculation of the transform in a very efficient way. When the fast flow diagram is directly implemented in software there is no need to keep the original data and each consecutive butterfly requires geometrically smaller number of operations on the transformed data. Each time only binary/ternary data has to be stored in the memory. Though the symmetric butterfly structure of the new transform is similar to sign Walsh transform, it requires lower computational costs than sign Walsh transform. Hence, the modified sign Haar transform is better suited to be implemented by simpler hardware. In addition, this new transform can replace the butterfly diagram related to Haar transform part inside the combined sign Hadamard-Haar transform [5] that will reduce the complexity of design in its hardware implementation. The relationships between logic functions and modified sign Haar spectral representations of Boolean functions have been stated in this article. Those properties of logic functions for different codings of incompletely specified functions will be useful in the application of modified Haar transform in logic synthesis.

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