PERGAMON

Computers and Electrical Engineering 29 (2003) 1–12

Computers and Electrical Engineering

www.elsevier.com/locate/compeleceng

Operations on logic functions and variables through sign Haar spectra

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Received 9 November 2000; accepted 18 January 2001

Abstract

A non-linear transform, called "sign Haar transform" has recently been introduced. The transform is unique and converts binary/ternary vectors into ternary spectral domain. Recursive definitions for the calculation of sign Haar transform are developed. Essential properties of logic functions and variables in the spectral domain of a quantized transform based on Haar functions are presented. Sign Haar transform has the smallest computational cost of all the quantized transforms. The properties of logic functions are listed for two different codings of incompletely specified functions.

Keywords: Spectral techniques; Sign Haar transform; Quantized transforms; Cyclic and dyadic addition

1. Introduction

The various discrete transformations have been used in analysis, synthesis and testing of digital circuits [1,9,12]. Another areas of their applications are signal processing, especially image processing and pattern recognition [1,15]. Spectral techniques have also been used for data transmission, especially in the theory of error correcting codes and for digital filtering [1,9].

Most of the used transforms are orthogonal and provide one-to-one mapping of an input vector onto an output spectrum. They are unique (canonical) and the inverse transform of the output spectrum yields back the original input vector. There are at least two transforms which are based on square-wave-like functions that are well suitable for Boolean functions: Walsh and Haar

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PII: S0045-7906(01)00017-9

transforms [1,4,11–15]. The Walsh functions are global like the Fourier functions and consist of a set of irregular rectangular waveforms with only two amplitude values +1 and −1. Walsh functions served as the underlying basis functions of new transform with restricted coefficient values called "sign transform" [2]. Each but two basis functions in Haar transform consists of a square-wave pulse located on an otherwise zero amplitude interval. Computation of fast Haar transform requires order O(N) (N is a number of spectral coefficients) additions and subtractions, which makes it faster than fast Walsh transform [1,5,6,10,12,13,15]. A new orthogonal transform which assumes only +1 and −1 values has recently been proposed and some of their properties and applications were discussed in Ref. [6]. This transform [6] may be treated as a Walsh-like function, it can be used to as an intermediation transformation to allow conversion between Haar and Walsh spectra. Both Walsh and Haar spectra of Boolean functions have easy interpretation and efficient methods of calculation of such spectra directly from reduced representation of Boolean functions have recently been introduced [3,8,13]. Similar efficient methods of calculating sign Walsh spectrum from reduced representations were shown in Ref. [7] and can be extended to sign Haar spectrum.

Different quantized transforms have been known to be extremely effective both in terms of memory requirements and processing time and hence very advantageous in the design of binary/ternary switching circuits. More work on the application of sign Walsh transform in logic design have been done [2,12], eventhough it has higher computational cost than sign Haar transform [4,6,10] and the latter has very similar properties and potential applications. In this article, we present a detailed properties of lesser known sign Haar transform. The basic definitions for this transform have been given in Refs. [4,5], and for a related quantized transform in Ref. [6]. Besides applications in logic design a new transform can be used when there is a need for a unique coding of binary/ternary vectors into the special domain of the same dimensions. One of the possible applications would be security coding in cryptographic systems using sign Haar-χ Walsh-γ transform [6] and ternary communication systems with sign Haar-χ transform [10]. An important property of sign Haar transform is that the computer memory required to store functional and spectral data is exactly the same since both of them operate on ternary values. It is in high contrast to traditional Haar spectrum where signs together with magnitudes have to be stored in spectral domain which then significantly increases requirements for storage space in spectral domain versus functional when only binary/ternary are considered.

2. General definitions of quantized transforms

S-coding is frequently used for representing Boolean functions when different spectra of such functions are calculated. The truth vector for S-coding is represented in the following way: the true minterms (minterms for which Boolean function has logical values 1) are denoted by −1, false minterms (minterms for which Boolean functions has logical values 0) by +1, and do not care minterms (minterms for which Boolean function can have arbitrary logical values 0 or 1) by 0. Hence binary vectors formed of only $\{+1, -1\}$ represent logical values of completely specified Boolean functions, and formed of $\{+1, 0, -1\}$ the values of incompletely specified Boolean functions. In the continuation, to shorten the notation, functional and spectral data will be represented by either $\{+,-\}$ or $\{+,0,-\}$. The data in functional domain can be arbitrary binary/ternary vectors or S-coded completely (binary) or incompletely (ternary) specified Boolean functions. The following symbols will be used, let $R_1 = \{+,-\}$, $R_2 = \{+,0,-\}$, R_l^n means n-space Cartesian product of a set R_l $(l = 1,2)$.

Definition 1. An n-variable S-coded completely specified Boolean function is the mapping $f_1\colon R_1^n \to R_1$.

Definition 2. An n-variable S-coded incompletely specified Boolean function is the mapping $f_2\colon R_1^n \to R_2$.

Definition 3. An invertible sign Haar transform h and its inverse transform h^{-1} are the mappings $h\colon R_2^N \to R_{2(h)}^N$ and $h^{-1}\colon R_{2(h)}^N \to R_2^N$, where $N = 2^n$. In the above equations, symbol $R_{2(h)}^N$ represents a set with the elements from R_2^N permuted by the mapping h of all the elements of the set R_2^N. When only completely specified Boolean functions are considered, the symbol R_2^N is replaced with R_1^N and $R_{2(h)}^N$ with $R_{1(h)}^N$ where the latter represents a proper subset of set $R_{2(h)}^N$ generated by the h mapping of all the elements of set R_1^N. In order to obtain the sign Haar spectrum h (an element of set $R_{2(h)}^N$), and its inverse (a corresponding element of the original data set R_2^N), the results of each fast forward or inverse Haar butterfly block are quantized first. In the above equations, the cardinality of the original data set R_2^N and its transformed spectrum $R_{2(h)}^N$ is equal to 3^N.

When some permutation is performed on the elements of set R_2^N the same permutation happens to the elements in $R_{2(h)}^N$ spectrum of the original set. Fast flow diagrams for calculation of forward and inverse sign Haar transform h are shown for $N = 8$ in Fig. 1(a) and (b) accordingly. The number of operations required to perform forward sign Haar transform h for a single element of set R_2^N and inverse sign Haar transform h^{-1} for a single element of set $R_{2(h)}^N$ and for transform

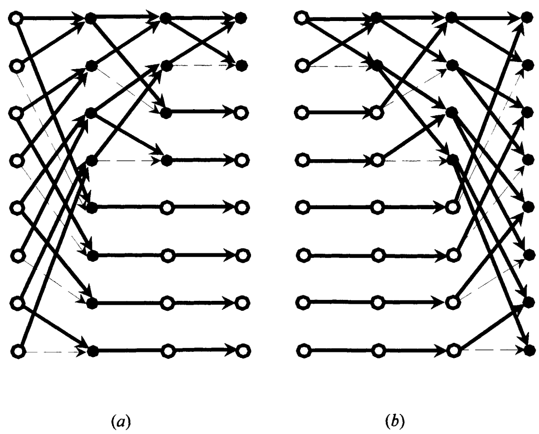

Fig. 1. Butterfly diagram for (a) forward and (b) inverse sign Haar transform, $n = 3$. (●) the sign function, (○) lack of any operation, the solid lines and dotted lines represent addition and subtraction respectively.

matrix of order $N = 2^n$ is equal to $4(2^n - 1)$. Besides calculation of sign Haar transform by using fast flow diagrams, sign Haar spectra can be calculated directly from recursive definitions that involve data and transform domain variables.

3. Recursive definitions of sign Haar transform

The following symbols are used: Let $\vec{x}_n = \{x_n, x_{n-1}, \dots, x_p, \dots, x_2, x_1\}$, and $\vec{\omega}_n = \{\omega_n, \omega_{n-1}, \dots, \omega_p, \dots, \omega_2, \omega_1\}$ be n-tuples over GF(2). The symbol x_p stands for a data variable, α_p represents a sign Walsh transform variable, and ω_p a sign Haar transform variable, p is an integer and $1 \leqslant p \leqslant n$. Let $\vec{F} = [F_0, F_1, \dots, F_j, \dots, F_{N-2}, F_{N-1}]$ be a ternary vector. For example, it can be the S-coded truth vector of $f: (0,1)^n \to (-1,0,1)$ where the value of F_j $(0 \leqslant j < N)$ is given by $F(\vec{x}_n)$ when $\sum_{p=1}^{n} x_p 2^{p-1} = j$. The truth vector for S-coding is defined as in Section 2. In R-coding [8], the original logical values of 0 and 1 are kept, and do not cares are represented by the value 0.5. Let $\vec{W}_F = [w_0, w_1, \dots, w_j, \dots, w_{N-2}, w_{N-1}]$ and $\vec{H}_F = [h_0, h_1, \dots, h_j, \dots, h_{N-2}, h_{N-1}]$ be the vectors corresponding to sign Walsh spectrum of $\vec{F}$ and sign Harr spectrum of $\vec{F}$, accordingly. The value of w_j $(0 \leqslant j < N)$ is given by $W_F(\alpha)$ when $\sum_{p=1}^{n} \alpha_p 2^{p-1} = j$. The value of h_j $(0 \leqslant j < N)$ is given by $H_F(\omega)$ when $\sum_{i=1}^{n} \omega_i 2^{i-1} = j$. Let $\vec{O}_i$ represent the vector of i zeros, $1 \leqslant i \leqslant n$. Let the symbol $\oplus_c$ represent cyclic addition, the symbol $\oplus_d$ represent dyadic addition, and the symbol $\wedge$ represent bit-by-bit logical AND.

When the above operations are applied to two vectors $\vec{A}_l$ and $\vec{B}_k$, $1 \leqslant l < k$, l and k are two different integer numbers, they result in the vector $\vec{C}_k$ of the length k. Only l elements of $\vec{B}_k$ and all elements of $\vec{A}_l$ are manipulated on, remaining $(k-1)$ elements of the resulting vector $\vec{C}_k$ are not affected by the applied operation and are simply the same as the elements of the vector $\vec{B}_k$ between positions k and $l+1$.

Definition 4 [2]. An invertible forward sign Walsh transform w is

$$w(\vec{a}_n) = \mathrm{sign}\left\{ \sum_{x_n=0}^{1} \mathrm{sign}\left[\sum_{x_{n-1}=0}^{1} \mathrm{sign}\left(\dots \mathrm{sign} \sum_{x_1=0}^{1} f(\vec{x}_n)(-1)^{\sum_{p=1}^{n} \alpha_p x_p} \right) \right] \right\} \tag{1}$$

The inverse sign Walsh transform is

$$f(\vec{x}_n) = \mathrm{sign}\left\{ \sum_{a_0=0}^{1} \mathrm{sign}\left[\sum_{a_1=0}^{1} \mathrm{sign}\left(\dots \mathrm{sign} \sum_{a_1=0}^{1} w(\vec{a}_n)(-1)^{\sum_{p=1}^{n} \alpha_p x_p} \right) \right] \right\} \tag{2}$$

In Eqs. (1) and (2), $1 \leqslant p \leqslant n$.

Definition 5 [4,5,10]. An invertible forward sign Haar transform h is:

$$h\left(\vec{O}_n \oplus_{\mathrm{d}} \omega_1\right) = \mathrm{sign} \sum_{x_n=0}^{1} \left[\mathrm{sign} \sum_{x_{n-1}=0}^{1} \left[\dots \mathrm{sign} \sum_{x_1=0}^{1} \left\{ (-1)^{x_n \omega_1} f\left(\vec{x}_n\right) \right\} \dots \right] \right] \tag{3}$$

and

$$h\left(\vec{O}_n \oplus_{\mathrm{d}} \vec{\omega}_i \oplus_{\mathrm{d}} 2^i\right) = \mathrm{sign} \sum_{x_{n-i}=0}^{1} \left[\mathrm{sign} \sum_{x_{n-i-1}=0}^{1} \left[\ldots \mathrm{sign} \sum_{x_1=0}^{1} (-1)^{x_{n-1}} \ldots f\left\{\left[\left(\vec{O}_n \oplus_{\mathrm{d}} \vec{\omega}_i\right) \oplus_{\mathrm{c}} (n-i)\right] \oplus_{\mathrm{d}} \vec{x}_{n-i}\right\}\right]\right] \tag{4}$$

where $1 \leqslant i < n$.

The inverse transform is:

$$\begin{aligned} f\left(\vec{x}_n\right) = \mathrm{sign}\Bigg\{ &(-1)^{x_1} h\left\{\left[\left(\vec{O}_1 \wedge \vec{x}_n\right) \oplus_{\mathrm{c}} 1\right] \oplus_{\mathrm{d}} 2^{n-1}\right\} \\ &+ \mathrm{sign}\Bigg\{(-1)^{x_2} h\left\{\left[\left(\vec{O}_2 \wedge \vec{x}_n\right) \oplus_{\mathrm{c}} 2\right] \oplus_{\mathrm{d}} 2^{n-2}\right\} \\ &+ \cdots + \mathrm{sign}\Bigg\{(-1)^{x_i} h\left\{\left[\left(\vec{O}_i \wedge \vec{x}_n\right) \oplus_{\mathrm{c}} i\right] \oplus_{\mathrm{d}} 2^{n-i}\right\} \\ &+ \cdots + \mathrm{sign}\Bigg\{(-1)^{x_{n-1}} h\left\{\left[\left(\vec{O}_{n-1} \wedge \vec{x}_n\right) \oplus_{\mathrm{c}} (n-1)\right] \oplus_{\mathrm{d}} 2\right\} \\ &+ \mathrm{sign}\left[\sum_{\omega_1=0}^{1} (-1)^{x_n \omega_1} h\left(\vec{O}_n \oplus_{\mathrm{d}} \omega_1\right)\right]\Bigg\} \ldots \Bigg\}\Bigg\}\Bigg\} \end{aligned} \tag{5}$$

In Eqs. (1)–(5),

$$\mathrm{sign}\, z = \begin{cases} -1 & z < 0 \\ 0 & z = 0 \\ +1 & z > 0 \end{cases} \tag{6}$$

Proof of uniqueness. Let us prove the uniqueness of sign Haar transform for a single variable and the general case for n variables follows by induction. The output function $f(x_1) \in \{-1, 0, +1\}$ where $x_1 \in \{0, 1\}$. From Eq. (3), forward sign Haar transform,

$$\begin{aligned} h\left(\vec{O}_1 \oplus_{\mathrm{d}} \omega_1\right) = h(\omega_1) &= \mathrm{sign}\left[\sum_{x_1=0}^{1} (-1)^{x_1 \omega_1} f(\vec{x}_1)\right] = \mathrm{sign}\left[\sum_{x_1=0}^{1} (-1)^{x_1 \omega_1} f(x_1)\right] \\ &= \mathrm{sign}[f(0) + (-1)^{\omega_1} f(1)] \end{aligned} \tag{7}$$

From Eq. (5), inverse sign Haar transform,

$$\begin{aligned} f\left(\vec{x}_1\right) = f(x_1) &= \mathrm{sign}\left[\sum_{\omega_1=0}^{1} (-1)^{x_1 \omega_1} h\left(\vec{O}_1 \oplus_{\mathrm{d}} \omega_1\right)\right] = \mathrm{sign}\left[\sum_{\omega_1=0}^{1} (-1)^{x_1 \omega_1} h(\omega_1)\right] \\ &= \mathrm{sign}[h(0) + (-1)^{x_1} h(1)] \end{aligned} \tag{8}$$

For the case of $x_1 = 0$, the right-hand side of Eq. (8) yields,

$$\mathrm{sign}[h(0) + h(1)] = \mathrm{sign}[\mathrm{sign}\{f(0) + f(1)\} + \mathrm{sign}\{f(0) - f(1)\}] = f(0)$$

The case $x_1 = 1$ is proved similarly. □

Example 1. For $n = 3$ the definitions of forward sign Haar transform h become:

$$h(\vec{O}_3 \oplus_{\mathrm{d}} \omega_1) = h(0,0,\omega_1) = \mathrm{sign}\sum_{x_3=0}^{1}\left[\mathrm{sign}\sum_{x_2=0}^{1}\left[\sum_{x_1=0}^{1}\left\{(-1)^{x_3\omega_1} f\left(\vec{x}_3\right)\right\}\right]\right]$$

and

$$h\left(\vec{O}_3 \oplus_{\mathrm{d}} \vec{\omega}_i \oplus_{\mathrm{d}} 2^i\right) = \mathrm{sign}\sum_{x_{3-i}=0}^{1}\left[\mathrm{sign}\sum_{x_{3-i-1}=0}^{1}(-1)^{x_{3-i}} f\left\{\left[\left(\vec{O}_3 \oplus_{\mathrm{d}} \vec{\omega}_i\right) \oplus_{\mathrm{c}} (3-i)\right] \oplus_{\mathrm{d}} \vec{x}_{3-i}\right\}\right]$$

for $1 \leqslant i < 3$.

In particular, for $i = 2$,

$$\begin{aligned} h\left(\vec{O}_3 \oplus_{\mathrm{d}} \vec{\omega}_2 \oplus_{\mathrm{d}} 2^2\right) = h(1,\omega_2,\omega_1) &= \mathrm{sign}\sum_{x_1=0}^{1}\left\{(-1)^{x_1} f\left\{\left[\left(\vec{O}_3 \oplus_{\mathrm{d}} \vec{\omega}_2\right) \oplus_{\mathrm{c}} 1\right] \oplus_{\mathrm{d}} \vec{x}_1\right\}\right\} \\ &= \mathrm{sign}\sum_{x_1=0}^{1}\left\{(-1)^{x_1} f\left[(\omega_2,\omega_1,0) \oplus_{\mathrm{d}} \vec{x}_1\right]\right\} \\ &= \mathrm{sign}\sum_{x_1=0}^{1}\{(-1)^{x_1} f(\omega_2,\omega_1,x_1)\} \end{aligned}$$

For $i = 1$,

$$\begin{aligned} h\left(\vec{O}_3 \oplus_{\mathrm{d}} \vec{\omega}_1 \oplus_{\mathrm{d}} 2^1\right) &= h(0,1,\omega_1) \\ &= \mathrm{sign}\sum_{x_2=0}^{1}\left[\mathrm{sign}\sum_{x_1=0}^{1}\left\{(-1)^{x_2} f\left\{\left[\left(\vec{O}_3 \oplus_{\mathrm{d}} \vec{\omega}_1\right) \oplus_{\mathrm{c}} 2\right] \oplus_{\mathrm{d}} \vec{x}_2\right\}\right\}\right] \\ &= \mathrm{sign}\sum_{x_2=0}^{1}\left[\mathrm{sign}\sum_{x_1=0}^{1}\{(-1)^{x_2} f(\omega_1,\omega_2,x_1)\}\right] \end{aligned}$$

In a similar manner definitions for inverse transform from Eq. (5) when $n = 3$ can be derived.

4. Properties of sign Haar spectra of logic functions and variables

Sign Haar spectra for common logic functions and the major properties of the transform are presented. There is no direct relationship between sign Haar spectra calculated for *S*- and *R*-coded Boolean functions, what differs from other transforms used in logic design (i.e., Walsh, Haar) [1,3,9,12]. Therefore, basic properties for logic operators have to be derived separately for both codings.

Table 1 shows the listing of all the 81 functions of two variables, and their corresponding sign Haar and sign Walsh spectra. In the following presentation of the properties, let function f and transform h be defined as in the previous section. Let a and b be ternary variables, where $a, b \in \{-1, 0, 1\}$. In order to illustrate better investigated properties let us introduce a sign domain map.

Definition 6. A sign domain map is a graphical two-dimensional representation of sign Haar spectrum and is an equivalent of a Karnaugh map in Boolean domain where spectral variables listed in gray code order are used to indicate all the cells of the map and sign spectral coefficients' values are entered into the cells.

Property 1. *The number of cells in sign domain map of the spectrum of an n-variable Boolean function is exactly the same as the number of minterms (cells on Karnaugh map) of such a function.*

Property 2. *For arbitrary ternary variables a and b:*

$$\text{sign}[\text{sign}(a+b) + \text{sign}(a-b)] = a \tag{9}$$

and

$$\text{sign}[\text{sign}(a+b) - \text{sign}(a-b)] = b \tag{10}$$

Property 3. *Let function $f(\vec{x}_n)$ be a constant, such that its ternary vector $\vec{F}$ has all the coefficients equal and F_j $(0 \leqslant j < 2^n)$; $F_j \in \{-, 0, +\}$. Then,*

$$f\left(\vec{x}_n\right) = 0 \Longleftrightarrow h\left(\vec{\omega}_n\right) = 0, \quad x_i, \omega_i \in \{0, 1\} \text{ and } 1 \leqslant i \leqslant n \tag{11}$$

$$f(\vec{x}_n) = \pm 1 \Longleftrightarrow h(\vec{\omega}_n) = \pm \prod_{j=0}^{n-1} (1 - \omega_j) \tag{12}$$

Example 2. For $n = 3$, $f_1(x) = (+, +, +, +, +, +, +, +) \Longleftrightarrow h_1(\omega) = (+, 0, 0, 0, 0, 0, 0, 0)$. Karnaugh map of the function $f_1(\vec{x}_3)$ and sign domain map of the spectrum $h_1(\vec{\omega}_3)$ are shown in Fig. 2(a).

Property 4. *When S-coded n-variable function is functionally dependent on a single Boolean variable in affirmation, i.e.*

$$f(\vec{x}_n) = f(x_n, \ldots, x_1) = x_j, \; j \in \{1, \ldots, n\}, \; x_j \in \{+1, -1\}$$

sign Haar transform,

$$h(\vec{\omega}_n) = +1 \times \left(\omega_{n-j+1} \wedge \left\{ \bigwedge_{k=n-j+2}^{n} \overline{\omega_k} \right\} \right) \tag{13}$$

where $\omega_k \in \{0, 1\}$ and the logic AND operations in brackets ( ) will yield value 1 or 0. If for some j, $k > n$, by definition the expression $\wedge_{k=n-j+2}^{n} \overline{\omega_k} = 1$, otherwise the symbol $\overline{\omega_k}$ represents the logical

Table 1
All incompletely specified *S*-coded two-variable Boolean functions, their sign Haar and sign Walsh spectra

Function ⟺	Sign Haar spectrum ⟺	Sign Walsh spectrum	Function ⟺	Sign Haar spectrum ⟺	Sign Walsh spectrum	Function ⟺	Sign Haar spectrum ⟺	Sign Walsh spectrum
−−−−	−000	−000	−−−0	−00−	−−0+	−−−+	−−0−	−−−+
−−0−	−00+	−+0−	−−00	−−00	−0−0	−−0+	0−0−	0−−+
−−+−	−−0+	−+−−	−−+0	0−0+	0+−−	−−++	0−00	00−0
−0−−	−0−0	−−0−	−0−0	−0−−	−−00	−0−+	−−−−	−−−0
−00−	−0−+	−00−	−000	−−−0	−−−−	−00+	0−−−	0−−0
−0+−	−−−+	−0−−	−0+0	0−−+	00−−	−0++	0−−0	0−−−
−+−−	−+−0	−−+−	−+−0	−+−−	−−+0	−+−+	00−−	0−00
−+0−	−+−+	−0+−	−+00	00−0	0−0−	−+0+	+−−−	+−−0
−++−	00−+	000−	−++0	+−−+	+0−−	−+++	+−−0	+−−−
0−−−	−0+0	−+0+	0−−0	−0+−	−00+	0−−+	−−+−	−0−+
0−0−	−0++	−+00	0−00	−−+0	−+−+	0−0+	0−+−	00−+
0−+−	−−++	−+−0	0−+0	0−++	0+−0	0−++	0−+0	0+−+
00−−	−+00	−0+0	00−0	−+0−	−−++	00−+	000−	0−0+
000−	−+0+	−++−	0000	0000	0000	000+	+−0−	+−−+
00+−	000+	0+0−	00+0	+−0+	++−−	00++	+−00	+0−0
0+−−	0+−0	0−+−	0+−0	0+−−	0−+0	0+−+	++−−	+−+0
0+0−	0+−+	00+−	0+00	++−0	+−+−	0+0+	+0−−	+−00
0++−	++−+	+0+−	0++0	+0−+	+00−	0+++	+0−0	+−0−
+−−−	−++0	−+++	+−−0	−++−	−0++	+−−+	00+−	000+
+−0−	−+++	−++0	+−00	00+0	0+0+	+−0+	+−+−	+0−+
+−+−	00++	0+00	+−+0	+−++	++−0	+−++	+−+0	++−+
+0−−	0++0	0+++	+0−0	0++−	00++	+0−+	+++−	+0++
+00−	0+++	0++0	+000	+++0	++++	+00+	+0+−	+00+
+0+−	++++	+++0	+0+0	+0++	++00	+0++	+0+0	++0+
++−−	0+00	00+0	++−0	0+0−	0−++	++−+	++0−	+−++
++0−	0+0+	0++−	++00	++00	+0+0	++0+	+00−	+−0+
+++−	++0+	+++−	+++0	+00+	++0−	++++	+000	+000

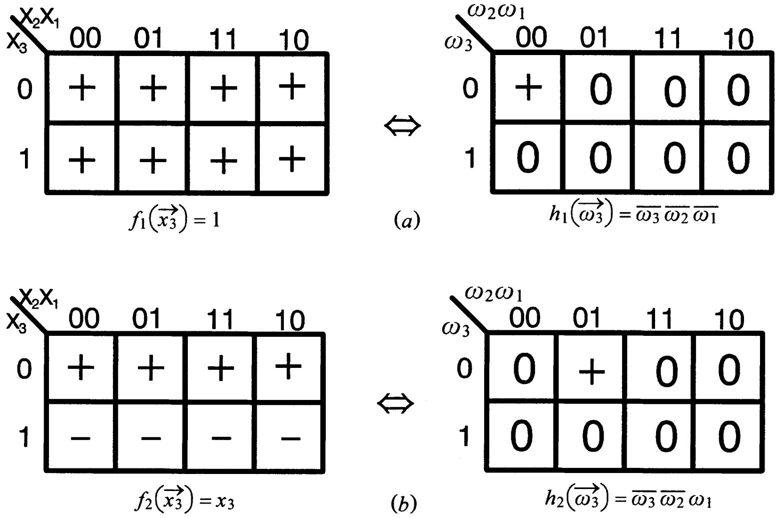

Fig. 2. Karnaugh and sign domain maps for S-coded functions f_1 and f_2.

inversion of the transform variable ω_k. *The meaning of the symbol* $\overline{\omega_k}$ *and the restriction on the value of* $\wedge_{k=n-j+2}^{n}\overline{\omega_k}$ *for some* j, *when* $k > n$ *is the same as above also for Properties* 6–8.

Example 3. For $n = 3$, when $f_2(\vec{x}_3) = x_3$, sign Haar transform is

$$h_2(\vec{\omega}_3) = +1\left(\omega_{3-3+1} \wedge \left\{\bigwedge_{k=3-3+2}^{3} \overline{\omega_k}\right\}\right) = +1(\omega_1 \wedge \overline{\omega_2} \wedge \overline{\omega_3}) = \overline{\omega_3}\;\overline{\omega_2}\omega_1$$

Hence $f_2(\vec{x}_3) = (+,+,+,+,-,-,-,-) \Longleftrightarrow h_2(\vec{\omega}_3) = (0,+,0,0,0,0,0,0)$.

The function $f_2(\vec{x}_3)$ and its corresponding spectrum are shown on the maps in Fig. 2(b).

Property 5. *When S-coded n-variable function is functionally dependent on a single Boolean variable in negation, i.e.,*

$$f(\vec{x}_n) = f(x_n, \ldots, x_1) = \bar{x}_j, \quad j \in \{1, \ldots, n\}, \; x_j \in \{+1, -1\}$$

sign Haar transform is

$$h(\vec{\omega}_n) = -1 \times \left(\omega_{n-j+1} \wedge \left\{\bigwedge_{k=n-j+2}^{n} \overline{\omega_k}\right\}\right) \tag{14}$$

Property 6. *For S-coded n-variable Boolean function* $f(\vec{x}_n)$ *whose sign Haar spectrum is* $h(\vec{\omega}_n)$, *the spectrum of the negated function is derived simply by inverting all the signs of the original spectra. Hence, when*

$$f(\vec{x}_n) \Longleftrightarrow h(\vec{\omega}_n) \quad \text{then} \quad \overline{f(\vec{x}_n)} \Longleftrightarrow -h(\vec{\omega}_n) \tag{15}$$

Property 7. *When R-coded n-variable function is functionally dependent on a single Boolean variable in affirmation, i.e.,*

$$f(\vec{x}_n) = x_j \quad \textit{where } j \in \{1, 2, \ldots, n\} \text{ and } x_j \in \{0, 1\}$$

Its sign Haar transform,

$$h(\vec{\omega}_n) = \bigwedge_{k=1}^{n} \overline{\omega_k} - \left(\omega_{n-j+1} \wedge \left\{ \bigwedge_{k=n-j+2}^{n} \overline{\omega_k} \right\} \right) \tag{16}$$

where $\omega_k \in \{0, 1\}$ *and the logical AND operation in the bracket ( ) will yield value* 1 *or* 0.

Example 4. For $n = 4$, when $f_3(\vec{x}_4) = f_3(x_4, x_3, x_2, x_1) = x_2$ then by Eq. (11) sign Haar transform is

$$h_3(\vec{\omega}_4) = \bigwedge_{k=1}^{4} \overline{\omega_k} - \left(\omega_3 \wedge \left\{ \bigwedge_{k=4}^{4} \overline{\omega_k} \right\} \right) = \overline{\omega_4}\,\overline{\omega_3}\,\overline{\omega_2}\,\overline{\omega_1} - \overline{\omega_4}\omega_3$$

Hence,

$$f_3(\vec{x}_4) = (0, 0, +, +, 0, 0, +, +, 0, 0, +, +, 0, 0, +, +) \Longleftrightarrow h_3(\vec{\omega}_4)$$
$$= (+, 0, 0, 0, -, -, -, -, 0, 0, 0, 0, 0, 0, 0, 0)$$

Property 8. *When R-coded n-variable function is functionally dependent on a single Boolean variable in negation, i.e.,*

$$f(\vec{x}_n) = \overline{x_j} \quad \textit{where } j \in \{1, \ldots, n\} \textit{ and } x_j \in \{0, 1\}$$

Then sign Haar transform,

$$h(\vec{\omega}_n) = \bigwedge_{k=1}^{n} \overline{\omega_k} + \left(\omega_{n-j+1} \wedge \left\{ \bigwedge_{k=n-j+2}^{n} \overline{\omega_k} \right\} \right) \tag{17}$$

5. Conclusion

The essential relationships between classical (Karnaugh maps, logic functions and their variables) and spectral (sign Haar recursive expansions, fast transforms) representations of binary and ternary networks have been stated. It should be noticed that sign Haar transform can be applied to both completely and incompletely specified Boolean functions. Hence, the essential properties of sign Haar spectra of logic functions and variables shown in this article are important to facilitate computer-aided design processing for such logic networks. Similarly to Haar transform used in logic design, sign Haar spectrum is based on local basis functions and is especially well suited to spectral processing of weakly defined locally grouped multi-variable incompletely specified Boolean functions [3]. In order to calculate sign Haar spectrum in an efficient way, it is possible to modify the procedure to convert disjoint cube representation shown for sign Walsh transform [7] to calculate sign Haar spectrum. It is also possible to modify the results presented for efficient calculation of Haar spectrum [3,8,13,14] to sign Haar spectrum. With the use of sign Haar spectra, the presented results may also be useful for ternary digital communication systems which operate on streams of ternary data that with recoding are equivalent to incompletely specified Boolean functions. An important property of all quantized transforms is that the computer memory required to store functional and spectral data is exactly the same since both of them operate on ternary values. This highlights the main advantage of the quantized transforms over the traditional spectral methods in digital logic design.

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