General characteristic of logic synthesis methods for reversible logic

Very little has been published

Sasao and Kinoshita - Cascade circuits - ***small garbage , high delay***

Picton - binary and multiple-valued PLAs, ***high garbage, high delay, high gate cost***

Toffoli, Fredkin, Margolus - ***examples of good circuits, no systematic methods***

De Vos, Kerntopf - new gates and their properties, ***no systematic methods***

Knight, Frank, De Vieira, Athas, Svenson - circuit design, ***no systematic methods***

Joonho Lim, Dong-Gyu Kim and Soo-Ik Chae
School of Electrical Engineering, Seoul National University- circuit design, ***no systematic methods***

- We introduce regular structures to realize arbitrary functions.

- **Controlled-Controlled NOT**

= **Toffoli Gate**

The Toffoli Gate

- The Toffoli gate $Q^{(3)}$ is universal in the sense that we can build a circuit to compute any reversible function using Toffoli gates alone (if we can x input bits and ignore output bits).
- It will be instructive to show this directly, without relying on our earlier argument that NAND/NOT is universal for Boolean functions.
- In fact, we can show the following:
  - ***From the NOT gate and the Toffoli gate $Q^{(3)}$, we can construct any invertible function on n bits, provided we have one extra bit of scratchpad space available***.

Use of Toffoli Gate

- From three-bit Toffoli-Gate $Q^{(3)}$

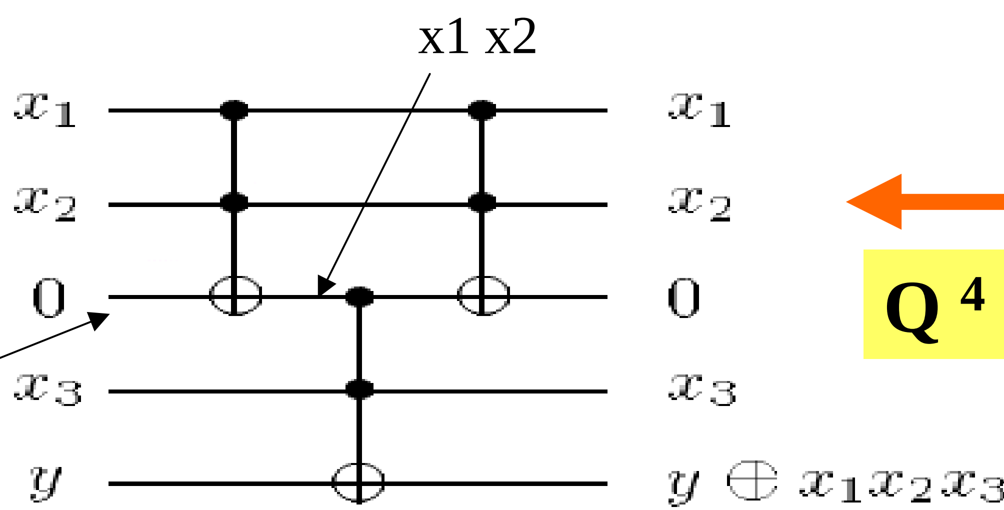
The first step is to show that from the three-bit Toffoli Gate Q $^{(3)}$ we can construct an n-bit Toffoli Gate Q $^{(n)}$.

The n-bit gate works as follows:

$$\mathbf{(x_1,x_2,\ldots,x_{n-1}, x_n) ==> (x_1,x_2,\ldots,x_{n-1}\ y@x_1\ x_2\ldots x_{n-1}\ )}$$

The construction requires one extra bit of scratch space.

For example, we construct Q $^{(4)}$ circuit from Q $^{(3)}$ circuits as follows:



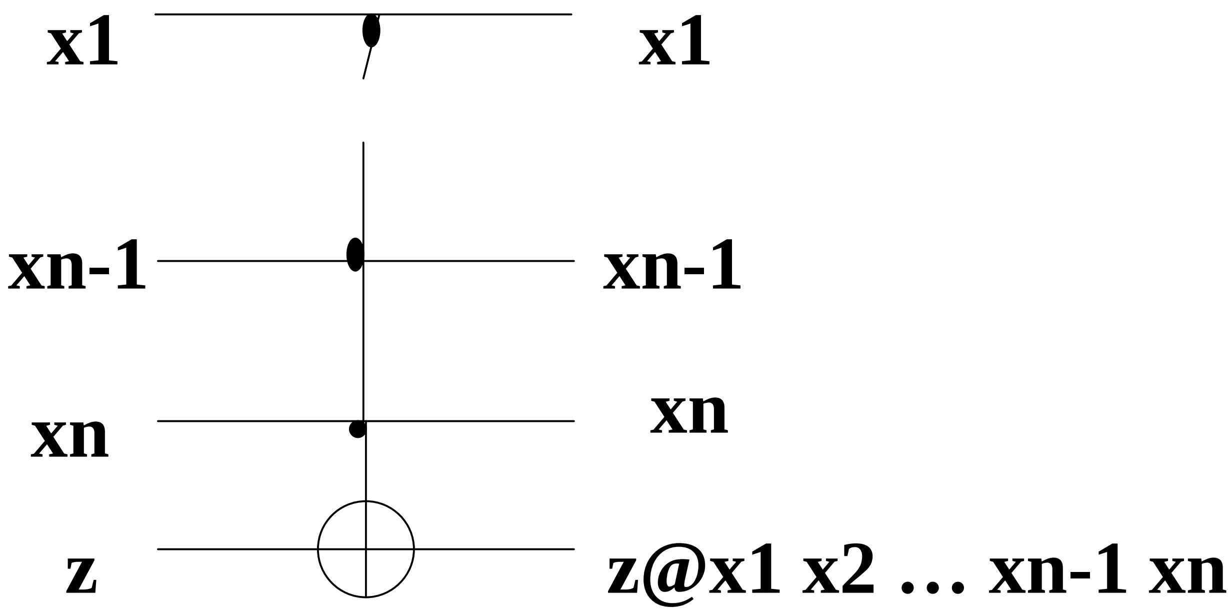
Scratch space

Q ^{4}

Q ^{4} from three Q ^{3}

Simple Idea

- If we generalize the Toffoli Gate, we can realize any binary function in a very efficient way



Food for Thought!

One can build Toffoli gate with 3 inputs

Can one build Toffoli gate with n inputs??????

Of course, from many gates, but directly???

Simple Idea

Karnaugh Maps

- A 4-variable K-map.

WX \ YZ	00	01	11	10
00	0 **0**	1 **0**	3 **1**	2 **0**
01	4 **0**	5 **0**	7 **1**	6 **0**
11	12 **1**	13 **1**	15 **0**	14 **1**
10	8 **0**	9 **0**	11 **1**	10 **0**

Simple Idea

SOP Cover

WX \ YZ	00	01	11	10
00	0 **0**	1 **0**	3 **1**	2 **0**
01	4 **0**	5 **0**	7 **1**	6 **0**
11	12 **1**	13 **1**	15 **0**	14 **1**
10	8 **0**	9 **0**	11 **1**	10 **0**

Simple Idea

ESOP = Positive RM cover

F=wx @ yz

WX \ YZ	00	01	11	10
00	0 **0**	1 **0**	3 **1**	2 **0**
01	4 **0**	5 **0**	7 **1**	6 **0**
11	12 **1**	13 **1**	15 **0**	14 **1**
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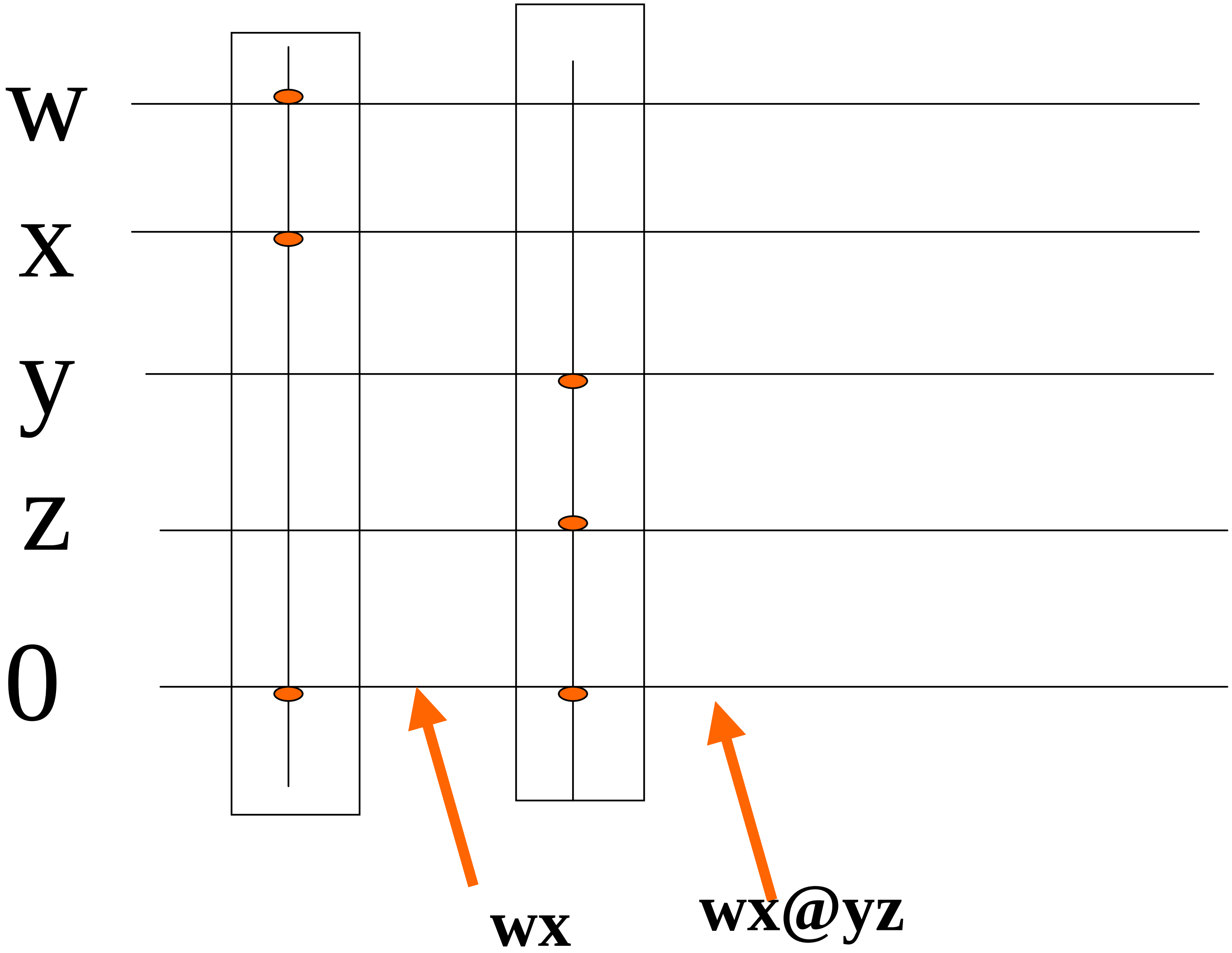
Simple Idea



Simple Idea

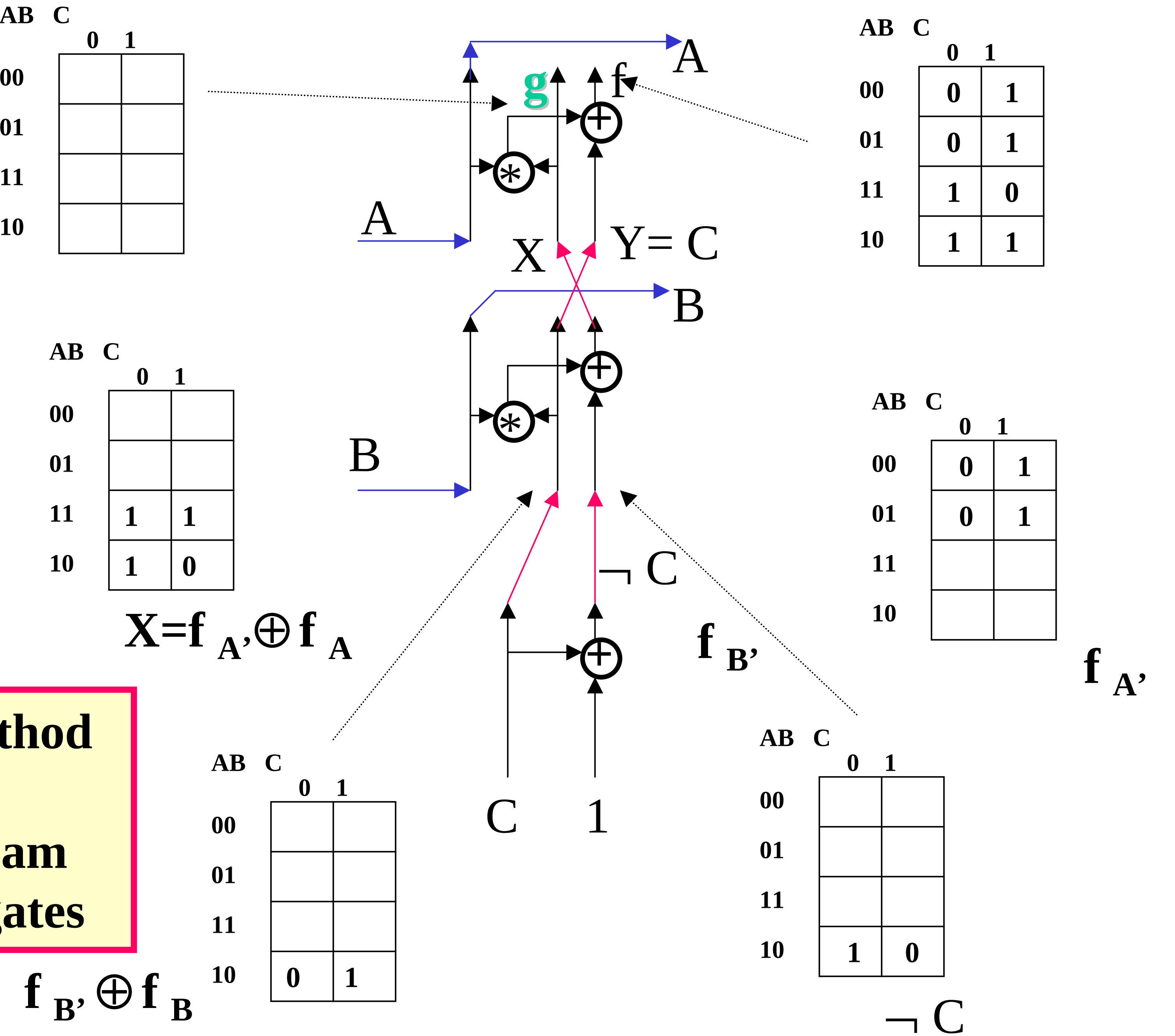
- **This idea can be generalized to:**
  - **Fixed Polarity Reed-Muller Expansions**
  - **Generalized Reed-Muller Expansions**
  - **Exclusive Sum of Products**
  - **Galois Sum of Galois Products Expansions**
  - **Boolean Ring based logic**
  - **Min-Modsum based logic**
  - **Any Quasi-Group based logic**
  - **Arithmetic Logic**

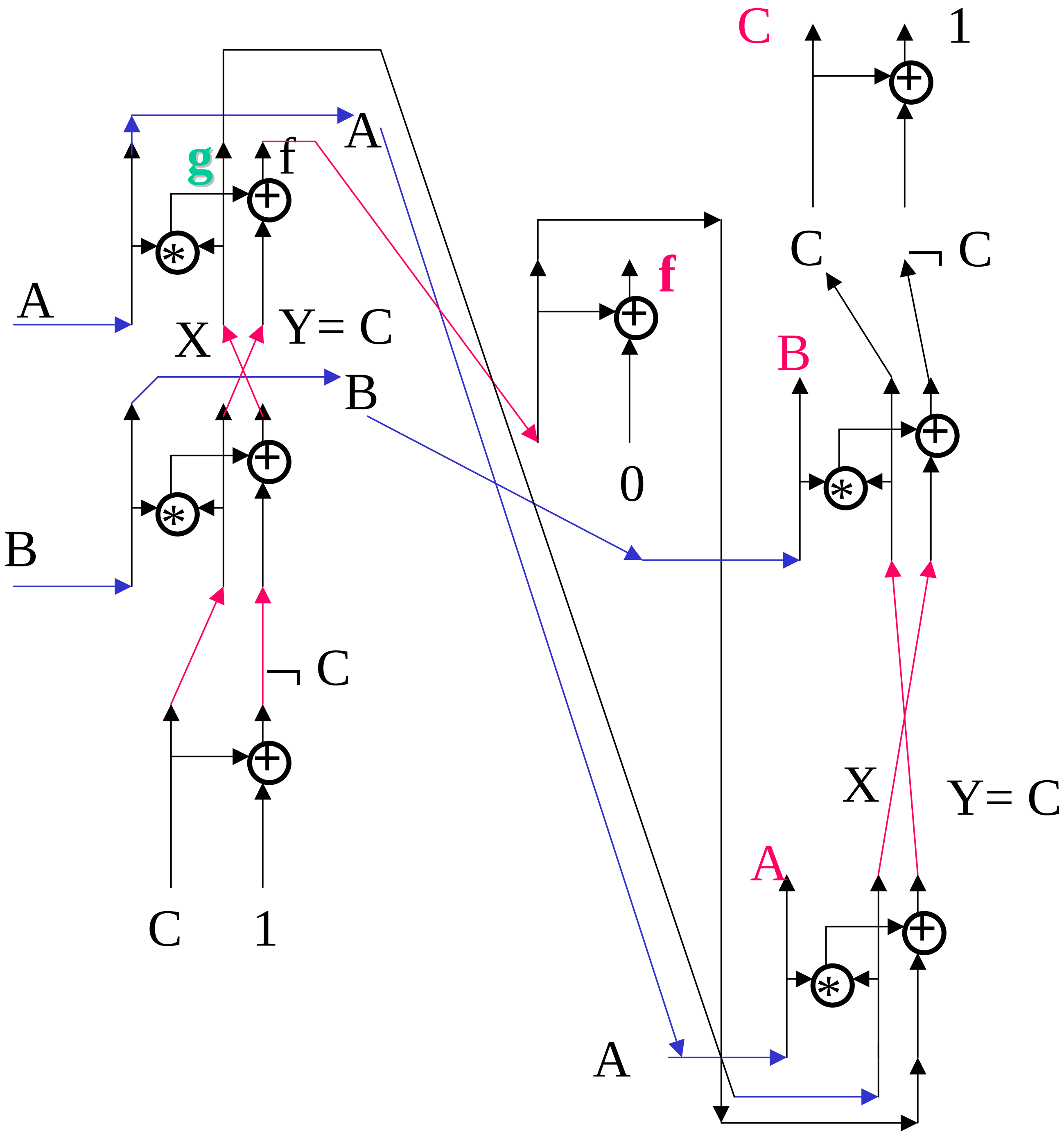
Realizations of binary logic with Toffoli and reversible logic with Toffoli-like circuits

- Kronecker functional Diagram
- Kronecker function-driven Diagram
- ESOP
- Kronecker Lattice Diagram
- PPRM-like forms
- other canonical

Reversible Fuzzy Diagrams

Graphical method to calculate decision diagram from Toffoli gates



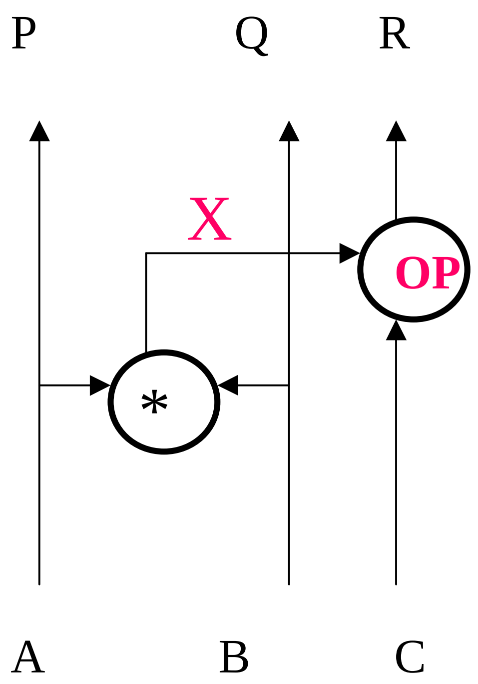


C
1
A
g
f
A
X
Y= C
B
B
¬ C
C
1
f
0
C
¬ C
B
X
Y= C
A
A

Fuzzy Reversible Logic using Toffoli-like gates

Building reversible fuzzy Toffoli gate

- We have to design such gate that knowing P,Q,R we will be able to find unique values of A,B and C.
- If we know values of P and Q we can find uniquely the value of X.
- We need now to find a fuzzy operator that can find in an uniqe way C from X and R and that will make the whole gate reversible.
- Observe that this must be continuous operator, but similar in operation to modulo, XOR or Latin Square.

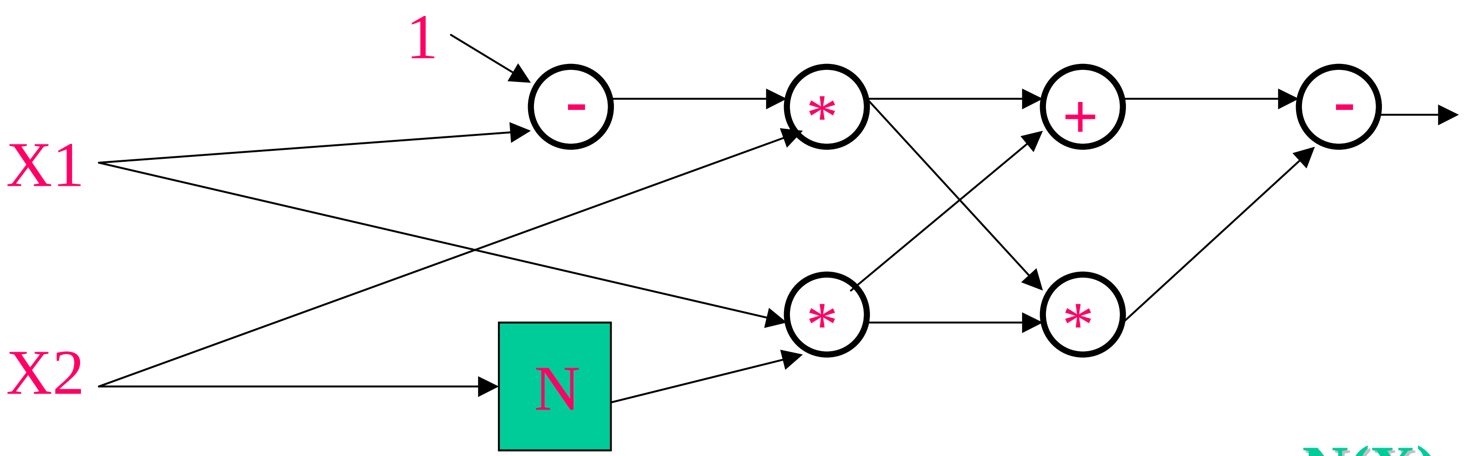


Assume arithmetic addition

What should be the OP?

Solution to OP

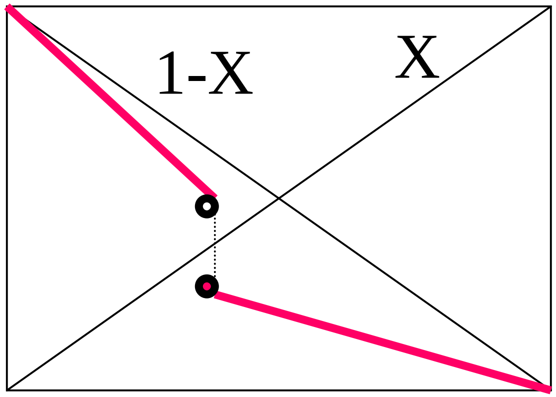
- Several solutions to OP are discussed in R. Rovatti, G. Baccarani, “Fuzzy Reversible Logic”, *Proc. 1998 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE'98)*

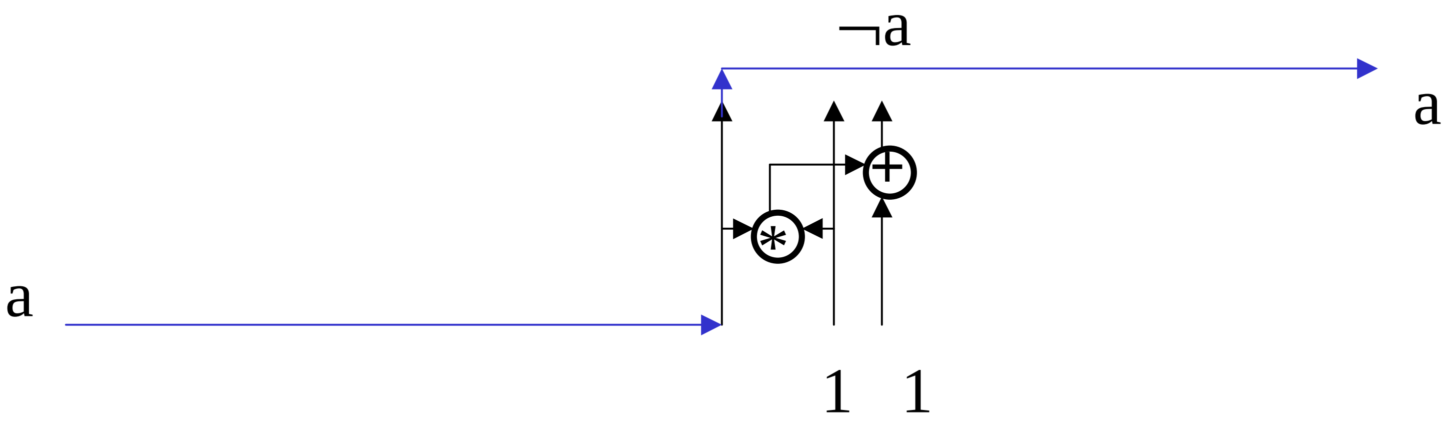


Operations * , - and + are arithmetic

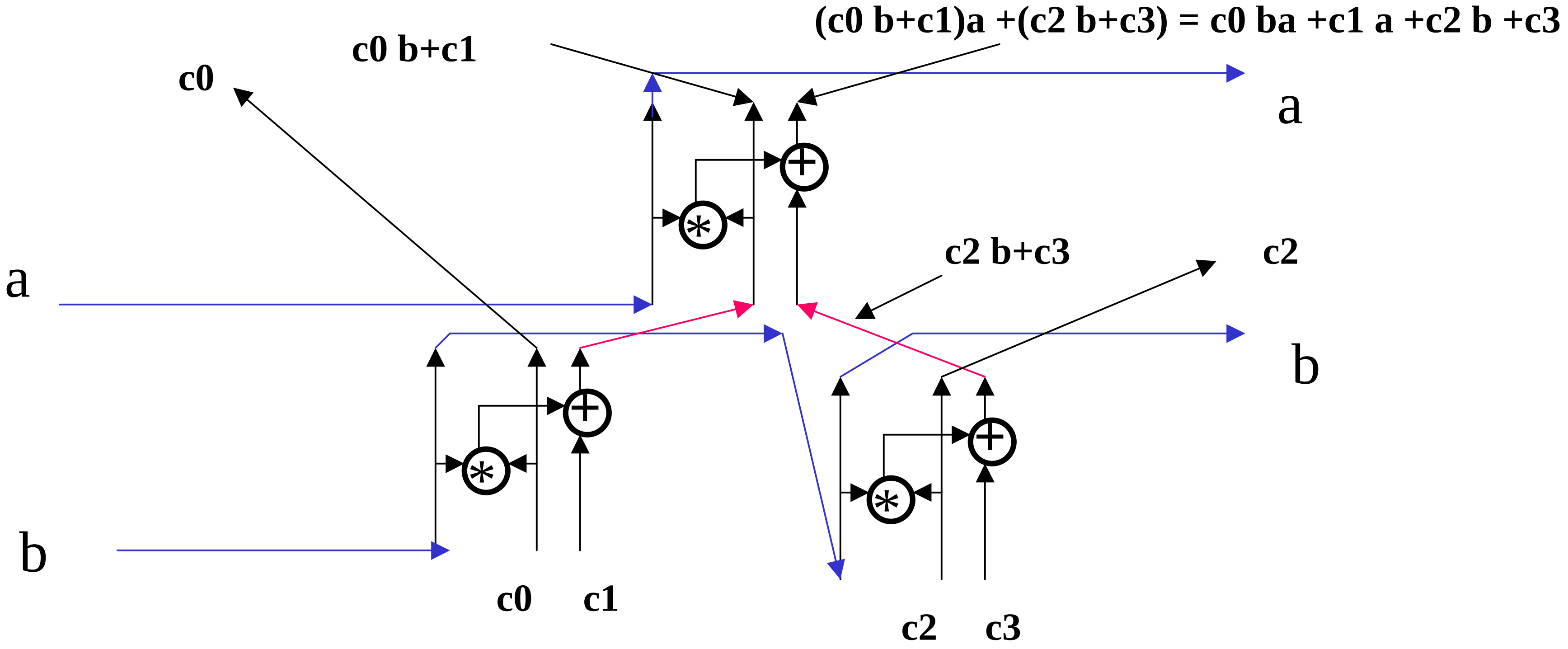
Generalized inverter **N** needs discontinuity

N(X)



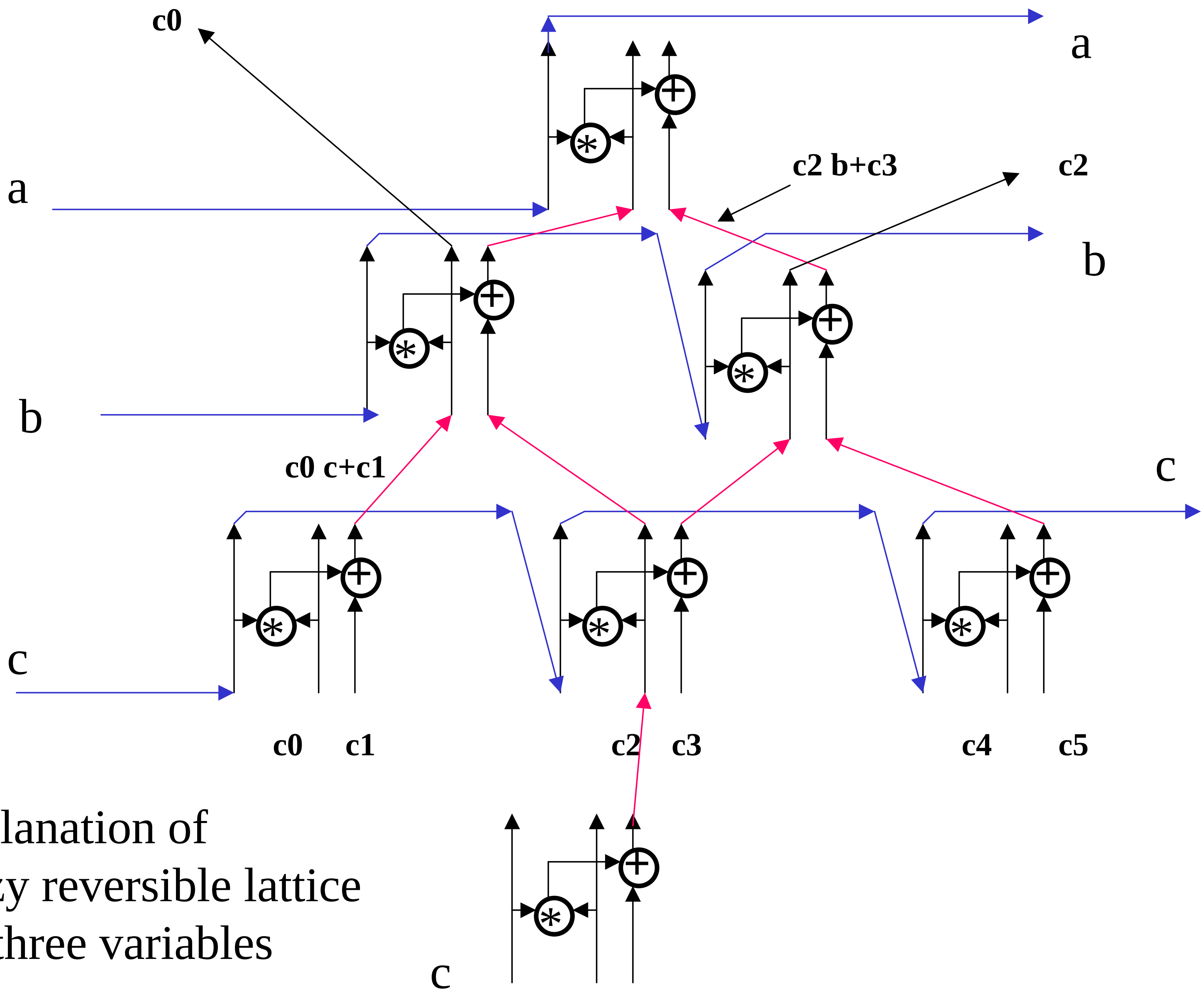


Now we can generate all fuzzy functions of a single variable



Realization of Positive Polarity Reed-Muller for functions of two variables

The same circuit for fuzzy reversible functions of two variables



Explanation of
fuzzy reversible lattice
for three variables

Kerntopf Gate and Regular Structures for Symmetric Functions

Kerntopf Gate

- The Kerntopf gate is described by equations:

  P = 1 @ A @ B @ C @ AB,

  Q = 1 @ AB @ B @ C @ BC,

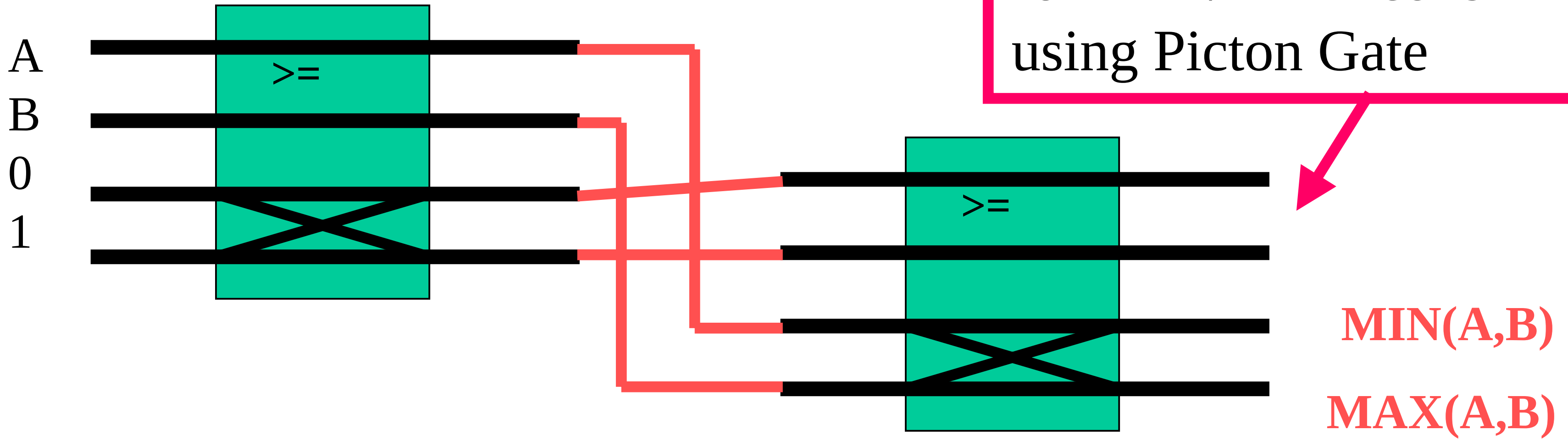
  R = 1 @ A @ B @ AC.

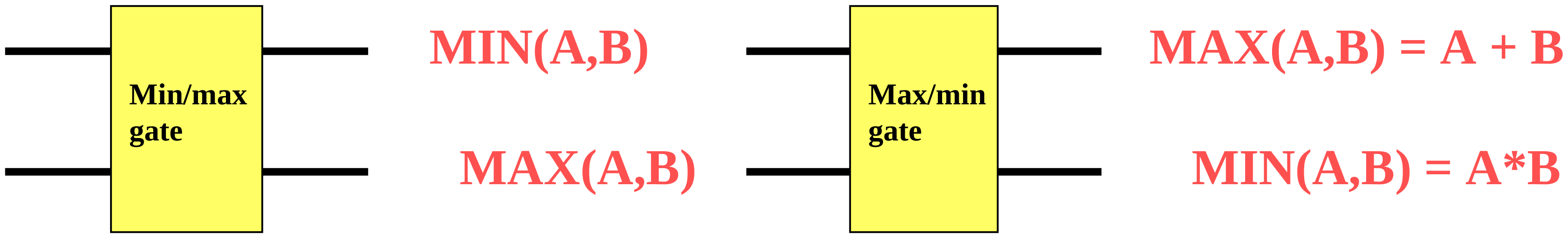
- When ***C=1*** then ***P = A + B, Q = A * B, R = !B,*** so ***AND/OR*** gate is realized on outputs ***P*** and ***Q*** with ***C*** as the controlling input value.
- When ***C = 0*** then ***P = !A * ! B, Q = A + !B, R = A @ B.***

Despite theoretical advantages of Kerntopf gate over classical Fredkin and Toffoli gates, so far there are no published results on optical or CMOS realizations of this gate.

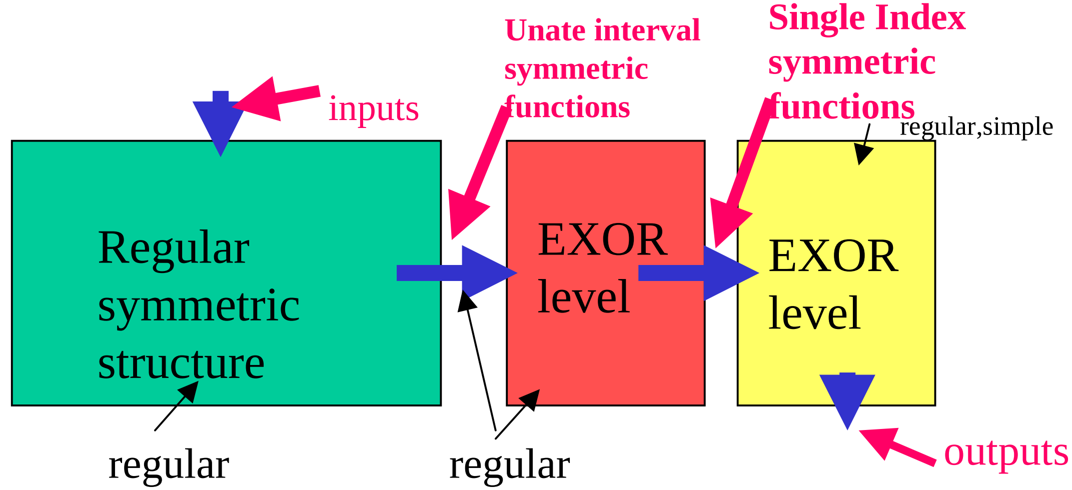
Use of two Multi-valued Fredkin (Picton) Gates to create MIN/MAX gate

Two garbage outputs for MIN/MAX cells using Picton Gate

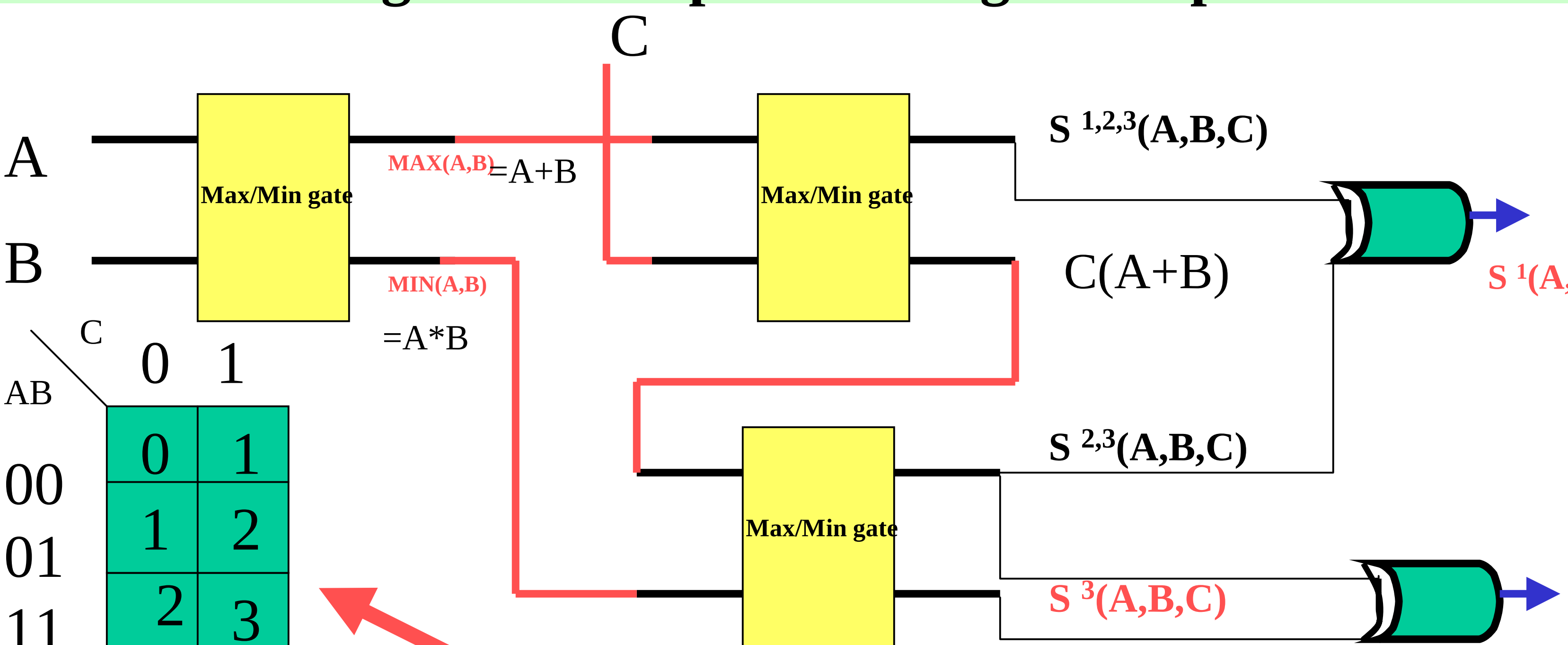




Regular Structure for Symmetric Functions



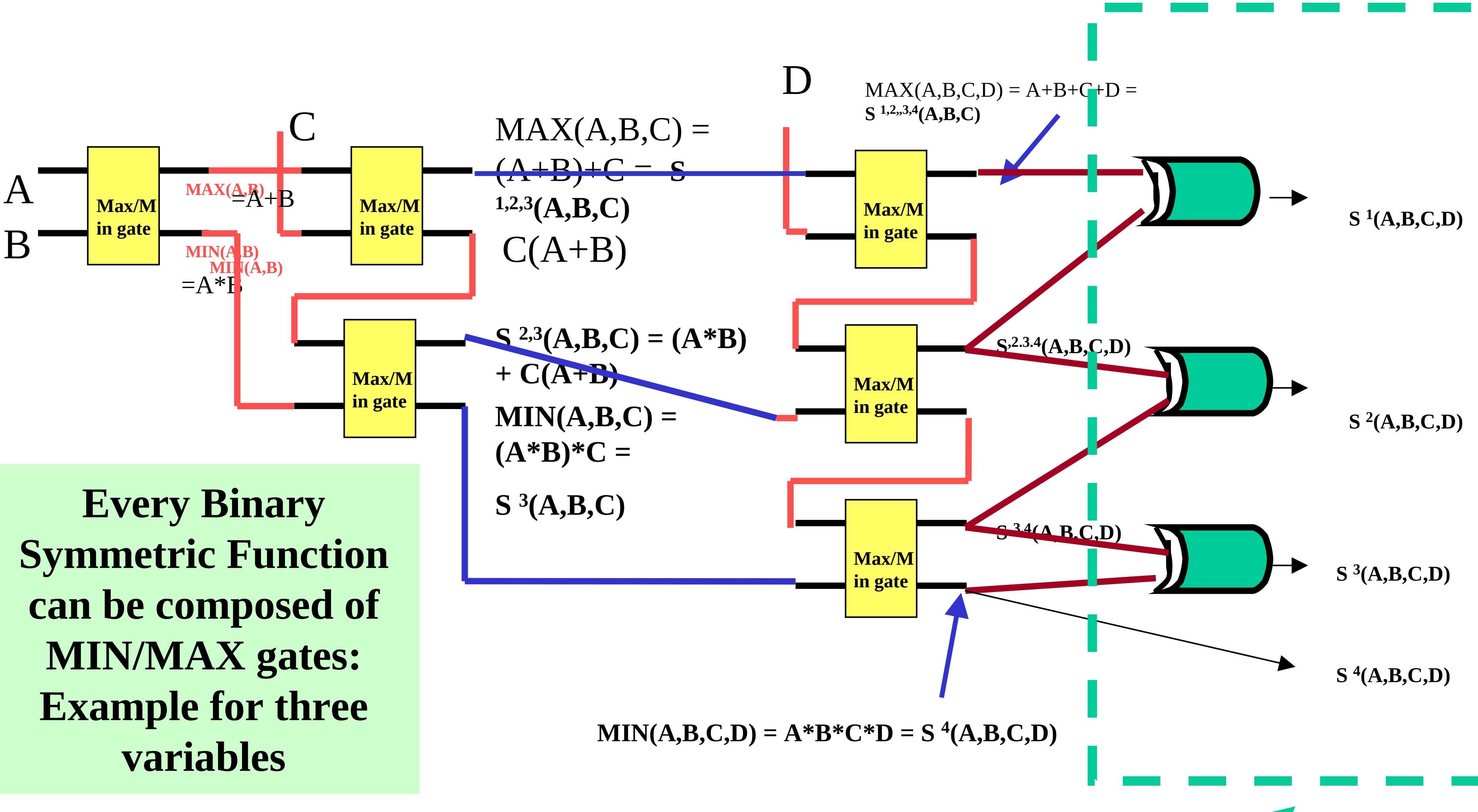
Every single index Symmetric Function can be created by EXOR-ing last level gates of the previous regular expansion structure



AB \ C	0	1
00	0	1
01	1	2
11	2	3
10	1	2

Indices of symmetric binary functions of 3 variables

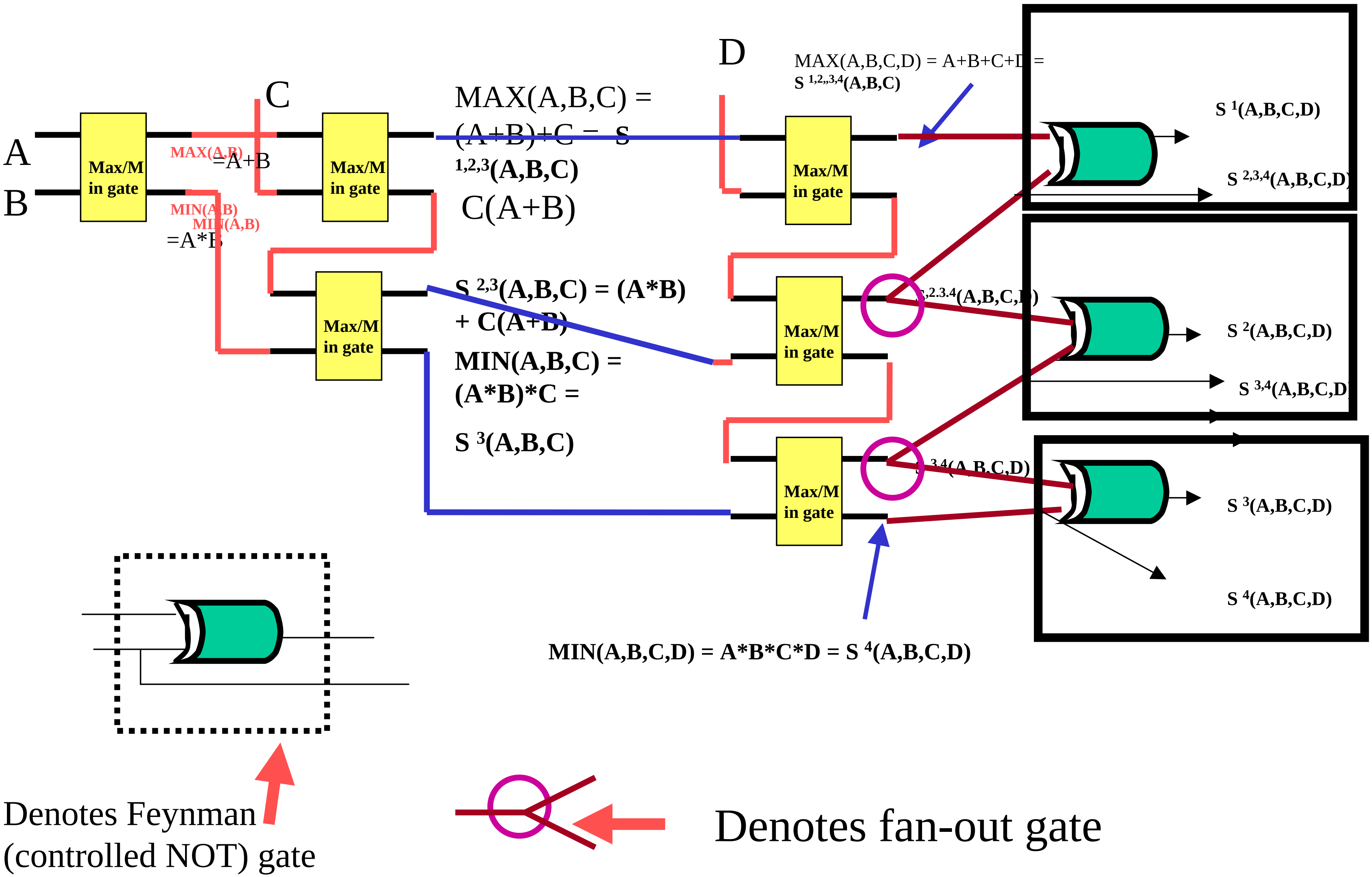
Example for four variables, EXOR level added



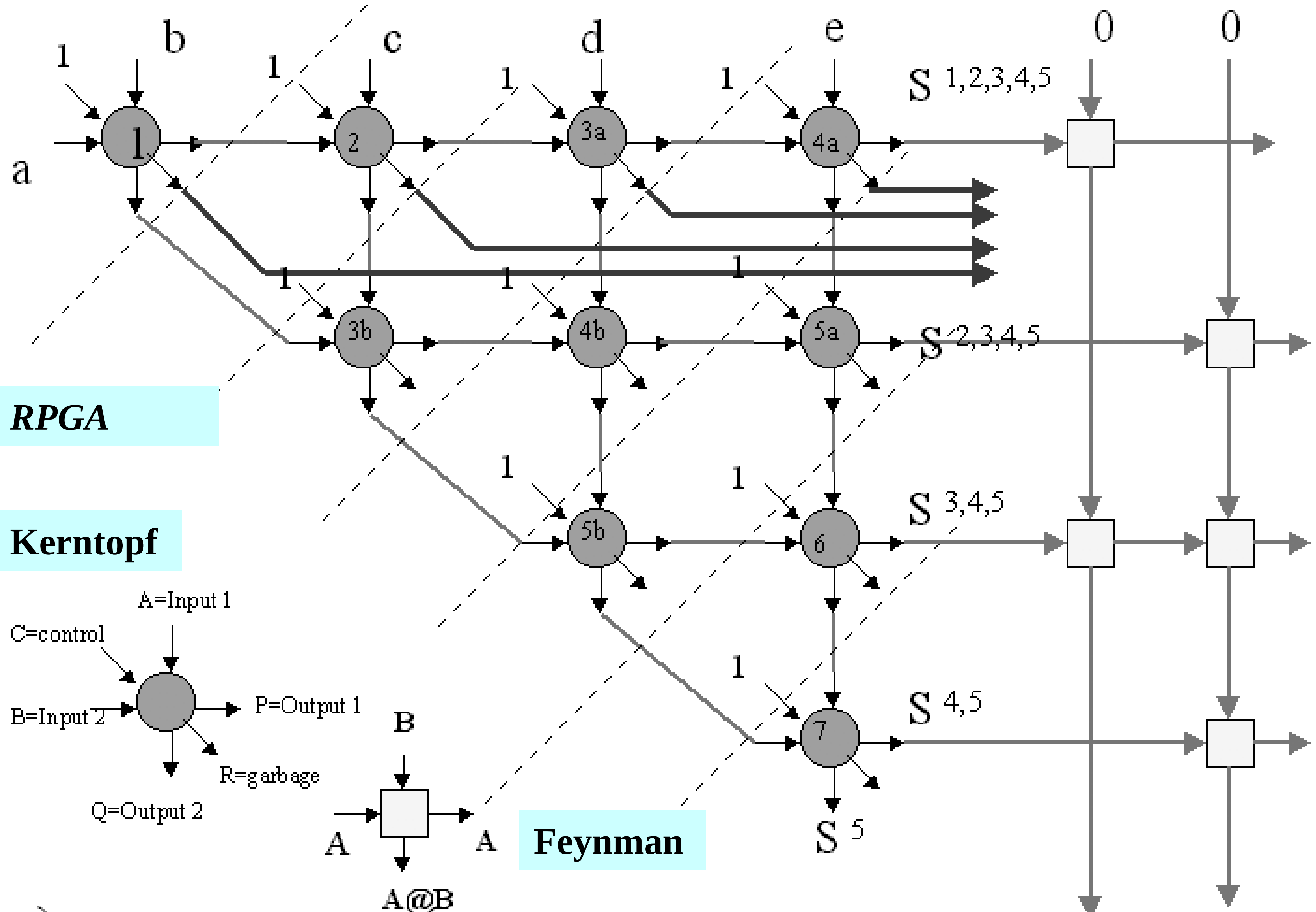
Every Binary Symmetric Function can be composed of MIN/MAX gates: Example for three variables

It is obvious that any multi-output function can be created by OR-ing the outputs of **EXOR level**

Now we extend to Reversible Logic



Using Kerntopf and Feynman Gates in **Reversible** ***Programmable Gate Array***



Arbitrary symmetric function can be created by exoring single indices

Generalizations and Current Work

- Arbitrary symmetric function can be realized in a net without repeated variables.
- Arbitrary (non-symmetric) function can be realized in a net with ***repeated variables*** (so-called ***symmetrization***).
- Many non-symmetric functions can be realized in a net without ***repeated variables***.
- We work on the characterization of the functions realizable in these structures without repetitions and respective synthesis algorithms.

Very many new circuit types, which are reversible and multi-valued generalizations of Shannon Lattices, Kronecker Lattices, and other regular structures introduced in the past.

- *Layout-driven synthesis* to regular structures
- **CMOS**, Optical, **Quantum dot** technologies.
- Software