ME 441/541, Fall 2004, Wed. Oct. 6

Homework #2, approx. due Wed. Oct. 13

Problems: Asymptotic Expansions (see lecture notes)

1. For the problem of the oscillating liquid column where viscous forces are small have

$$h_{tt} + B\,h_t + h = 0 \tag{1}$$

$$h(0) = -1 \quad h_t(0) = 0\,.$$

By expanding $h = h_0 + B\,h_1 + B^2\,h_2 + O(B^3)$, solve eq. (1) to $O(B^2)$ and compare to exact solution

$$h = -\exp\left[-\frac{B}{2}t\right]\left(\cos\omega t + \frac{B}{2\omega}\sin\omega t\right),$$

expanded to $O(B^2)$, where

$$\omega \equiv \left(1 - \frac{B^2}{4}\right)^{1/2}.$$

2. A capillary driven flow is modeled by the following equation for a slender fluid column of $H/L \equiv \varepsilon$ sketched below:

$$h_t = 2\,h_z^2 + h\,h_{zz} \tag{2}$$

$h(t,z)$ is the height of the meniscus measured along the bisector of the corner, z is the corner axis coordinate, and the remaining variables (α, θ, μ, σ) are folded into the time scale used to nondimensionalize the problem. The axial coordinate z is scaled by L and h is scaled by a known liquid height H. Eq. (2) is accurate to $O(\varepsilon)$, or in other words is appropriate when $\varepsilon^2 << 1$. It is reasonable to assume a leading order solution form for h

$$h = h_0 + \varepsilon\,h_1 + O(\varepsilon^2)\,. \tag{3}$$

An important problem is to determine the settling time for a disturbance to a stationary liquid column in the corner. Thus the 'base state' for this 'perturbation' might be a solution of constant height. Picking $h_0 = const. = 1$ in eq. (3) solve eq. (2) to $O(\varepsilon)$ and determine the characteristic settling time for a disturbance of dimensional wavelength $2L$.