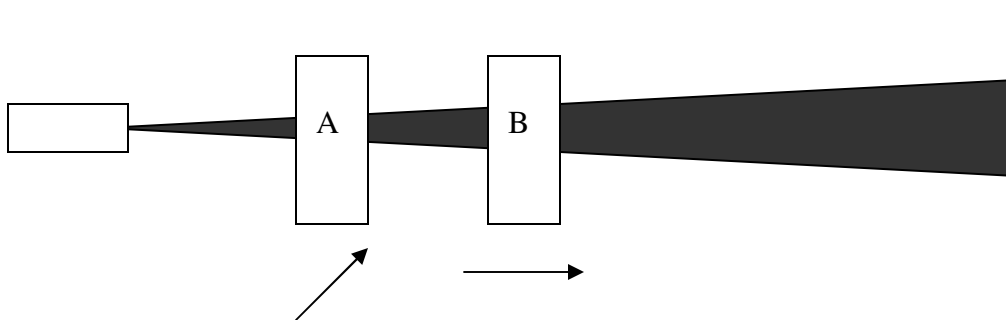


CS 410/510
Nonstandard Computation
Spring, 2008

Homework 3: Quantum Computation

Due Tuesday, May 13

1. Consider the photon polarization experiment described in Section 2.1 of the Rieffel and Polak paper. Suppose the experiment is altered as pictured below, with filter A being polarized at 45 degrees to the horizontal, and filter B being polarized horizontally. The light reaches a screen after travelling through both filters.



What fraction of the original light will appear on the screen? Why?

2. Consider the quantum key distribution scheme described in Section 3.1 of the Rieffel and Polak paper. (a) Explain in a short paragraph or two how Alice and Bob can detect an eavesdropper (Eve), and why this detection scheme will work. (b) Explain why Eve can't defeat the scheme by intercepting (and changing when necessary) the parity bits used by Alice and Bob to detect errors.

3. Let

$$\varphi = \frac{1}{2}(|0001\rangle + |0011\rangle + |1111\rangle + |1100\rangle)$$

be a quantum state. Suppose you measure the first two qubits of this state (but not the last two), and get the value 11. What is φ at this point?

4. Construct a quantum circuit, using combinations of the Toffoli (controlled-controlled not) gate, to implement the XOR function on two qubits. Give your gate array in a format similar to that of the 1-bit full adder on page 20 of the Rieffel and Polak paper.

5. For this problem, you will have to understand the example of quantum error correction given in Section 8.3 of the Rieffel and Polak paper. Suppose you have a quantum bit

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),$$

and you encode it with the error correcting code

$$C|\psi\rangle = |\phi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle).$$

Now, suppose it is subject to the error

$$E = \frac{3}{5}(X \otimes I \otimes I) + \frac{4}{5}(I \otimes I \otimes X).$$

Write down the steps, analogous to those in the

paper, that show how you recover the original state $|\phi\rangle$ from $E|\phi\rangle$.

6. **Extra Credit:** Computer exercise: Implement a simulation of the quantum coin-flipping protocol described in class. (Also described in section 3.1 of G. Brassard and C. Crépeau, Quantum Bit Commitment and Coin Tossing Protocols, URL on class web page.)

Let \mathbf{B} (random vector of bits) be of length 20.

(a) Simulate 10 quantum coin flips and show that the protocol works when neither party cheats. For each coin flip, report b (the value of the coin flip), \mathbf{B} (Alice's vector of random bits), θ , Bob's vector of random bases, \mathbf{B}' , Bob's measurements of \mathbf{B} according to θ , b' , Bob's choice of heads or tails, and whether or not Bob accepts the outcome.

(b) Simulate 10 quantum coin flips in which Alice cheats (the protocol is as in part (a), except before sending b to Bob, she sets it to the opposite value of Bob's choice b'). Report the same things as requested in part (a).

Details will be discussed in class.