1. Given the training set plotted below: Sketch the hyperplane (i.e., line) that maximally separates the two classes (open circle and solid circle). Also sketch a line that indicates the margin, and label it as “margin”. Also circle the support vectors.

![Margin Diagram](image)

2. Answer the following in one or two sentences.

(1) What are the inputs to the SVM learning algorithm?

**Training examples**

(2) What does the SVM learning algorithm output?

**Support vectors, \( \alpha_i \) for each support vector \( i \), bias**

3. Consider the following three points, \( x_1, x_2, \) and \( x_3 \), which have been identified as support vectors for a training set, along with their corresponding \( \alpha \) values.

\[
\begin{align*}
\alpha_1 &= -4 \\
\alpha_2 &= -4 \\
\alpha_3 &= 8
\end{align*}
\]

The bias is \( b = 0 \).
(a) Using the formula

\[ h(x) = \text{sgn}\left( \sum_{i=1}^{m} \alpha_i (x_i \cdot x) + b \right). \]

give the classification of the new instance \( x = (1, 2) \).

\[ h(1,2) = \text{sgn}\left[ -4(2,1) \cdot (1,2) - 4(4,3) \cdot (1,2) + 8(2,3) \cdot (1,2) \right] \]
\[ = \text{sgn}[-4 \cdot 4 - 4 \cdot 1 + 8 \cdot 8] = \text{sgn}[8] = 1 \]

(b) What is the decision value for this instance?

8

(c) Use the \( x_i \)'s and \( \alpha_i \)'s to find the weight vector \( w \) associated with the separating hyperplane, where \( w = \sum_i \alpha_i x_i \).

\[ w = -4(2,1) - 4(4,3) + 8(2,3) = (-8, 8) \]

(d) Using the weight vector you obtained in part (c) and the bias \( b = 0 \), find the equation of the separating hyperplane. Give the equation in the slope-intercept form: \( x_2 = (\text{slope} \cdot x_1) + y\text{-intercept} \).

\[-8x_1 + 8x_2 = 0 \]
\[ x_2 = x_1 \]

(e) Using the graph below, plot the support vectors and the separating hyperplane. Also draw a line to show the margin. Finally, plot the new point \( (1, 2) \) from part (a) to confirm that it is in the class you found in part (a).
4. Consider the following training set:

<table>
<thead>
<tr>
<th>Instance</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$x_2$</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

and let $k(y, z)$ be the polynomial kernel: $k(y, z) = ((y \cdot z) + 1)^2$. Give the kernel (Gram) matrix for this kernel function using this training set.

$$K = \begin{pmatrix} k(x_1, x_1) & k(x_1, x_2) & k(x_1, x_3) \\ k(x_2, x_1) & k(x_2, x_2) & k(x_2, x_3) \\ k(x_3, x_1) & k(x_3, x_2) & k(x_3, x_3) \end{pmatrix}$$

$$= \begin{pmatrix} ((1,0)\cdot(1,0)+1)^2 & ((1,0)\cdot(-1,-1)+1)^2 & ((1,0)\cdot(0,1)+1)^2 \\ ((-1,-1)\cdot(1,0)+1)^2 & ((-1,-1)\cdot(-1,-1)+1)^2 & ((-1,-1)\cdot(0,1)+1)^2 \\ ((0,1)\cdot(1,0)+1)^2 & ((0,1)\cdot(-1,-1)+1)^2 & ((0,1)\cdot(0,1)+1)^2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 & 1 \\ 0 & 9 & 0 \\ 1 & 0 & 4 \end{pmatrix}$$