1. (a) Give weights for a perceptron that can correctly classify the examples in the following dataset (there are many possible correct answers).

<table>
<thead>
<tr>
<th>Example</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**SOLUTION:**

One possible answer:

$$w_0 = -.6, w_1 = .5, w_2 = .5, w_3 = .5$$

(b) Give the equation of the separating hyperplane obtained from your weights.

**SOLUTION:**

$$.5x_1 + .5x_2 + .5x_3 - .6 = 0$$

2. Consider the two-layer neural network given below. The input layer consists of two input nodes and one bias node. The activation function at the hidden and output layers is the sigmoid function:

$$o = \sigma(w \cdot x), \quad \text{where} \quad \sigma(z) = \frac{1}{1 + e^{-z}}$$

The weight value for each connection is 1.0, as shown in the figure below.
Suppose the input \((x_1, x_2) = (1, -1)\) is given to the network. What is the resulting activation at the output node \(o_1\)? Show your work.

**SOLUTION:**

\[
\begin{align*}
    h_1 &= \sigma(1 - 1 + 1) = \sigma(1) = .731 \\
    h_2 &= \sigma(1 - 1 + 1) = \sigma(1) = .731 \\
    o_1 &= \sigma(.731 + .731) = \sigma(1.462) = .812
\end{align*}
\]
3. The following data was collected on several different days, along with whether people rated the day as “nice” or “not nice”. Temperature can be either “warm” or “cold”, Precipitation can be either “low” or “high”, and Foggy can either be “yes” or “no”.

<table>
<thead>
<tr>
<th>Day</th>
<th>Temperature</th>
<th>Precipitation</th>
<th>Foggy</th>
<th>Rating (Class)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>warm</td>
<td>low</td>
<td>no</td>
<td>nice</td>
</tr>
<tr>
<td>D2</td>
<td>cold</td>
<td>low</td>
<td>no</td>
<td>nice</td>
</tr>
<tr>
<td>D3</td>
<td>warm</td>
<td>high</td>
<td>no</td>
<td>nice</td>
</tr>
<tr>
<td>D4</td>
<td>cold</td>
<td>high</td>
<td>yes</td>
<td>nice</td>
</tr>
<tr>
<td>D5</td>
<td>cold</td>
<td>high</td>
<td>no</td>
<td>not nice</td>
</tr>
<tr>
<td>D6</td>
<td>cold</td>
<td>low</td>
<td>yes</td>
<td>not nice</td>
</tr>
</tbody>
</table>

Suppose now you are given a new description of a day:

D7  warm  high  yes  ?

How would a Naive Bayes classifier, trained on D1-D6, classify D7? Use probabilities with Laplace smoothing. Show your work on how to calculate this.

SOLUTION:

\[
P(nice) = \frac{4}{6} \quad P(not \ nice) = \frac{2}{6}
\]

\[
P(T = warm | nice) = \frac{2}{4} \rightarrow \frac{3}{6} \quad P(T = warm | not \ nice) = \frac{0}{2} \rightarrow \frac{1}{4}
\]

\[
P(P = high | nice) = \frac{2}{4} \rightarrow \frac{3}{6} \quad P(P = high | not \ nice) = \frac{1}{2} \rightarrow \frac{2}{4}
\]

\[
P(F = yes | nice) = \frac{1}{4} \rightarrow \frac{2}{6} \quad P(F = yes | not \ nice) = \frac{1}{2} \rightarrow \frac{2}{4}
\]

nice: \[\frac{4}{6} \left(\frac{2}{3}\right) \left(\frac{3}{6}\right) \left(\frac{2}{3}\right) = 0.056\]

not nice: \[\frac{2}{6} \left(\frac{1}{4}\right) \left(\frac{3}{4}\right) \left(\frac{2}{4}\right) = 0.021\]

Class = nice
4. Robot R is shown in the grid below. Some squares have black circles and some are empty. She can sense the contents of her current cell as well as the cell directly south. The contents are either “empty”, “circle”, or “wall”. (Note that her current cell cannot be “wall”.)

For example, her current state is:
**Current:** empty
**South:** circle

Her possible actions are MoveNorth, MoveSouth, PickUpCircle.

She receives a reward of 10 for picking up a circle, and –5 for crashing into a wall. Once a circle is picked up, it disappears from the grid, and the square becomes empty.

(a) List all the possible states in R’s state space.

**SOLUTION**

<table>
<thead>
<tr>
<th>Current</th>
<th>South</th>
</tr>
</thead>
<tbody>
<tr>
<td>empty</td>
<td>empty</td>
</tr>
<tr>
<td>empty</td>
<td>wall</td>
</tr>
<tr>
<td>empty</td>
<td>circle</td>
</tr>
<tr>
<td>circle</td>
<td>empty</td>
</tr>
<tr>
<td>circle</td>
<td>circle</td>
</tr>
<tr>
<td>circle</td>
<td>wall</td>
</tr>
</tbody>
</table>

(b) Assume the Q table is initialized to all zeros. During episode 1, R performs the following four actions, updating the Q table after each action:

MoveSouth
PickUpCircle
MoveSouth
PickUpCircle

Given the update formula \( Q(s,a) \leftarrow Q(s,a) + \eta (r + \gamma \max_{a'} Q(s',a') - Q(s,a)) \), and setting \( \eta = 1 \) and \( \gamma = 0.9 \), give the non-zero entries of the Q table at the end of this episode.

**SOLUTION:**

\( t = 1 \)
\( s = \text{(empty, circle)} \)
\( a = \text{MoveSouth} \)
\( r = 0 \)
s’ = (circle, circle)
Q((empty, circle), MoveSouth) = 0 + (0 + (.9)(0) − 0) = 0

t = 2
s = (circle, circle)
a = PickUpCircle
r = 10
s’ = (empty, circle)
Q((circle, circle), PickUpCircle) = 0 + (10 + (.9)(0) − 0) = 10

s = (empty, circle)
a = MoveSouth
r = 0
s’ = (circle, circle)
Q((empty, circle), MoveSouth) = 0 + (0 + (.9)(10) − 0) = 9

s = (circle, circle)
a = PickUpCircle
r = 10
s’ = (empty, circle)
Q((circle, circle), PickUpCircle) = 10 + (10 + (.9)(9) − 10) = 18.1

Final nonzero Q table entries:
Q((empty, circle), MoveSouth) = 9
Q((circle, circle), PickUpCircle) = 18.1