Suppose the neural network above is initialized with the following weights:

**Input to hidden layer:**

\[ w_{h_1, x_0} = -0.4 \quad w_{h_1, x_1} = 0.2 \quad w_{h_1, x_2} = 0.1 \]
\[ w_{h_2, x_0} = -0.2 \quad w_{h_2, x_1} = 0.4 \quad w_{h_2, x_2} = -0.1 \]

**Hidden to output layer:**

\[ w_{o_1, h_0} = 0.1 \quad w_{o_1, h_1} = -0.2 \quad w_{o_1, h_2} = 0.1 \]
\[ w_{o_2, h_0} = 0.4 \quad w_{o_2, h_1} = -0.1 \quad w_{o_2, h_2} = 0.1 \]

(a) What are the activations of the hidden and output units after forward-propagation of the input \( x^1 = (1, 0) \)?

**Solution:**

\[ h_1 = \sigma \left( (-0.4)(1) + (0.2)(1) \right) = \frac{1}{1 + e^{-0.4}} = 0.45 \]
\[ h_2 = \sigma \left( (-0.2)(1) + (0.4)(1) \right) = \frac{1}{1 + e^{-0.2}} = 0.55 \]
\[ o_1 = \sigma \left( (0.1)(1) + (-0.2)(0.45) + (0.1)(0.55) \right) = \frac{1}{1 + e^{-0.055}} = 0.52 \]
\[ o_2 = \sigma \left( (0.4)(1) + (-0.1)(0.45) + (0.1)(0.55) \right) = \frac{1}{1 + e^{-0.41}} = 0.6 \]
(b) Suppose $\mathbf{x}^1$ has target output $\mathbf{t} = (.9, .1)$. What are the weights after back-propagation on this example? Use learning rate $\eta = 0.1$ and momentum $\alpha = 0.9$.

$$\delta_{o_1} = (.52)(1-.52)(.9-.52) = .095 \quad \text{(target for } o_1 \text{ is } .9)$$

$$\delta_{o_2} = (.6)(1-.6)(.1-.6) = -.12 \quad \text{(target for } o_2 \text{ is } .1)$$

- $\delta_{h_1} = (.45)(1-.45)[(.095)(-.2) + (-.12)(-.1)] = -.002$

- $\delta_{h_2} = (.55)(1-.55)[(.095)(.1) + (-.12)(.1)] = -.0006$

$$\Delta \mathbf{w}_{o_1,h_0} = (.1)(.095)(1) + (.9)(0) = .01 \quad w_{o_1,h_0} = .1 + .01 = .11$$

$$\Delta \mathbf{w}_{o_1,h_1} = (.1)(.095)(.45) + (.9)(0) = .004 \quad w_{o_1,h_1} = -2 + .004 = -.196$$

$$\Delta \mathbf{w}_{o_1,h_2} = (.1)(.095)(.55) + (.9)(0) = .005 \quad w_{o_1,h_2} = 1 + .005 = 1.005$$

$$\Delta \mathbf{w}_{o_2,h_0} = (.1)(-.12)(1) + (.9)(0) = -.012 \quad w_{o_2,h_0} = .4 - .012 = .388$$

$$\Delta \mathbf{w}_{o_2,h_1} = (.1)(-.12)(.45) + (.9)(0) = -.0054 \quad w_{o_2,h_1} = -.1 - .0054 = -.1054$$

$$\Delta \mathbf{w}_{o_2,h_2} = (.1)(-.12)(.55) + (.9)(0) = -.0066 \quad w_{o_2,h_2} = 1 - .0066 = .0934$$

$$\Delta \mathbf{w}_{h_1,x_0} = (.1)(-.002)(1) + (.9)(0) = -.0002 \quad w_{h_1,x_0} = -.4 - .0002 = -.4002$$

$$\Delta \mathbf{w}_{h_1,x_1} = (.1)(-.002)(1) + (.9)(0) = -.0002 \quad w_{h_1,x_1} = .2 - .0002 = .1998$$

$$\Delta \mathbf{w}_{h_1,x_2} = (.1)(-.002)(0) + (.9)(0) = 0 \quad w_{h_1,x_2} = 1 + 0 = 1$$

$$\Delta \mathbf{w}_{h_2,x_0} = (.1)(-.0006)(1) + (.9)(0) = -.00006 \quad w_{h_2,x_0} = -.2 - .00006 = -.20006$$

$$\Delta \mathbf{w}_{h_2,x_1} = (.1)(-.0006)(1) + (.9)(0) = -.00006 \quad w_{h_2,x_1} = .4 - .00006 = .39994$$

$$\Delta \mathbf{w}_{h_2,x_2} = (.1)(-.0006)(0) + (.9)(0) = 0 \quad w_{h_2,x_2} = -.1 + 0 = -.1$$