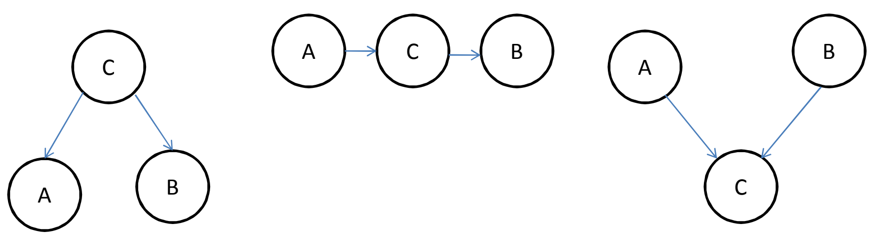
More on conditional independence and "Markov blankets"

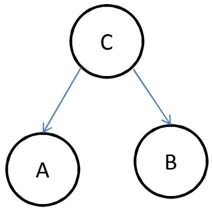
(from C. Bishop, Pattern Recognition and Machine Learning , Ch. 8)

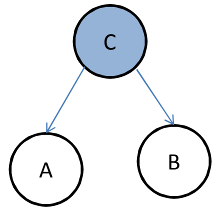
- A random variable A is conditionally independent of a random variable B iff $p(A,B) = p(A)p(B)$
- A random variable A is conditionally independent of a random variable B given a third random variable C iff $p(A,B|C) = p(A|C)p(B|C)$

Consider three Bayesian networks:



When is A conditionally independent of B , not given C?
When is A conditionally independent of B, given C?





(1) C is not given (observed)

$$p(A,B,C) = p(A \mid C)P(B \mid C)P(C)$$
$$p(A,B) = \sum_C p(A \mid C)P(B \mid C)P(C)$$
$$\neq p(A)p(B)$$

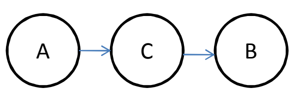
In this case, A and B are **conditionally dependent.**

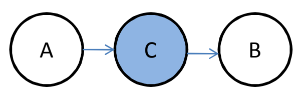
(2) C is given (observed)

$$p(A,B \mid C) = \frac{p(A,B,C)}{p(C)}$$
$$= p(A \mid C)p(B \mid C)$$

In this case, A and B are **conditionally independent.**

C is "tail to tail" with respect to the path between A and B.
A and B are conditionally dependent, unless C is observed, which "blocks" the path from A to B and they become conditionally independent, given C.





(1) C is not given (observed)

$$p(A,B,C) = p(A)P(C \mid A)P(B \mid C)$$
$$p(A,B) = p(A)\sum_C p(C \mid A)P(B \mid C)$$
$$= p(A)p(B \mid A)$$
$$\neq P(A)P(B)$$

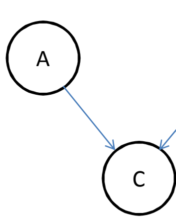
In this case, A and B are **conditionally dependent.**

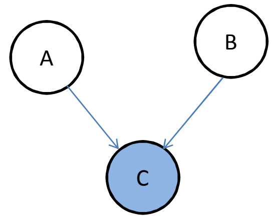
(2) C is given (observed)

$$p(A,B \mid C) = \frac{p(A,B,C)}{p(C)}$$
$$= \frac{p(A)p(C \mid A)p(B \mid C)}{p(C)}$$
$$= p(A \mid C)p(B \mid C)$$

In this case, A and B are **conditionally independent.**

C is "head to tail" with respect to the path between A and B.
A and B are conditionally dependent, unless C is observed, which "blocks" the path from A to B and they become conditionally independent, given C.





(1) C is not given (observed)

$$p(A,B,C) = p(A)p(B)p(C \mid A,B)$$
$$p(A,B) = p(A)p(B)\sum_{C} p(C \mid A,B)$$
$$= p(A)p(B)$$

In this case, A and B are **conditionally independent.**

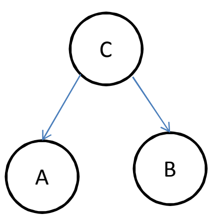
(2) C is given (observed)

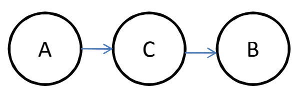
$$p(A,B \mid C) = \frac{p(A,B,C)}{p(C)}$$
$$= \frac{p(A)p(B)p(C \mid A,B)}{p(C)}$$
$$\neq p(A \mid C)p(B \mid C)$$

In this case, A and B **are conditionally dependent.**

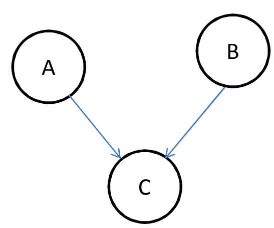
C is "head to head" with respect to the path between A and B.
A and B are conditionally independent, unless C is observed, which "unblocks" the path from A to B and they become conditionally dependent.

Summary





C tail-to-tail or C head-to-tail: C leaves path unblocked unless C is observed, in which case it blocks path



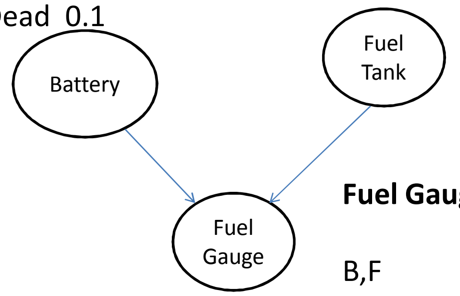
C head-to-head:
C blocks path unless C is observed, in which case it unblocks path

Battery

Charged 0.9   Dead 0.1

Fuel Tank

Full 0.9   Empty 0.1



Fuel Gauge

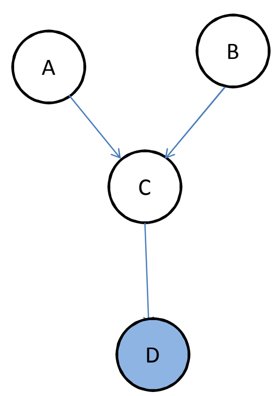
	Full	Empty
B,F	.8	.2
B,¬F	.2	.8
¬ B, F	.2	.8
¬ B, ¬ F	.1	.9

1. **What is probability that fuel tank is empty?**
2. **What is probability that fuel tank is empty given gauge**
3. **reads Empty?**
4. **What is probability that fuel tank is empty given gauge reads**
5. **Empty and battery is dead?**

Notion of "explaining away".

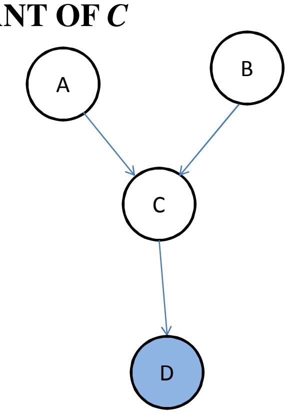
Summary

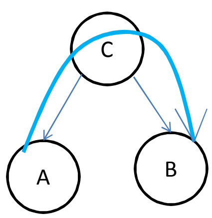
C head-to-head:
C blocks path unless *C*
is observed, in which case it unblocks
path



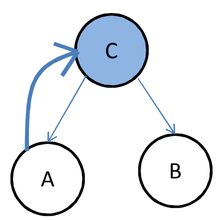
Summary

C head-to-head:
C blocks path unless *C* OR **A DESCENDANT OF *C*** is observed, in which case it unblocks path

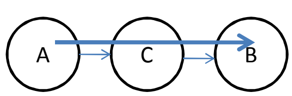


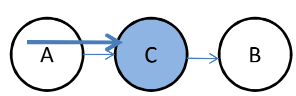


C unobserved: Path is unblocked from A to B and vice versa. Conditionally dependent.

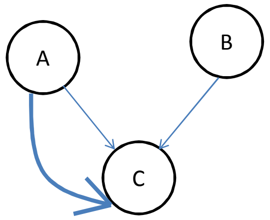


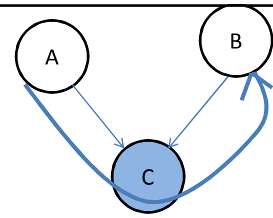
C observed: Path is blocked. Conditionally independent





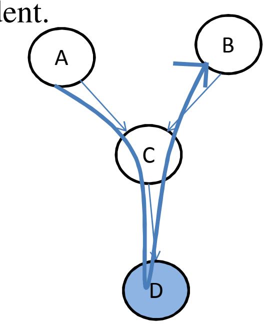
Examples?





C observed:Path is unblocked from A to B and vice versa. Conditionally dependent.

C unobserved: Path is blocked. Conditionally independent.

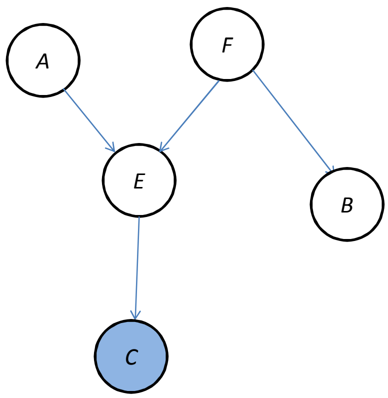


Examples?

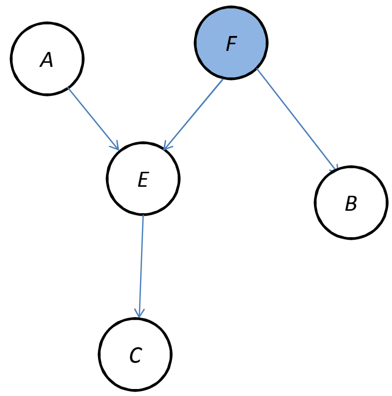
d-separation

- d = “directed” — applies to directed graphs
- If all paths between node *A* and node *B* are blocked, then they are said to be **d-separated**, and are conditionally independent.

Examples



Is A d-separated from B?



Is A d-separated from B?

Markov Blanket

- Prove that the conditional distribution for a node *X* in a directed graph, conditioned on all of the nodes in its Markov blanket, is independent of the remaining variables in the graph.