Quiz 2

Please write all answers on these pages.

1. Given the training set plotted below: Sketch the hyperplane (i.e., line) that maximally separates the two classes (open circle and solid circle). Also sketch a line that indicates the margin, and label it as "margin". Also circle the support vectors.

![Graph with annotated margin and support vectors]

Other depictions of the margin are possible.

2. Answer the following in one or two sentences.

(1) What are the inputs to the SVM algorithm?

*Training examples*

(2) What does the SVM algorithm output?

*di's, support vectors, bias*
3. Consider the following three points, \( x_1, x_2, \) and \( x_3 \), which have been identified as support vectors for a training set. Here, \( y_i \) is the class of the point, and \( a_i \) is the support vector coefficient.

\[
\begin{align*}
x_1 &= (2, 1) & y_1 &= -1 & a_1 &= -4 \\
x_2 &= (4, 3) & y_2 &= -1 & a_2 &= -4 \\
x_3 &= (2, 3) & y_3 &= +1 & a_3 &= 8
\end{align*}
\]

The bias is \( b = 0 \).

(a) Using the formula

\[
h(x) = \text{sgn} \left( \sum_{i=1}^{n} a_i (x_i \cdot x) + b \right),
\]

give the classification of the new instance \( x = (1, 2) \). Show your work.

\[
h(1, 2) = \text{sgn} \left[ -4 \,(2,1) \cdot (1,2) -4 \,(4,3) \cdot (1,2) + 8 \,(2,3) \cdot (1,2) \right]
\]

\[
= \text{sgn} \left[ -4 \cdot 4 - 4 \cdot 10 + 8 \cdot 8 \right]
\]

\[
= \text{sgn} \left[ -56 + 64 \right] = \text{sgn} \left[ 8 \right] = +1
\]

(b) Use the \( x_i \)'s and \( a_i \)'s to find the weight vector \( w \) associated with the separating hyperplane, where \( w = \sum a_i x_i \).

\[
w = -4 \,(2,1) - 4 \,(4,3) + 8 \,(2,3)
\]

\[
= (-8, -4) + (-16, -8) + (16, 24)
\]

\[
= (-8, 8)
\]
(c) Using the weight vector you obtained in part (b) and the bias $b = 0$, find the equation of the separating hyperplane. Give the equation in the slope-intercept form: $x_2 = (slope \cdot x_1) + y$-intercept.

$$-8x_1 + 8x_2 = 0$$

$$x_2 = x_1$$

(d) Using the graph below, plot the support vectors and the separating hyperplane. Also draw a line to show the margin. Finally, plot the new point $(1, 2)$ from part (a) to confirm that it is in the class you found in part (a).
4. Consider the following training set:

<table>
<thead>
<tr>
<th>Instance</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$x_2$</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

and let $k(y, z)$ be the polynomial kernel: $k(y, z) = ((y \cdot z) + 1)^2$. Give the kernel (Gram) matrix for this kernel function using this training set.

\[
K' = \begin{pmatrix}
  k(x_1, x_1) & k(x_1, x_2) & k(x_1, x_3) \\
  k(x_2, x_1) & k(x_2, x_2) & k(x_2, x_3) \\
  k(x_3, x_1) & k(x_3, x_2) & k(x_3, x_3)
\end{pmatrix}
\]

\[
= \begin{pmatrix}
  (1,0) \cdot (1,0) + 1)^2 & (1,0) \cdot (-1,-1) + 1)^2 & (1,0) \cdot (0,1) + 1)^2 \\
  (-1,-1) \cdot (1,0) + 1)^2 & (-1,-1) \cdot (-1,-1) + 1)^2 & (-1,-1) \cdot (0,1) + 1)^2 \\
  (0,1) \cdot (1,0) + 1)^2 & (0,1) \cdot (-1,-1) + 1)^2 & (0,1) \cdot (0,1) + 1)^2
\end{pmatrix}
\]

\[
= \begin{pmatrix}
  4 & 0 & 1 \\
  0 & 9 & 0 \\
  1 & 0 & 4
\end{pmatrix}
\]