So far we’ve done...

- Dynamics and chaos
- Thermodynamics, statistical mechanics, entropy, information
- Computation, Turing machines, halting problem
- Evolution, genetics

Can we integrate these ideas in order to understand complexity?

First, what is “complexity”? Can we measure it?
Complexity definitions and measures discussed in book

• Complexity as size
• Complexity as entropy / information content
• Complexity as algorithmic information content
• Complexity as logical depth
• Complexity as thermodynamic depth
• Statistical complexity
• Complexity as fractal dimension
• Complexity as hierarchy
### Complexity as size?

<table>
<thead>
<tr>
<th>Species</th>
<th>Size of genome</th>
<th>Number of genes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human</td>
<td>2.9 billion base pairs</td>
<td>30,000</td>
</tr>
<tr>
<td>Fruit fly (<em>Drosophila melanogaster</em>)</td>
<td>120 million base pairs</td>
<td>13,601</td>
</tr>
<tr>
<td>Baker's yeast (<em>Saccharomyces cerevisiae</em>)</td>
<td>12 million base pairs</td>
<td>6,275</td>
</tr>
<tr>
<td>Worm (<em>Caenorhabditis elegans</em>)</td>
<td>97 million base pairs</td>
<td>19,000</td>
</tr>
<tr>
<td><em>E. coli</em></td>
<td>4.1 million base pairs</td>
<td>4,800</td>
</tr>
<tr>
<td>Arabidopsis (<em>Arabidopsis thaliana</em>)</td>
<td>125 million base pairs</td>
<td>25,000</td>
</tr>
</tbody>
</table>
Complexity as information content?

Perfectly ordered genome (A A A A A A A A ... A)

Human genome

Random genome (G A A G T C C T A G C G A A G ... )

Information content?? Complexity???
Complexity as information content?

Perfectly ordered genome  (A A A A A A A A ... A)

Human genome

Random genome (G A A G T C C T A G C G A A G ... )

Information content??? Complexity???

Algorithmic information content??? Logical Depth?

Thermodynamic depth???
Complexity as Fractal Dimension
Fractals and Fractal Dimension

Measuring the Coastline of Great Britain
Koch Curve

1. Start with a single line.

2. Apply the Koch curve rule: “For each line segment, replace its middle third by two sides of a triangle, each of length $1/3$ of the original segment.” Here there is only one line segment; applying the rule to it yields:
3. Apply the Koch curve rule to the resulting figure. Keep doing this forever. For example, here are the results from a second, third, and fourth application of the rule:

Notion of “self-similarity”
Koch curve

- How long is it?
- What is its dimension?
Ordinary dimension
What happens when you continually bisect the sides of lines, squares, cubes, etc.?

Dimension 1: Each level is made up of two half-sized copies of previous level

Dimension 2: Each level is made up of four quarter-sized copies of previous level

Dimension 3: Each level is made up of eight 1/8-sized copies of previous level

Dimension $n$: ?
Generalized definition of dimension

Create a geometric structure from an original object by repeatedly dividing the length of its sides by a number $x$. Then each level is made up of $x^{dimension}$ copies of the previous level.
Generalized definition of *dimension*

Let $M$ be the magnification factor of a side to get from level $n+1$ to level $n$.

Let $N$ be the number of copies at level $n+1$ of each object in level $n$.

Then

$$M^{\text{Dimension}} = N$$

or

$$\text{Dimension} = \log N / \log M$$
Generalized definition of **dimension**

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Generalized definition of dimension

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Then

$$M^{Dimension} = N$$

or

$$Dimension = \log N / \log M$$

Here, $M = 3$, $N = 4$

So Fractal Dimension $\approx 1.26$
Cantor set

\[ n = 0 \]

\[ n = 1 \]

\[ n = 2 \]

\[ n = 3 \]

\[ n = 4 \]

etc.
Cantor set

Here, $M = 3$, $N = 2$

So Fractal Dimension = $\log 2 / \log 3 \approx .63$
Sierpinski triangle
Approximate dimension of coastlines  
(Shelberg, Moellering, and Lam)

<table>
<thead>
<tr>
<th>Curve</th>
<th>Slope ($\beta$)</th>
<th>$D$ $(1-\beta)$</th>
<th>New $D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>West Coast of Great Britain</td>
<td>-0.25</td>
<td>1.25</td>
<td>1.2671</td>
</tr>
<tr>
<td>Coast of Australia</td>
<td>-0.13</td>
<td>1.13</td>
<td>1.1490</td>
</tr>
<tr>
<td>Coast of South Africa</td>
<td>-0.02</td>
<td>1.02</td>
<td>1.0356</td>
</tr>
<tr>
<td>Land-frontier between Spain</td>
<td>-0.14</td>
<td>1.14</td>
<td>1.1014</td>
</tr>
<tr>
<td>and Portugal</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

See [Google Maps](https://www.google.com/maps)
Approximate dimension of Cauliflower

Each branch contains ~13 subbranches, each three times smaller.
Fractal geometry in crumpled paper balls

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(Received 11 December 1985; accepted for publication 6 October 1986)

The geometry of crumpled paper balls is examined. The analysis stresses some physical, mathematical, and intuitive aspects of the problem, introducing the concept of fractal dimension which underlies many areas of modern physics.

Fractals are now a topic of wide interest and here we describe an interesting example of fractal dimension defined via the mass-size exponent. In what follows, we discuss this example which has been used with success in the last two years in the freshman course Experimental Physics 1 at UFPE.

With two sheets of paper divided in the way indicated in Fig. 1 we construct after crumpling \((n+1)\) handmade balls of different sizes and masses (Fig. 2). We assign mass \(M = 1\) to the smallest ball and mass \(M = 2^n\) for the \((n+1)\)th ball, with \(n\) increasing with the size of the paper (see Fig. 1). In our case \(1 < M < 21\), with the largest mass corresponding to a sheet of paper of 98 cm \(\times\) 65 cm. Typical log-log graphs of the average diameter \((L)\) versus mass \((M)\) for such crumpled paper balls are quite well described by \(L = kM^{1/D}\). \(D\) is interpreted as the fractal dimension of the balls and \(k\) \(\sim (1/\rho)^{1/D}\) is a measure of the average mass-density \(\rho\) on these fractal structures. The values obtained for \(D\) and \(k\) were \(D = 2.51 \pm 0.19\), \(k = 5.75 \pm 0.71\) for writing paper of surface density \(\sigma \sim 80 \text{ g/m}^2\). The fractal dimension \(D\) in this case tells about the complexity or degree of contortion of the area, since a fixed measure of rounded smooth area can enclose a larger volume than a complicated one can. The values of \(D\) and \(k\) are statistically independent of the students' weight and height. It is experimentally evident that \(k\) (lacunarity) has a percent mean square deviation approximately two times larger than that of \(D\), and \((\Delta \rho/\rho) = D(\Delta k/k) \approx 0.31\). These values show that \(D\) is much less affected by the way of crumpling (pressure applied, haste or not, etc.) than the density is. The topological dimension of these balls is \(D_T = 2\), since they are made of sheets of paper, which conform with \(D_T = 2\).

On the other hand, they are embedded in the Euclidean view our crumpled paper balls are self-avoiding surfaces. In this case the Flory argument predicts that the size \(L\) obeys \(L \sim r^\nu\), where \(\nu = 4/(E+2)\), \(r\) is the linear size of the uncrumpled paper, and \(E\) is the dimension of the embedding space. Since the mass scales with \(r\) according to \(M \sim r^D\), we obtain \(L \sim M^{\nu/2} = M^{2/(E+2)} = M^{1/D}\), with \(D = [(E + 2)/2] = 2.5\), for \(E = 3\), as experimentally obtained.

On the other hand, the experimental dependence of \(D\) with the surface density \(\sigma\) (g/m²) of the paper is shown in

Fig. 2. A typical set of crumpled paper balls with masses 1, 2, 4, ..., 64.
This week’s reading assignment: “The Calculus of Intricacy”

Seth Lloyd (Professor at MIT)
Barry’s Paradox:

What is “the smallest number that requires more than sixty-eight symbols to be specified”?
ing jackass. Harik! Harik! Harik! The rose is white in the darik! And Sunfella’s nose has got rhinoceritis from haunting the roes in the parik! So all rogues lean to rhyme. And contradrinking themselves about Lillytrilly law pon hilly and Mrs. Niall of the Nine Corsages and the old markiss their besterfar, and, arrah, sure there was never a marcus at all at all among the manlies and dear Sir armoury, queer sir rumoury, and the old house by the churpelizod, and all the goings on so very wrong long before when they were going on retreat, in the old gammeldags, the four of them, in Milton’s Park under lovely Father Whisperer and making her love with his stuffustuff in the languish of flowers and feeling to find was she mushymushy, and wasn’t that very both of them, the saucicissters, a drahereen o machree!, and (peep!) meeting waters most improper (peepette!) ballround the garden, trickle trickle trickle triss, please, miman, may I go flirtling? farmers gone with a groom and how they used her, mused her, licksed her and cuddled. I differ with ye! Are you sure of yourself now? You’re a liar, excuse me! I will not and you’re another! And Lully holding their breach of peace for them. Pool loll Lolly! To give and to take! And to forego the pasht! And

From *Finnegan’s Wake* (James Joyce)
Netlogo

• Models library → Fractals:
  • Koch curve
  • Sierpinski simple
  • Tree simple
  • Mandelbrot
L systems
(Aristid Lindenmayer)

Grammars for generating fractals (and other shapes)

Need “axiom” and “grammar rules”

Alphabet for rules: \{F,f,+,-\}

+ Turn counterclockwise by a specified angle $q$
- Turn clockwise by a specified angle $q$
F Move forward one step while drawing a line
f Move forward one step without drawing a line
Example 1: Cantor middle thirds set

axiom:
\[ S = F \]

rules:
\[ F \rightarrow FfF \]
\[ f \rightarrow fff \]
Example 2: Koch curve

axiom:
S = F

rules:
F → F - F++F - F
+ → +
- → -

q = 60 degrees
Netlogo L-systems lab

- Go through Koch curve code