Cellular Automata, Part 2
What is the relationship between the dynamics of cellular automata and their ability to compute?
Phase transitions

temperature

steam
water
ice

gas
fluid
solid

chaotic
complex

control parameter
ordered
Hypothesis (Langton, Packard, Kauffman, others):

Need maximally “fluid” state to maximize potential for

- information processing
- complexity of dynamics
- ability to adapt
Recall the “logistic map” model of population dynamics

\[ x_{t+1} = rx_t (1 - x_t) \]

Order parameter is population growth rate, \( r \)
Recall the “logistic map” model of population dynamics

\[ x_{t+1} = r x_t (1 - x_t) \]

Order parameter is population growth rate, \( r \)

Fixed point
Recall the “logistic map” model of population dynamics

\[ x_{t+1} = r x_t (1 - x_t) \]

Order parameter is population growth rate, \( r \)

Fixed point

Period 2
Recall the “logistic map” model of population dynamics

\[ x_{t+1} = r x_t (1 - x_t) \]

Order parameter is population growth rate, r

Period 4
Recall the "logistic map" model of population dynamics

\[ x_{t+1} = r x_t (1 - x_t) \]

Order parameter is population growth rate, \( r \)
Recall the “logistic map” model of population dynamics

\[ x_{t+1} = rx_t (1 - x_t) \]

Order parameter is population growth rate, \( r \)

**Period 16**
Recall the “logistic map” model of population dynamics

\[ x_{t+1} = r x_t (1 - x_t) \]

Order parameter is population growth rate, \( r \)

**Period 16**

**Chaos**

The onset (or “edge”) of chaos is

\( r = 3.569946 \)
The ‘edge of chaos’ in cellular automata


• Langton devised an “order parameter” for cellular automata called \( \lambda \).

• For binary-state CAs, \( \lambda \) is defined as follows:

\[
\lambda = \frac{\text{number of 1s in rule table's output bits}}{\text{number of entries in rule table}}
\]
Rule 110

Rule:

\[ \lambda = ? \]
The ‘edge of chaos’ in cellular automata


• Building on Wolfram’s classes of behavior for CA, Langton found evidence that cellular automata can be “ordered” or “chaotic” roughly according to lambda

• He showed evidence that the “complexity” of patterns formed by cellular automata is maximized at the transition between order and chaos

• He argued that the potential for computation must be maximized at this “phase transition”
http://math.hws.edu/xJava/CA/EdgeOfChaosCA.html
How to define “potential for computation”?