Examples of Complex Networks

Neural Network
(C. Elegans)


Food Web

http://1.bp.blogspot.com/_vIFBm3t8boU/SBhzqbchIeI/AAAAAAAAAXk/RsC-Pj45Avc/s400/food%2Bweb.bmp
Figure 1. Bipartite graph of the metabolic network of *Giardia lamblia* oocysts. Dark gray and white nodes represent enzymes and light gray nodes represent metabolites (Lemire et al., 2004).

**Metabolic Network**


Figure 2. Bipartite graph of the genetic network of *Giardia lamblia* oocysts. Blue nodes represent genes and red nodes represent gene products.

**Genetic Regulatory Network**

Tong et al., *Science* 294:2364-2368 (2001)

http://expertvoices.nsdl.org/cornell-info204/files/2009/03/figure-3.jpeg
Airline Routes

http://virtualskies.arc.nasa.gov/research/tutorial/images/12routemap.gif

US Power Grid

http://images.encarta.msn.com/xrefmedia/aencmed/targets/maps/map/000a5302.gif
Internet

http://www.visualcomplexity.com/vc/images/270_big01.jpg

World Wide Web (small part)

Social Network

http://ucsdnews.ucsd.edu/graphics/images/2007/07-07socialnetworkmapLG.jpg

The Science of Networks

Are there properties common to all complex networks?

If so, why?

Can we formulate a general theory of the structure, evolution, and dynamics of networks?
<table>
<thead>
<tr>
<th>Observed common properties:</th>
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</thead>
<tbody>
<tr>
<td>– Small world property</td>
</tr>
<tr>
<td>– Scale-free structure</td>
</tr>
<tr>
<td>– Clustering and community structure</td>
</tr>
<tr>
<td>– Robustness to random node failure</td>
</tr>
<tr>
<td>– Vulnerability to targeted hub attacks</td>
</tr>
<tr>
<td>– Vulnerability to cascading failures</td>
</tr>
</tbody>
</table>

Stanley Milgram

On average: “six degrees of separation”
The Small-World Property  
(Watts and Strogatz)

The network has relatively few “long-distance” links but there are short paths between most pairs of nodes, usually created by “hubs”.

Notion of average path length

Notion of clustering coefficient

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Scale-Free Structure  
(Albert and Barabási, 1998)

Typical structure of a randomly connected network

Typical structure of World Wide Web  
(nodes = web pages, links = links between pages)

http://www.dichotomistic.com/images/random%20network.gif
Concept of “Degree Distribution”

A node with degree 3

Degree

Number of Nodes

Number of nodes

Degree

random network

part of WWW

Number of nodes

Degree
12/5/2009

The Web's approximate Degree Distribution

"Scale-free" distribution

The probability that a node will have degree k is proportional to $\frac{1}{k^2}$

"power law"

Relation to fractals?

"Scale-free" distribution = "power law" distribution

Other examples of power-laws in nature

- Magnitude vs. frequency of earthquakes
- Magnitude vs. frequency of stock market crashes
- Income vs. frequency (of people with that income)
- Populations of cities vs. frequency (of cities with that population)
- Word rank vs. frequency in English text
How are scale-free networks created?

Barabási and Albert: **Preferential attachment**

Netlogo demos

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**Gutenberg-Richter Law**

![Gutenberg-Richter Law graph](image)

By: Bak [1]
Regularity of Biological Extinctions

By: Bak [1]

More examples of scale-free networks

http://www.orgnet.com/cases.html
Robustness of Scale-Free Networks

- **Vulnerable** to targeted “hub” failure
- **Robust** to random node failure

unless....

nodes can **cause** other nodes to fail

Can result in *cascading failure*

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**August, 2003 electrical blackout in northeast US and Canada**

http://earthobservatory.nasa.gov/images/imagerecords/3000/3719/NE_US_OLS2003227.jpg

- **9:29pm**
  - 1 day before

- **9:14pm**
  - Day of blackout
We see similar patterns of cascading failure in biological systems, ecological systems, computer and communication networks, wars, etc.
Risk Distributions

Normal ("bell-curve") distributions

http://www.canadianfundwatch.com/image-browser/sharpe-ratio/distribution-of-returns.gif

Power law ("scale free") distribution

http://www.marketoracle.co.uk/images/mauldin_16_10_07image003.gif
“Few economists saw our current crisis coming, but this predictive failure was the least of the field’s problems. More important was the profession’s blindness to the very possibility of catastrophic failures in a market economy.


Power law ("scale free") distribution

http://www.marketoracle.co.uk/images/mauldin_16_10_07image003.gif