Complexity measures discussed in book

- Complexity as size
- Complexity as entropy
- Complexity as algorithmic information content
- Complexity as logical depth
- Complexity as thermodynamic depth
- Statistical complexity
- Complexity as fractal dimension
- Complexity as hierarchy

Algorithmic Information Content
(also called “Kolmogorov Complexity”)

Let $K(s)$ denote the algorithmic information content of string $s$.

It can be proved that $K(s)$ is incomputable – that is, there is no Turing machine that can compute $K(s)$ for any $s$.

Related paradox (due to Barry):

"Let $n$ be the smallest positive integer that cannot be defined in fewer than twenty English words."
Fractals and Fractal Dimension


Measuring the Coastline of Great Britain
Koch curve

How long is it?
What is its dimension?

Definition of \textbf{dimension}

Generalized definition of dimension:

Let $M$ be the magnification factor of side to get from level $n+1$ to level $n$.

Let $N$ be the number of copies at level $n+1$ of each object in level $n$.

Then

\[ M^{\text{Dimension}} = N \]

or

\[ \text{Dimension} = \log N / \log M \]
Fractal dimension of Koch curve

Fractal dimension of Cantor set
Fractal dimension of box fractals

Another example

Initiator

Generator

replaced by
Fractal dimension of Sierpinski triangle

An exercise!

Approximate dimension of coastlines
(Shelberg, Moellering, and Lam)

<table>
<thead>
<tr>
<th>Curve</th>
<th>Slope (β)</th>
<th>D (1-β)</th>
<th>New D</th>
</tr>
</thead>
<tbody>
<tr>
<td>West Coast of Great Britain</td>
<td>-.25</td>
<td>1.25</td>
<td>1.2671</td>
</tr>
<tr>
<td>Coast of Australia</td>
<td>-.13</td>
<td>1.13</td>
<td>1.1490</td>
</tr>
<tr>
<td>Coast of South Africa</td>
<td>-.02</td>
<td>1.02</td>
<td>1.0356</td>
</tr>
<tr>
<td>Land-frontier between Spain and Portugal</td>
<td>-.14</td>
<td>1.14</td>
<td>1.1014</td>
</tr>
</tbody>
</table>

See Google Maps
Approximate dimension of Cauliflower

Each branch contains ~13 subbranches, each three times smaller.

Fractal geometry in crumpled paper balls

M. R. F. de Souza
Departamento de Física, Universidade Federal de Pernambuco, 50600 Recife, PE, Brazil
(Received 11 December 1985; accepted for publication 6 October 1986)

The geometry of crumpled paper balls is examined. The analysis stresses some physical, mathematical, and intuitive aspects of the problem, introducing the concept of fractal dimension, which underlies many areas of modern physics.

Fractals are now a topic of wide interest and here we describe an interesting example of fractal dimension defined via the mass ratio exponent. In what follows, we discuss this example which has been used with success in the last two years in the freshman course Experimental Physics 1 at UFPE.

With two sheets of paper divided in the way indicated in Fig. 1, we can obtain after crumpling (n = 2) hand-made balls of different sizes and masses (Fig. 2). We assign mass $M = 1$ to the smallest ball and mass $M = 2^n$ for the $(n = 1, 2, 3, \ldots)$ ball, with $m$ increasing with the size of the paper. The typical log-log graphs of the average diameter $d$ versus mass $M$ for each crumpled paper ball are quite well described by $d = K M^{1/2}$. We interpreted as the fractal dimension of the balls and $K = (L_p)^{1/2}$ is a measure of the average mass density $\rho$ of the fractal structures. The value obtained for $D$ and $k$ were $D = 2.51 \pm 0.02$, $k = 5.75 \pm 0.71$ for writing paper of surface density $\rho = 80$ g/m². The fractal dimension $D$ in this case tells about the complexity or degree of corrugation of the surface, since a fixed measure of extended smooth area can include a larger volume than a complicated one can. The values of $D$ and $k$ are statistically independent of the students' weight and height. It is experimentally evident that $k$ (necessity) has a percent mass square deviation approximately two times larger than that of $D$. And $k = (\rho L_p)^{1/2}$, $D = 1.51$. These values show that $D$ is much less affected by the ways of crumpling (pressure applied, hands or not, etc.) than the density $\rho$. The reproducibility of these results at $\rho = 4$, since they are made of sheets of paper, which conforms with $D_f = 2$.

On the other hand, the experimental dependence of $D$ with the surface density $\rho$ of the paper is shown in Fig. 1. The approximate crumpled paper balls made with various $\rho$ are seen.
By request: Fractals and the Golden Ratio

- **Golden ratio**
  \[ \frac{a + b}{a} = \frac{a}{b} = \varphi. \]
  \[ \varphi = \frac{1 + \sqrt{5}}{2} \approx 1.6180339887 \ldots \]

- **Fibonacci sequence**
  \[ 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987 \ldots \]
  \[ \lim_{n \to \infty} \frac{F(n + 1)}{F(n)} = \varphi. \]
L systems
(Aristid Lindenmayer)

Grammars for generating fractals (and other shapes)

Need “axiom” and “grammar rules”

Alphabet for rules: \{F, f, +, −\}

+ Turn counterclockwise by a specified angle $\theta$
– Turn clockwise by a specified angle $\theta$
F Move forward one step while drawing a line
f Move forward one step without drawing a line
Example 1: Cantor middle thirds set

axiom:
\( S = F \)

rules:
\( F \rightarrow FfF \)
\( f \rightarrow fff \)

Example 2: Koch curve

axiom:
\( S = F \)

rules:
\( F \rightarrow F - F++F - F \)
\( + \rightarrow + \)
\( - \rightarrow - \)

\( q = 60 \) degrees

graphical representations of the two examples