CS 346U
Exploring Complexity in Science and Technology
Fall, 2009

Week 2 Reading Questions (Chapters 3 and 4)

Due Monday, October 12.

1. Consider the slot machine in Figure 3.2 of the textbook.

(a) How many microstates correspond to the macrostate "pictures contain at least one cherry"?

(b) How many microstates correspond to the macrostate "two pictures the same"?

2. Explain in your own words (one paragraph) how Boltzmann's definition of entropy, $S = klogW$, captures the notions of "order" and "disorder".

3. Explain in your own words (one paragraph) how the second law of thermodynamics defines the "arrow of time".

4. Suppose three-year-old Jake has a vocabulary of 500 words (including "um"). When talking, he will say the word "the" one-tenth of the time, the word "um" one sixth of the time, and the rest of the time all his other words will be used equally often. What is the average Shannon information of his side of a conversation? Show how you calculated this.

5. (a) Devise a set of rules for a Turing machine that reads a tape containing all blank cells except for some number of 1s (possibly zero) sandwiched between exactly two 0s, and determines whether or not there are more than three 1s. If so, the final output of the machine will be a single 1 with all other cells blank; if not, the final output will be a single 0 with all other cells blank.

(b) Using the encoding for Turing machines described on p. 64–65 of the textbook, give the code for the rules described in part (a).

6. Consider a Turing machine M whose tape starts with a integer in binary notation (and all other cells blank), and which decides whether or not that integer is prime.

(a) Is it possible to construct such a machine that could do this task in finite time for any finite integer? Why or why not?

(b) Supposing the answer to part (a) is "yes", will this machine halt when run on its own code? Why or why not?

7. In your own words, explain (in one paragraph) what Turing's halting problem says about the limits of computation using Turing machines.