

Exam 2 Solutions
November 17, 1999

ECE 171: Introduction to Digital Design
Dr. McNames

Write your 6-digit identification number and student identification numbers below. Do not begin the exam or look at the problems until instructed to do so.

Once you begin, write your student ID at the top of each page. This is worth 5 points.

Do not use separate scratch paper. If you need more space, use the backs of the exam pages.

Problem 1: _____ /15

Problem 2: _____ /25

Problem 3: _____ /10

Problem 4: _____ /25

Problem 5: _____ /10

Student ID: _____ / 5

Total: _____ /85

6-Digit Identification Number: _____

Student Identification Number: _____

1. Boolean Algebra & Minimal Products (15 Points)

Simplify each of the expressions below to a minimal sum **using Boolean algebra**. You must show all of your work to receive full credit.

a. (4 pts) $Y = \bar{A}\bar{B}CD + \bar{A}BCD + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}C\bar{D} + A\bar{B}CD + ABC\bar{D} + ABCD$

$$\begin{aligned} Y &= A'CD + AB'C + ABC' + ABC \\ &= A'CD + AC + AB \\ &= C(A + A'D) + AB \\ &= C(A + D) + AB \\ &= AC + CD + AB \end{aligned}$$

b. (4 pts) $Y = B \oplus CA + BC(A + BC + BA) + (BC)$

$$\begin{aligned} Y &= B'AC + (AC)'B + ABC + BC + ABC + BC \\ &= AB'C + A'B + BC' + BC \\ &= AB'C + A'B + B(C' + C) \\ &= AB'C + A'B + B \\ &= AB'C + B \\ &= AC + B \end{aligned}$$

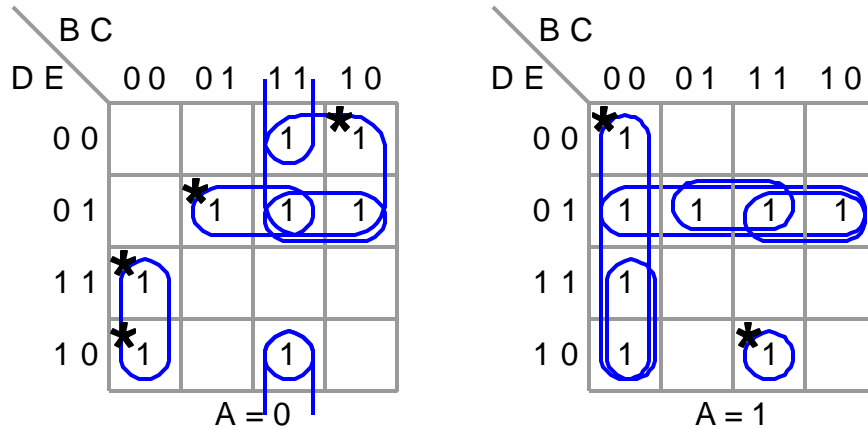
c. (4 pts) $Y = \overline{(A + B + C)} \overline{(AB + C)} + \bar{A}(B + C)$

$$\begin{aligned} Y &= (A'B'C')(AB'C') + A'B + A'C \\ &= (A'B'C')(A'C' + B'C') + A'B + A'C \\ &= A'B'C' + A'B'C' + A'B + A'C \\ &= A'B'C' + A'B + A'C \\ &= A'(B'C' + B + C) \\ &= A'(C' + B + C) \\ &= A'(1) \\ &= A' \end{aligned}$$

d. (3 pts) Find the minimal product for the function in part a. Hint: start with your simplified expression from part a.

$$\begin{aligned} Y' &= (AC + CD + AB)' \\ &= (AC)'(CD)'(AB)' \\ &= (A' + C')(C' + D')(A' + B') \\ &= (A'C' + C' + A'D' + C'D')(A' + B') \\ &= (C' + A'D')(A' + B') \\ &= A'C' + A'D' + B'C' + A'B'D' \\ &= A'C' + A'D' + B'C' \\ Y &= (A'C' + A'D' + B'C')' \\ &= (A'C')'(A'D')'(B'C')' \\ &= (A + C)(A + D)(B + C) \end{aligned}$$

2. Five Variable Karnaugh Maps (25 points)



- a. (2 pts) Circle all of the prime implicants for the 5-variable Karnaugh map above.
- b. (4 pts) Put an asterisk (*) in the upper left corner of each distinguished 1 cell.
- c. (8 pts) Write the boolean expression for each prime implicant below.

$B'C'D$

$A'BD'$

$AB'C'$

$CD'E$

$A'BCE'$

$AD'E$

$BD'E$

$BCDE'$

- d. (4 pts) Circle each of the essential prime implicants in part c.
- e. (6 pts) Write the expression for each minimal sum.

$Y = B'C'D + CD'E + A'BD' + BCDE' + AB'C' + AD'E$

$Y = B'C'D + CD'E + A'BD' + BCDE' + AB'C' + BD'E$

Y = _____

Y = _____

- f. (1 pt) How many minimal sums are there? Circle one of the options below.

1 2 3 4

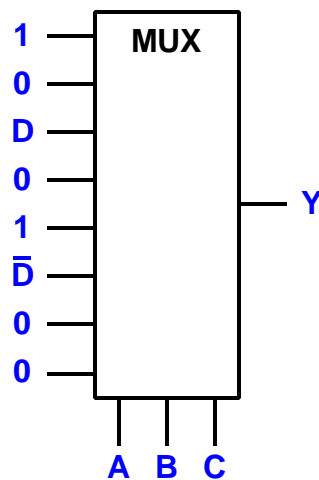
3. Multiplexers (10 Points)

- a. (3 pts) Fill in the following truth table for the function:

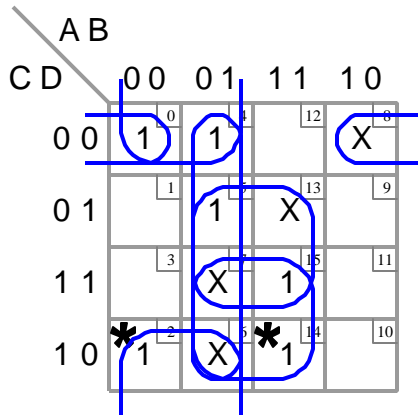
$$Y = \sum_{ABCD} (0,1,5,8,9,10)$$

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

- b. (7 pts) Show how to use the multiplexer below to implement this function. Label all 12 lines going into and coming out of the multiplexer.



4. Karnaugh Maps – Don't Cares & Static Hazards (25 points)



- a. (2 pts) For each cell in the Karnaugh map above, write the decimal equivalent of that input combination in the upper right hand corner box of that cell.
- b. (5 pts) For the following function, fill in the corresponding 1's and X's in the Karnaugh map above.
 $Y = \Sigma_{ABCD} (0,2,4,5,14,15) + d(6,7,8,13)$
- c. (2 pts) Circle all of the prime implicants in the Karnaugh map.
- d. (4 pts) Put an asterisk (*) in the upper left corner of each distinguished 1 cell.
- e. (2 pts) How many essential prime implicants are there? Please circle one of the choices below.

1 2 3 4 5 6 7 8

- f. (5 pts) Write the expression for each minimal sum. Underline the essential prime implicants.

f1. $Y = \underline{A'D'} + \underline{BC} + BD$

f3. $Y =$

f2. $Y = \underline{A'D'} + \underline{BC} + A'B$

f4. $Y =$

- g. (1 pt) How many minimal sums are there? Circle one of the options below.

1 2 3 4

- h. (2 pts) Which, if any, of the expressions in part f. contain a potential static-1 hazard? Assume the circuit is implemented in a 2-level NAND-NAND structure. Please circle zero or more of the options below.

f1 f2 f3 f4

- i. (2 pts) Repeat part h. for potential static-0 hazards.

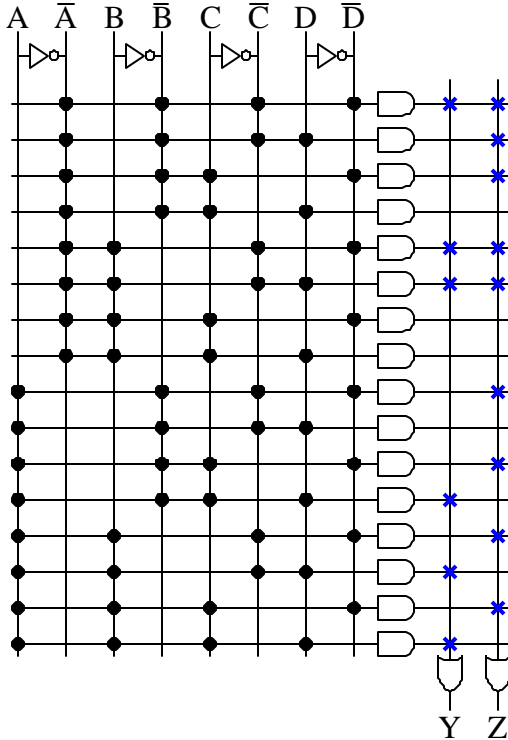
f1 f2 f3 f4

5. PLD's (10 Points)

$$Y = \sum_{ABCD} (0,4,5,11,13,15)$$

$$Z = \sum_{ABCD} (0,1,2,4,5,8,10,12,14)$$

- a. (5 pts) For the two functions shown above, Y and Z, show which connections should be made on the PROM array below. Show your connections with an **X** at the connection points.



- b. (5 pts) Show which connections should be made on the PAL device below to implement the functions F and G. The minimal sums for these expressions are given below. Show your connections with an **X** at the connection points.

$$F = \bar{A}\bar{C}\bar{D} + B\bar{C}D + ACD$$

$$G = \bar{C}\bar{D} + \bar{A}\bar{C} + A\bar{D} + \bar{A}\bar{B}\bar{D}$$

