Implications of FDs and IDs

Haven't discussed whether relation instances can be infinite

- Examples have all been finite sets of tuples
- Hasn't mattered for implication so far

It does make a difference when dealing with FDs and IDs together

SAT

Go back to implication as containment of sets of instances

SAT(M, R) = all instances over scheme R that satisfy all dependencies in M

R will usually be understood, so we write SAT(M)

Have: M implies dependency d if and only if SAT(M) $\subseteq$ SAT({d})

Finite Implication

Let FSAT(M) = all finite instances that satisfy all dependencies in M

Definition: M *finitely implies* dependency d if FSAT(M) $\subseteq$ FSAT({d}).

Can have "finitely implies" without (regular) "implies"

First, a Fact

If r is finite and r satisfies A $\rightarrow$ B, the the number of distinct A-values in r is greater or equal to the number of distinct B-values.

$|r[A]| \geq |r[B]|$

A	B
a1	b1
a2	b2
a3	b1

Finite Implication without Implication

Relation r(AB)

M = {A → B, r[A] ⊆ r[B]}
d = r[B] ⊆ r[A]

Part 1. Assume r is finite

Since A → B, then |r[A]| ≥ |r[B]|
Since r[A] ⊆ r[B], |r[A]| ≤ |r[B]|

So, |r[A]| = |r[B]| and the inclusion must be an equality: r[A] = r[B]

Thus, r[B] ⊆ r[A]

Part 2. Allow r to be infinite

Here is a relation instance that satisfies {A → B, r[A] ⊆ r[B]}, but not r[B] ⊆ r[A]

A	B
1	0
2	1
3	2
…	
i	i-1
…	

What Gives?

It is hard to express "r is finite" in normal logic

Thus generally hard to define FSAT[M] and reason about finite implication

Finite implication is often undecidable