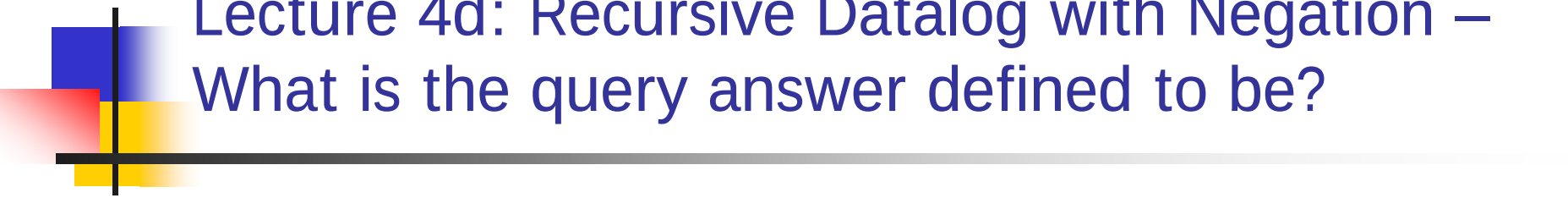

CS589 Principles of DB Systems
Fall 2008
Lecture 4d: Recursive Datalog with Negation – What is the query answer defined to be?

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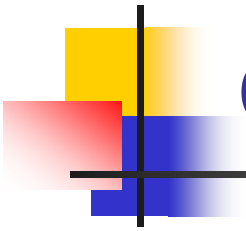


Goals for today

- Introduce the problem with recursion and negation in Datalog.
- Introduce stratification – a syntactic restriction that avoids the problem.
- Mention several ways to define the semantics of a Datalog program.

Several Datalog language

This one is often referred to as "Datalog"; it was defined first.

This one is often referred to as "Datalog¬" or "Datalog-neg"

- Datalog – multiple rules, no negation, with recursion. *Recursion but not relationally complete.*
- Datalog – multiple rules, with negation, no recursion. *Relationally complete but no recursion.*
- Datalog – multiple rules, with negation, with recursion. *Relationally complete with recursion but some queries are ambiguous!*

Our focus for QL equiv.

Our focus now

Datalog program and it's dependency graph

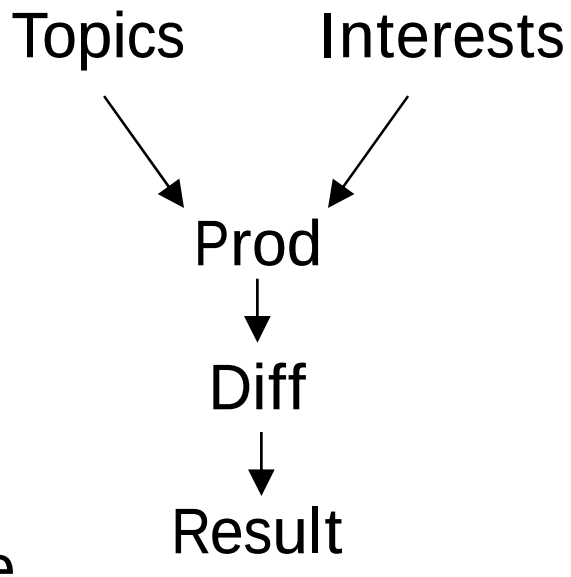
Given a DB with two relations:
Topics(Topic) and Interests(Person,Topic)

What is this query computing?

Result(a):-Interests(a,b,), ¬DIFF(a).

Diff(a) :- Prod(a,b), ¬Interests(a,b).

Prod(a,b) :- Interests(a,c), Topics(b).



Create one node for each body predicate.

Draw an arrow from body predicate
to corresponding head predicate.

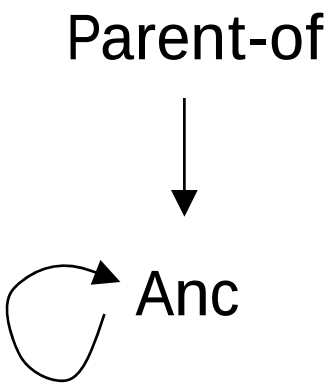
Acyclic dependency graph ≡ not recursive.

Another example

Anc(x,y) :- Parent-of(x,y).

Anc(x,z) :- Anc(x,y), Parent-of(y,z).

Recursive Datalog program
has a cycle in the dependency graph.



Now consider Datalog with recursion & negation

Person(Dan). (one tuple in base relation)

Student(x) :- Person(x), ¬Employee(x).

Employee(x) :- Person(x), ¬Student(x).

What is the query answer?

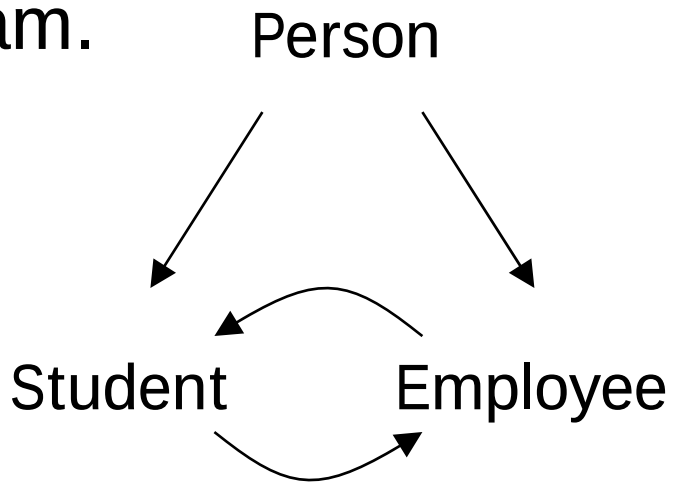
What does the dependency graph look like?

Person(Dan). (one tuple in base relation)

Student(x) :- Person(x), ¬Employee(x).

Employee(x) :- Person(x), ¬Student(x).

This is a recursive program.



More about this dependency graph

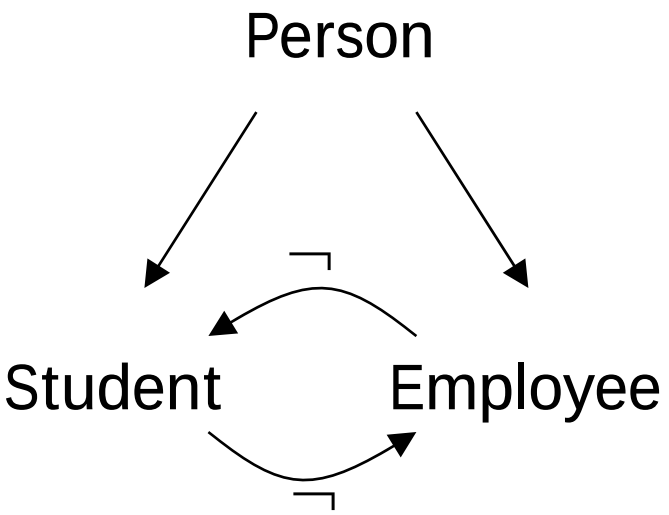
Person(Dan). (one tuple in base relation)

Student(x) :- Person(x), ¬Employee(x).

Employee(x) :- Person(x), ¬Student(x).

Cycle in dependency graph → recursion.

Label negative predicates in dependency graph.



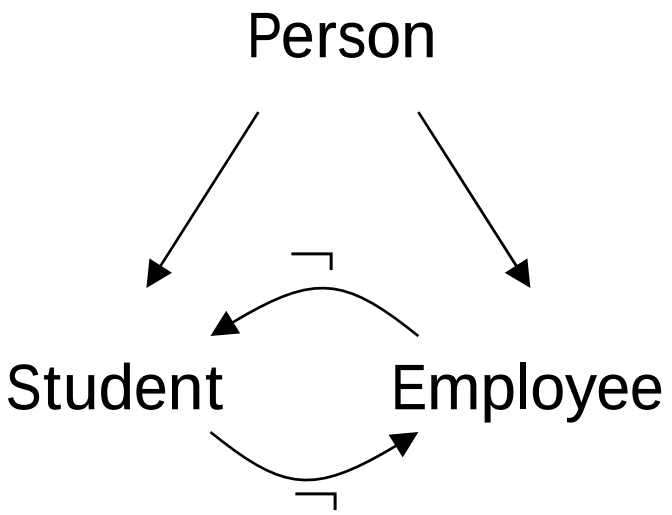
Now consider recursion & negation

Person(Dan). (one tuple in base relation)

Student(x) :- Person(x), ¬Employee(x).

Employee(x) :- Person(x), ¬Student(x).

We have a problem when a cycle in a dependency graph includes (at least) one negative edge.



Another example with negation & recursion

Node(a).

Node(b).

Node(c).

Node(d). These are facts from the database.

Arc(a,b).

Arc(c,d).

Reachable(a).

Reachable(y) :- Reachable(x), Arc(x,y).

Unreachable(x) :- Node(x), ¬Reachable(x).

Stratification – for Datalog with negation and recursion

A stratification of a Datalog program P is a partition of P into strata or layers where, for every rule, H :- $A_1, \ldots, A_m, \neg B_1, \ldots, B_q$

- The rules that define the predicate H are all in the same strata.
- For all positive predicates in this rule, their strata is less than or equal to the strata of this rule. $Strata(A_i) \leq Strata(H)$.
- For all negative predicates in this rule, their strata is strictly less than the strata of this rule. $Strata(B_i) < Strata(H)$.

Exercise: Define a stratification for this datalog program.

Node(a).

Node(b).

Node(c).

Node(d).

Arc(a,b).

Arc(c,d).

Reachable(a).

Compute all Reachable facts (positively) in Strata 1. Then, use Reachable negatively in Strata 2.

Strata 1 Reachable(y) :- Reachable(x), Arc(x,y).

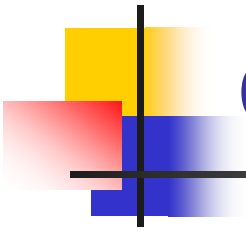
Strata 2 Unreachable(x) :- Node(x), ¬Reachable(x).

Can you define a stratification for this program?

Person(Dan). (one tuple in base relation)

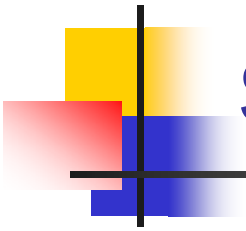
Student(x) :- Person(x), ¬Employee(x).

Employee(x) :- Person(x), ¬Student(x).



Comments

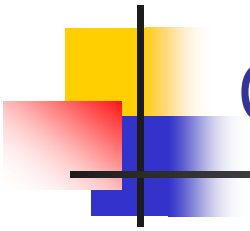
- Stratified Datalog programs have a unique answer.
- Not all Datalog programs can be stratified.



Semantics of a Datalog Program

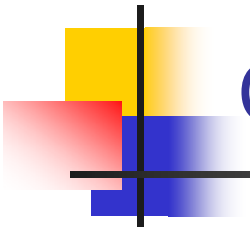
- The fixed point of the “immediate consequence” operator, applied to the Datalog program.
- The minimal model for the Datalog program.

Plus others, particularly for Datalog with negation.



Comments

- For Datalog with recursion, but NO negation, then:
  - The minimal model is unique.
  - The minimal model is always the intersection of all the models.
  - The minimal model is the same as the fixed point of the immediate consequence operator.
  - This language is monotonic (rules only add facts)
- For Datalog with recursion & negation:
  - There may not be a unique minimal model.



Questions

- If a predicate appears in the head of one rule and in the body of another, is the Datalog program necessarily recursive?
- If a datalog program is not recursive, how many times to we have to fire the rules?