

## Implications of FDs and IDs

Haven't discussed how infinite relations affect implication

- Examples have all been finite sets of tuples
- Hasn't mattered for combinations of dependencies seen so far

It does make a difference when dealing with FDs and IDs together

## SAT

Go back to implication as containment of sets of instances

$SAT(M, R)$  = all instances over scheme  $R$  that satisfy all dependencies in  $M$

$R$  will usually be understood, so we write  $SAT(M)$

Have:  $M$  implies dependency  $d$  if and only if  $SAT(M) \subseteq SAT(\{d\})$

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### Finite Implication

Let  $FSAT(M)$  = all finite instances that satisfy all dependencies in  $M$

**Definition:**  $M$  *finitely implies* dependency  $d$  if  $FSAT(M) \subseteq FSAT(\{d\})$ .

Can have "finitely implies" without (regular) "implies"  
But not vice versa

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### First, a Fact

If  $r$  is finite and  $r$  satisfies  $A \rightarrow B$ , then the number of distinct  $A$ -values in  $r$  is greater or equal to the number of distinct  $B$ -values.

$$|r[A]| \geq |r[B]|$$

<u>A</u>	<u>B</u>
a1	b1
a2	b2
a3	b1

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## Finite Implication without Implication

### Relation $r(AB)$

$$M = \{A \rightarrow B, r[A] \subseteq r[B]\}$$

$$d = r[B] \subseteq r[A]$$

### Part 1. Assume $r$ is finite

$$\text{Since } A \rightarrow B, \text{ then } |r[A]| \geq |r[B]|$$

$$\text{Since } r[A] \subseteq r[B], |r[A]| \leq |r[B]|$$

So,  $|r[A]| = |r[B]|$  and the inclusion must be an equality:

$$r[A] = r[B]$$

$$\text{Thus, } r[B] \subseteq r[A]$$

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### Part 2. Allow $r$ to be infinite

Here is a relation instance that satisfies  $\{A \rightarrow B, r[A] \subseteq r[B]\}$ , but not  $r[B] \subseteq r[A]$

<u>A</u>	<u>B</u>
1	0
2	1
3	2
...	
i	i-1
...	

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## What Gives?

It is hard to express "r is finite" in normal logic

Thus generally hard to define FSAT[M] and reason about finite implication

Finite implication is often undecidable