Implications of FDs and IDs

Haven't discussed how infinite relations affect implication

- Examples have all been finite sets of tuples
- Hasn't mattered for combinations of dependencies seen so far

It does make a difference when dealing with FDs and IDs together

SAT

Go back to implication as containment of sets of instances

SAT(M, R) = all instances over scheme R that satisfy all dependencies in M

R will usually be understood, so we write SAT(M)

Have: M implies dependency d if and only if $SAT(M) \subseteq SAT(\{d\})$

Finite Implication

Let FSAT(M) = all finite instances that satisfy all dependencies in M

Definition: M *finitely implies* dependency d if FSAT(M) $\subseteq$ FSAT({d}).

Can have "finitely implies" without (regular) "implies"

But not vice versa

First, a Fact

If r is finite and r satisfies A $\rightarrow$ B, then the number of distinct A-values in r is greater or equal to the number of distinct B-values.

|r[A]| ≥ |r[B]|

# of tuples (cardinality)

A	B
a1	b1
a2	b2
a3	b1

Finite Implication without Implication

Relation r(AB)

$M = \{A \to B,\ r[A] \subseteq r[B]\}$
$d = r[B] \subseteq r[A]$

Part 1. Assume r is finite

Since $A \to B$, then $|r[A]| \geq |r[B]|$
Since $r[A] \subseteq r[B]$, $|r[A]| \leq |r[B]|$

So, $|r[A]| = |r[B]|$ and the inclusion must be an equality: $r[A] = r[B]$

Thus, $r[B] \subseteq r[A]$

Part 2. Allow r to be infinite

Here is a relation instance that satisfies $\{A \to B,\ r[A] \subseteq r[B]\}$, but not $r[B] \subseteq r[A]$

A	B
1	0
2	1
3	2
…	
i	i-1
…	

$0 \notin r[A]$

What Gives?

It is hard to express “r is finite” in normal logic

Thus generally hard to define FSAT[M] and reason about finite implication

Finite implication is often undecidable