CS 589, Principles of Database Systems, Unit 2, Notes 1

Unit 2
Dependencies and Inference

Model Theoretic View of Databases

A database schema is a theory or specification

The database state is a model or instance that satisfies the specification

- When is this view useful?
  - discussing constraints and implication
  - analyzing representation equivalence of two schemata
  - modeling incomplete information

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"Specification" Part

relation name

`assigned( PILOT FLIGHT DATE TIME)`

relation scheme

attribute

Each attribute A has an dom(A)

So the relation scheme defines attributes and allowable values for a conforming relation instance

Relation Instance

relation instance

`assigned(`

PILOT	FLIGHT	DATE	TIME)
Cushing	83	9 Aug	10:15a
Cushing	116	10 Aug	1:25p
Clark	281	8 Aug	5:50a
Clark	301	12 Aug	6:35p
Clark	83	11 Aug	10:15a
Chin	83	13 Aug	10:15a
Chin	116	12 Aug	1:25p
Copley	281	9 Aug	5:50a
Copley	281	13 Aug	5:50a
Copley	412	15 Aug	1:25p

Expect a relation instance to satisfy requirements of the schema

Data dependencies: further constraints that the schema puts on a satisfying instance

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Functional Dependency

assigned(

PILOT	FLIGHT	DATE	TIME)
Cushing	83	9 Aug	10:15a
Clark	83	11 Aug	10:15a
Chin	83	13 Aug	10:15a
Cushing	116	10 Aug	1:25p
Chin	116	12 Aug	1:25p
Clark	281	8 Aug	5:50a
Copley	281	9 Aug	5:50a
Copley	281	13 Aug	5:50a
Clark	301	12 Aug	6:35p
Copley	412	15 Aug	1:25p

A generalization of keys: when tuples agree on certain attributes, must agree on others

```
FLIGHT → TIME                   (L → T)
PILOT DATE TIME → FLIGHT        (PDT → L)
FLIGHT DATE → PILOT             (LD → P)
```


Functional Dependency Definition

A relation instance r(R) satisfies the *functional dependency* (FD) X → Y (X and Y subsets of schema(R)), if

```
for every two tuples t,s in r,
if t[X] = s[X],
then t[Y] = s[Y]
```


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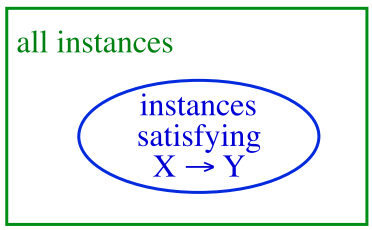
FDs as Specification

Usually, we don't ask which FDs a relation instance satisfies.

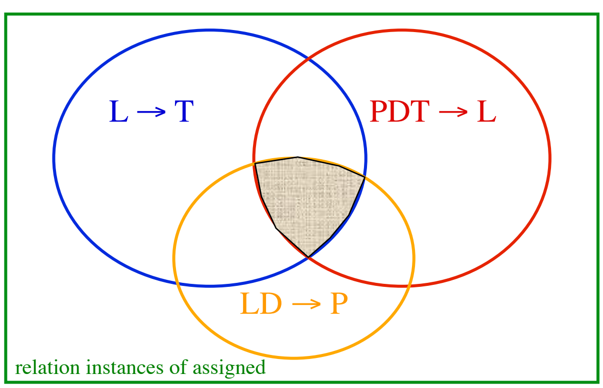
Rather, specify FDs that we expect all instances of to satisfy as part of the relation schema



Sets of FDs

A relation instance r satisfies a set of FDs F if it satisfies every FD in F

F = {L → T, PDT → L, LD → P}



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Implication

If an instance r satisfies FDs F, there may be another FD X → Y it necessarily satisfies

Say that F *implies* X → Y (F ⊨ X → Y)

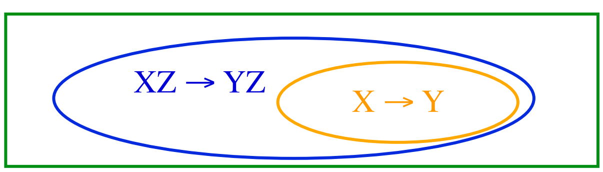
```
Consider L → T
Let r be a relation instance
  satisfying it
take t,s in r where t[PL] = s[PL]
so t[L] = s[L]
by L → T, t[T] = s[T]
thus r also satisfies PL → T
```


Inference Axioms

General patterns that describe implications. For X, Y, Z, W sets of attributes

F1. Reflexivity: X → X

F2. Augmentation: if X → Y, then XZ → YZ



F3. Additivity: if X → Y and X → Z, then X → YZ

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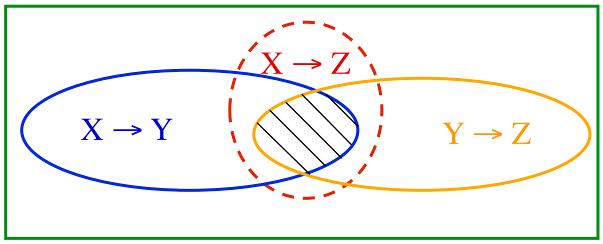
More Axioms

F4. Projectivity: if $X \rightarrow YZ$, then $X \rightarrow Y$

F5. Transitivity: if $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$



F6. Pseudotransitivity: if $X \rightarrow Y$ and $YZ \rightarrow W$, then $XZ \rightarrow W$

Can Do Proofs

```
F = {L → T, PDT → L, LD → P}

1. L → T (in F)
2. LD → TD (augmentation from 1.)
3. LD → P (in F)
4. LD → PTD (additivity from 2.
                             and 3.)
```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Derivation

Say that F *derives* X → Y if there is a proof of X → Y from FDs in F using inference axioms F1. - F6.

$F \vdash X \rightarrow Y$

Is implies the same as derives?

soundness

$F \models X \rightarrow Y$ if and only if $F \vdash X \rightarrow Y$

completeness

Soundness

Prove that each inference axiom is correct. That is, it holds in any relation instance.

F5. If X → Y and Y → Z, then X → Z

Take any tuples t,s in r, where r satisfies X → Y and Y → Z.

Suppose t[X] = s[X]. Then from X → Y, must have t[Y] = s[Y].

But by Y → Z, then have t[Z] = s[Z].

So r satisfies X → Z.

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Definitions

Closure of F, F^+, is F plus all FDs that can be derived from F by F1. - F6.

So $F \vdash X \rightarrow Y$ means $X \rightarrow Y$ in F^+

Closure of X, X^+, is largest Z such that $X \rightarrow Z$ is in F^+

{L → T, PDT → L, LD → P}

What is $(LD)^+$

What is T^+

Completeness

If it's true, you can prove it

or

If you can't prove it, it's not true.

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Instance to show not true

Take any $X \rightarrow Y$ not in F^+ over R.

Consider X^+ and let $X^- = R - X^+$.

	X+ atts	X- atts
t	a1 a2 … an	b1 b2 … bm
s	a1 a2 … an	c1 c2 … cn

bi ≠ ci

This table violates $X \rightarrow Y$.

- $t[X] = s[X]$, but claim $t[Y] \neq s[Y]$
- if $t[Y] = s[Y]$, then $Y \subseteq X^+$
- then since $X \rightarrow X^+$ in F^+, so is $X \rightarrow Y$ by projectivity, a contradiction

Principles of Database Systems

The instance satisfies F^+

Take any $W \rightarrow Z$ in F^+

Case 1. $W \not\subseteq X^+$.

- Then $t[W] \neq s[W]$, no problem

Case 2. $W \subseteq X^+$.

- Then $t[W] = s[W]$
- Have $X \rightarrow X^+$ in F^+, $X^+ \rightarrow W$ by reflexivity and projectivity, and $W \rightarrow Z$ in F^+.
- That gives $X \rightarrow Z$ by transitivity applied twice
- Additivity on $X \rightarrow X^+$ and $X \rightarrow Z$ gives $X \rightarrow X^+Z$

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Case 2, continued

We have proved $X \rightarrow X^{+}Z$ is in F^{+}

But wait! X^{+} is supposed to be the maximum set of attributes that X determines.

Must be that $Z \subseteq X^{+}$.

So t[Z] = s[Z]

Thus our instance satisfies $W \rightarrow Z$.

Armstrong Relation for F

An "exact model" for F:

- Satisfies F^{+}
- Violates every FD in F^{-}

Use the 2-tuple instances in the last proof, combined into one big instance

Example R = ABC, F = $\{A \rightarrow B, B \rightarrow C\}$

Only need FDs in F^{-} with one attribute on right side

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Armstrong Relation

$C \to A, C \to B, B \to A, BC \to A$

A	B	C
a1	b1	c1
a2	b2	c1
a3	b3	c2
a4	b4	c2
a5	b5	c3
a6	b5	c3
a7	b6	c4
a8	b6	c4

Join Dependency

Note the redundancy in the `assigned` table: repeat time for each flight.

Can split up

asgn1(

PILOT	FLT	DATE)
Cushing	83	9 Aug
Clark	83	11 Aug
Chin	83	13 Au
Cushing	116	10 Aug
Chin	116	12 Aug
Clark	281	8 Aug
Copley	281	9 Aug
Copley	281	13 Aug
Clark	301	12 Aug
Copley	412	15 Aug

asgn2(

FLT	TIME)
83	10:15a
116	1:25p
281	5:50a
301	6:35p
412	1:25p

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Can Recover with a Join

assigned = asgn1 ⋈ asgn2

assigned satisfies a *join dependency* (JD)

```
⋈[PLD, LT]

assigned =
 π_PLD(assigned) ⋈ π_LT(assigned)
```

$assigned = \pi_{PLD}(assigned) \bowtie \pi_{LT}(assigned)$

A "lossless" join

What about ⋈[PL, PDT]?

```
asgn3(PILOT   FLT) asgn4(PILOT    DATE   TIME)
     Cushing  83       Cushing   9 Aug 10:15a
     Cushing 116       Cushing 10 Aug  1:25p
       . . .             . . .


asgn3⋈asgn4(PILOT   FLT   DATE    TIME)
           Cushing  83   9 Aug  10:15a
           Cushing  83 10 Aug   1:25p
               . . .
```


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Join Dependency, General Form

r(R), with R1, R2, …, Rn subsets of R

r satisfies $\bowtie$[R1, R2, …, Rn] if

$r = \pi_{R1}(r) \bowtie \pi_{R2}(r) \bowtie \ldots \bowtie \pi_{Rn}(r)$

notice that always have

$r \subseteq \pi_{R1}(r) \bowtie \pi_{R2}(r) \bowtie \ldots \bowtie \pi_{Rn}(r)$

A *multivalued dependency* is a special case where n = 2.

Implications of FDs and JDs

There are some axioms

```
X → Y on R, Z = R - XY
FJ1. X → Y implies ⋈[XY, XZ]

Example
L → T, so ⋈[LT, LPD]
```

However, there is no finite, complete set of axioms for just FDs and JDs (or JDs alone).

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Are We Stuck?

No

- There are complete axiom sets for FDs and multi-valued dependencies.
- There are complete axiom sets for classes of constraints that include FDs and JDs.
- There is an *inference procedure* for FD and JD implication.
  not based on axioms

Need to Use Tableaux

A "template" for a relation

Has *rows* of variables instead of tuples of values

	U(P	L	D	T)
v	*p1*	*f1*	*d1*	*m1*
w	*p2*	*f1*	*d2*	*m2*

Treat tableau rows like tuples

w(L) = *f1*

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Valuation

A *valuation* ρ for tableau U maps its variables to domain values.

ρ(v) must be in dom(*A*) if v in *A* column

ρ(*p1*)=Chin ρ(*f1*)=86 ρ(*d1*)=1 June ρ(*m1*)=4p
ρ(*p2*)=Chao ρ(*d2*)=2 June ρ(*m2*)=4p

Extend Valuation to Rows and Tableaux

ρ(w) = apply ρ to each variable in w

ρ(w) **(***A***)** = ρ(w**(***A***)**)

ρ(*p1 f1 d1 m1*)=<Chin 86 1June 4p>

ρ(U) = {ρ(w) | w in U}

ρ(**U**)	**P**	**L**	**D**	**T**
ρ(v)	Chin	86	1June	4p
ρ(w)	Chao	86	2June	4p

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How About This Valuation?

ρ(*p1*)=Chin ρ(*f1*)=86 ρ(*d1*)=1 June ρ(*m1*)=4p
ρ(*p2*)=Chao ρ(*d2*)=2 June ρ(*m2*)=5p

ρ(U) (

	P	L	D	T)
ρ(v)	Chin	86	1June	4p
ρ(w)	Chao	86	2June	5p

Want to Enforce L → T

Need to make sure that ρ(m1) = ρ(m2).

Can we "fix" the tableau?

U	(P	L	D	T)
v	*p1*	*f1*	*d1*	*m1*
w	*p2*	*f1*	*d2*	***m1***

Have "applied" L → T to tableau U.

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The Chase

Apply FDs and JDs to a tableau, to reason about an example relation (or sub-relation)

Tableau rule for FD X → Y

For rows v and w

If v[X] = w[X], make v[Y] = w[Y] by equating variables in Y

[Book gives rules for replacing variables]

FD chase of tableau U for F: Apply FD rules for FDs in F to U until no change.

Example

{AB → E, AG → C, BE → D, E → G, DG → H}

A	B	C	D	E	G	H
a1	a2	a3	a4	a5	a6	a7
a1	b2	b3	b4	a5	b6	b7

Apply E → G

A	B	C	D	E	G	H
a1	a2	a3	a4	a5	a6	a7
a1	b2	b3	b4	a5	**a6**	b7

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Apply $AG \to C$

A	B	C	D	E	G	H
a1	a2	a3	a4	a5	a6	a7
a1	b2	**a3**	b4	a5	a6	b7

Testing FD Implication with the Chase

Method in book is ambiguous

Want to test $F \models X \to Y$ on scheme
$R = A1\ A2 \ldots An$

1. Set up a tableau U_X with rows v, w:
   v(Ai) = ai
   w(Ai) = ai if Ai in X
   = bi otherwise
2. Chase U_X with FDs in F
3. If v[Y] = w[Y], then $F \models X \to Y$
   (if not, we have a counterexample)

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Example

F = {AB → C, B → D, CD → E, CE → GH, G → A} ⊨ BG → C?

A	**B**	**C**	**D**	**E**	**G**	**H**
a1	*a2*	*a3*	*a4*	*a5*	*a6*	*a7*

You Try It

F = {AB → E, AG → C, BE → D, E → G, DG → H}

A	**B**	**C**	**D**	**E**	**G**	**H**
a1	*b1*	*c1*	*d1*	*e1*	*g1*	*h1*

Does F ⊨ BE → A?

Does F ⊨ BE → H? (can use same chase)

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