

3A. Assume schemas are $r(R)$, $s(S)$, $u(U)$.

$$\begin{aligned}
 r \bowtie (s \cup u) &= \text{expand} \{ t1[S] \mid t1[S] \in s \text{ or } t1[S] \in u \} \\
 &= \text{expand} \{ t2[RS] \mid t2[R] \in r \text{ and } t2[S] \in \{ t1[S] \mid t1[S] \in s \text{ or } t1[S] \in u \} \} \\
 &= \text{simplify} \{ t2[RS] \mid t2[R] \in r \text{ and } (t2[S] \in s \text{ or } t2[S] \in u) \} \\
 &= \text{distribute} \{ t2[RS] \mid (t2[R] \in r \text{ and } t2[S] \in s) \text{ or } (t2[R] \in r \text{ and } t2[S] \in u) \}^*
 \end{aligned}$$

$$\begin{aligned}
 (r \bowtie s) \cup (r \bowtie u) &= \text{expand} \{ t3[RS] \mid t3[R] \in r \text{ and } t3[S] \in s \} \cup \\
 &\quad \{ t4[RS] \mid t4[R] \in r \text{ and } t4[S] \in u \} \\
 &= \text{expand} \{ t5[RS] \mid t5[RS] \in \{ t3[RS] \mid t3[R] \in r \text{ and } t3[S] \in s \} \\
 &\quad \text{or } t5[RS] \in \{ t4[RS] \mid t4[R] \in r \text{ and } t4[S] \in u \} \} \\
 &= \text{simplify} \{ t5[RS] \mid (t5[R] \in r \text{ and } t5[S] \in s) \text{ or } \\
 &\quad (t5[R] \in r \text{ and } t5[S] \in u) \}
 \end{aligned}$$

which can be made equal to (*) by renaming of $t5$ to $t2$.

3Ba. Consider $r \begin{matrix} (A & B) \\ \hline 1 & 2 \end{matrix}$ and $s \begin{matrix} (B & C) \\ \hline 4 & 3 \end{matrix}$ with $X = A$

$$\Pi_{ABC}(r \bowtie s) = \begin{matrix} A & B & C \\ \hline & & \emptyset \end{matrix} \quad \Pi_A(r) \bowtie s = \begin{matrix} A & B & C \\ \hline 1 & 4 & 3 \end{matrix}$$

6. It's possible to show inequivalence here by choosing X to get different schemas on left and right, say by leaving C out of X . But even if the schemas match there can be problems

3Bc. cont.

Consider $r \left(\begin{array}{ccc} A & B & C \\ \hline 5 & b & c1 \\ 5 & b & c2 \end{array} \right)$ let $X = BCA, Y = B, Z = AB$

Then $\pi_{BCA} (G[B; CA = \text{sum}(A)](r)) = G[B; CA = \text{sum}(A)](\pi_{AB}(r)) =$

$$\begin{array}{cc} B & CA \\ \hline b & 10 \end{array} \quad \begin{array}{cc} B & CA \\ \hline b & 5 \end{array} \quad \text{since } \pi_{AB}(r) = \begin{array}{cc} A & B \\ \hline 5 & b \end{array}$$

3Bb. Consider $r \left(\begin{array}{cc} A & B \\ \hline 1 & 2 \end{array} \right) \quad s \left(\begin{array}{cc} B & C \\ \hline 2 & 3 \end{array} \right) \quad u \left(\begin{array}{cc} C & A \\ \hline 3 & 4 \end{array} \right)$

$r \text{ LOS } (s \text{ LOS } u) = (r \text{ LOS } s) \text{ LOS } u =$

$$\begin{array}{ccc} A & B & C \\ \hline 1 & 2 & 1 \end{array} \quad \begin{array}{ccc} A & B & C \\ \hline 1 & 2 & 3 \end{array}$$

3Bd. Consider $r \left(\begin{array}{cc} A & B \\ \hline 1 & 2 \end{array} \right) \quad u \left(\begin{array}{cc} A & B \\ \hline 1 & 3 \end{array} \right) \quad X = A$

$$\pi_A(r \bowtie u) = \begin{array}{c} A \\ \hline 1 \end{array} \quad \pi_A(r) \bowtie \pi_A(u) = \begin{array}{c} A \\ \hline \emptyset \end{array}$$

3C a. $X \supseteq R \cap S$

b. $X = Y \cup \{CA\}$ so schemas match

$Z \rightarrow R$ so no duplicates are eliminated

(note that $Z \supseteq YA$ is required by the expression being well formed)

3A^Da. Not equivalent

$$\text{Let } r = s = u = \frac{A \ B}{1 \ 2}$$

$$\text{then LHS is } \frac{A \ B}{1 \ 2} \quad \text{RHS is } \frac{A \ B}{1 \ 2}$$

b. Not equivalent. With relations from a.)

$$\text{LHS is } \frac{A \ B}{1 \ 2} \quad \text{RHS is } \frac{A \ B}{1 \ 2}$$

c. Take r as above Not equivalent

$$\text{LHS is } \frac{A \ B}{1 \ 2} \quad \text{RHS is } \frac{A \ B}{1 \ 2}$$

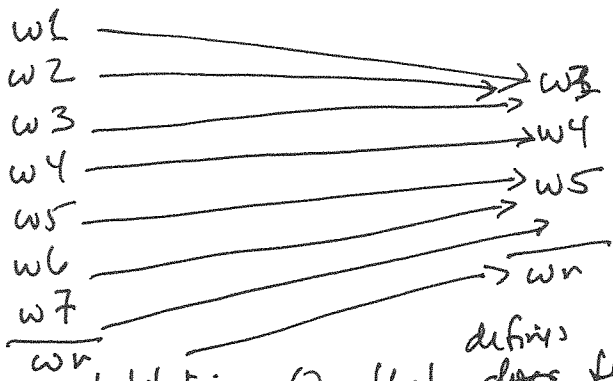
d. Holds — in both LHS and RHS, multiplicity of any tuple is ~~the multiplicity of the~~ sum of the multiplicities in the relations

e. Not equivalent let $r = \frac{A \ B}{1 \ 2}$ and $X = A$
 $1 \ 3$

$$\text{then LHS is } \frac{A}{1} \quad \text{RHS is } \frac{A}{1}$$

f. Holds. Since join doesn't introduce duplicates, ~~there~~ if there are none in the inputs, then neither side has duplicates

3E We can map T down to 3 of its rows



The substitution θ that ~~does~~ ^{defines} this containment mapping is

$$\theta(v) = \frac{a_5 \ b_3 \ d_5 \ e_4 \ b_4}{a_4 \ b_2 \ d_3 \ e \ b_2}$$

3F. Most people went through the minimization and attribute-removal process to get back where they started. However, you might end up with a different set of FDs

① Replace RHS with closures

- $A \rightarrow AB$
- $CD \rightarrow ABCD$
- $CB \rightarrow ABCD$
- $AE \rightarrow ABEG$
- $CE \rightarrow ABCDEG$

② Remove redundant ~~attributes~~ FDs: no change

③ Remove extraneous attributes

- $A \rightarrow B$
- $CD \rightarrow B$
- $CB \rightarrow AD$
- $AE \rightarrow G$
- $CE \rightarrow D$

⋮ Note the difference

④ ~~the~~ Create schemas $(\underline{A} B)$, $(\underline{C} D B)$, $(\underline{C} B A D)$, $(\underline{A} E \rightarrow G)$, $(\underline{C} E \rightarrow D)$

⋮ might close to combine these

⑤ Lossless join because CE is a key for R.