

1. $AB^+ = ABC$ (by $A \to BC$)
   $BC^+ = BC$
   $CD^+ = CD$
   $DE^+ = DE$
   $EF^+ = EF$
   $FG^+ = EFG$ (by $FG \to E$)
   $ABC^+ = ABC$
   $BCD^+ = ABCDE$ (by $BD \to E$, $EC \to A$)
   $CDE^+ = ABCDE$ (by $EC \to A$, $A \to BC$)
   $DEF^+ = DEF$
   $EFG^+ = EFG$

2. Let $Z = X \cap Y$, and suppose $Z^+ \neq Z$ (Z not closed). Since $Z \subseteq Z^+$, it must be that there is an $A \in Z^+$, where $A \notin Z$. By the definition of Z^+, $F \models Z \to Z^+$. By projectivity, $Z \to A$. Now $Z \subseteq X$, so
   $X \to X$ reflexivity
   $X \to Z$ projectivity
   $X \to A$ transitivity from $X \to Z$, $Z \to A$
   So A must be in X^+. Similarly, $A \in Y^+$. Thus $A \in X^+ \cap Y^+$. So
   But $X^+ \cap Y^+ = X \cap Y$ (Since $X = X^+$ and ~~X~~ $Y = Y^+$). So
   $A \in X \cap Y = Z$. So our supposition that $Z^+ \neq Z$ (and hence $A \notin Z$) must be incorrect, so $Z^+ = Z$ and Z is closed.

3. a. Bi-additivity is sound:
   1. $X \to Y$, $Z \to W$ (Given)
   2. $XZ \to YZ$, $XZ \to XW$ (Augmentation)
   3. $XZ \to XWYZ$ (Additivity)
   4. $XZ \to YW$ (Projectivity)

   b. Replacement is not sound
   Let $X = AB$ $Y = C$ $Z = DE$

A	B	C	D	E
1	2	3	4	5
1	2	8	4	5

   Satisfies $ABC \to DE$, ~~$AB \to C$~~, $DE \to AB$, ~~$C \to DE$~~
   not $DE \to C$

   c. Cascade is sound
   1. $X \to Y$, $Y \to Z$ (Given)
   2. $X \to Z$ (Transitivity)
   3. $X \to YZ$ (Additivity)

$\boxed{\text{Note}}$ ~~Of~~ Obviously, there are counterexamples with fewer attributes. I wanted to point out that X, Y, Z are sets of attributes, which most people glossed over

4. U(A B C D E)

A	B	C	D	E
a1	b1	a3	b2	a5
a1	b3	b4	a4	a5
b5	a2	a3	a4	b6

$A \to B \Rightarrow b1 = b3$ $\longrightarrow$ $E \to D \Rightarrow b2 = a4$

U1(A B C D E)

A	B	C	D	E
a1	b1	a3	a4	a5
a1	~~b3~~ b1	b4	a4	a5
b5	a2	a3	a4	b6

$\longrightarrow$ JD (twice) on rows 2+3

U2(A B C D E)

A	B	C	D	E
a1	b1	a3	a4	a5
a1	b1	b4	a4	a5
b5	a2	a3	a4	b6
a1	b1	b4	a4	b6
b5	a2	a3	a4	a5

$\longrightarrow$ JD on rows 1+3

U3(A B C D E)

A	B	C	D	E
a1	b1	a3	a4	a5
a1	b1	b4	a4	a5
b5	a2	a3	a4	b6
a1	b1	b4	a4	b6
b5	a2	a3	a4	a5
a1	b1	a3	a4	b6

U3 is the final chase.

[So we know $G \not\models \bowtie[ACE, ADE, BCD]$.]

5. a. $r \cup s$ need ~~to~~ not satisfy $A \to B$. Consider r(A B) / 1 2, s(A B) / 1 3.
Then ~~r∪s~~ $r \cup s$ has $\langle 1\ 2 \rangle$ and $\langle 1\ 3 \rangle$, a violation.

b, c. The result satisfies $A \to B$. Take tuples $t_1, t_2 \in r \cap s$ ($\in r - s$). Thus $t_1, t_2 \in r$. If $t_1[A] = t_2[A]$, since R satisfies $A \to B$, we must have $t_1[B] = t_2[B]$.
So $r \cap s$ ($r - s$) satisfies $A \to B$.

d. $r \bowtie u$ satisfies $A \to B$. Consider two tuples t_1, t_2 in $r \bowtie u$. Let $s_1 = t_1[AB]$ and $s_2 = t_2[AB]$. Now $s_1 + s_2$ must be in r, in order for t_1 and t_2 to exist in the join. If $t_1[A] = t_2[A]$, then $s_1[A] = s_2[A]$. Since r satisfies $A \to B$, we have $s_1[B] = s_2[B]$, hence $t_1[B] = t_2[B]$.
Hence $r \bowtie s$ satisfies $A \to B$.

e. Not necessarily satisfied:

r(A B)

A	B
1	2

u(B C)

B	C
3	4
5	6

$\pi_A(r) \bowtie u =$

A	B	C
1	3	4
1	5	6

violates $A \to B$