CS 410/584: Test 2, 10 June 2009 Name:____________

You may use the course text and course materials. You may not use computers, other books or other materials. Work individually.

100 points possible. Give your answers on these sheets. (You can ask for extra blank sheets if needed. Put your name on any extra sheets.)

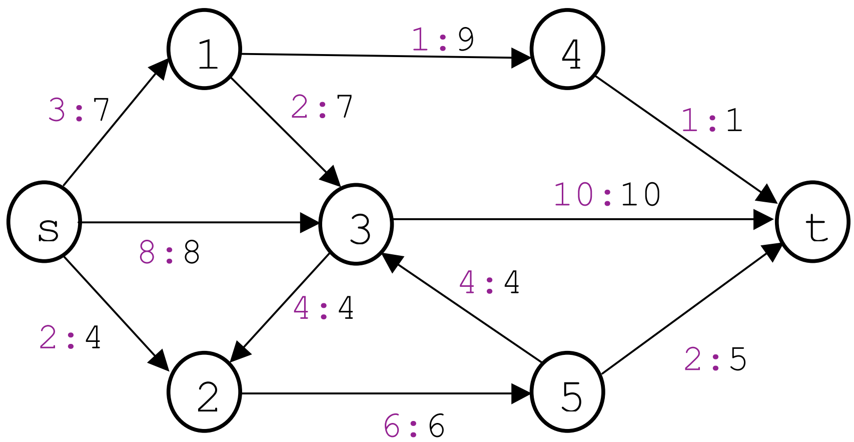
1. String matching (10 points): Give the Knuth-Morris-Pratt failure function for the pattern string
   aabaaab.

Show the state transitions of its matching machine when run against the text
   abaabaabaaabbb.

You can stop at the first match.

2. Network Flow (10 points): Say that a flow f in a flow network G has *useless movement* if there is a directed cycle $n_1, n_2, \ldots, n_k$ where the flow along each edge is positive. For example, the flow below has useless movement because of the cycle 2, 5, 3.

Given a flow $f1$ with useless movement, explain how to get a flow $f2$ with no useless movement with $|f1| = |f2|$.



3. Reductions: Consider the following variation on the long-path problem:

LONG(G, v, w, k): Is there a simple path from v to w in G with length k?

(Assume length is just the number of edges.)

(a) (5 points): Consider the problem

GET-LENGTH(G, v, w): Return the length k of the longest simple path from v to w in G.

Show that if there is a polynomial-time algorithm A for LONG, then there is a polynomial-time algorithm for GET-LENGTH.

(b) (15 points): Consider the problem

FIND-LONGEST(G, v, w): Return a simple path P (as a list of nodes) of maximum length from v to w in G.

Show that if there is a polynomial-time algorithm B for GET-LENGTH, then there is a polynomial-time algorithm for FIND-LONGEST.

4. (0 points) Can you reduce this exam to a previous exam in polynomial time?

5. Matrices: An n × n *permutation matrix P* is an $n \times n$ integer matrix with 0's and 1's such that every row has exactly one 1 and every column has exactly one 1. For example

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

(a) (10 points): Explain why the product of two $n \times n$ permutation matrices must be a permutation matrix.

(b) (10 points): Explain how to multiply two $n \times n$ permutation matrices in $O(n^2)$ time.

6. Polynomials (15 points): Consider polynomials of degree n. Propose a representation of polynomials in which two polynomials can be multiplied in O(1) time and a polynomial can be evaluated at point in O(n) time. Give an example of multiplication.
Hint 1: There is no requirement on how quickly polynomials can be added.
Hint 2: Think about ways we write polynomials other than coefficient and point-value forms.

7. Computational geometry:
(a) (5 points): Let P and Q be triangles given by points $[p_1, p_2, p_3]$ and $[q_1, q_2, q_3]$, respectively. Explain how to use cross product to determine which of P and Q has the larger area.

(b) (5 points): Use your method to decide which of $P = [(-1, 3), (0, 0), (5, 4)]$ and $Q = [(-2, 2), (0, 0), (5, 5)]$ has the larger area.

(c) (15 points): Now consider polygons $P = [p_1, p_2, \ldots, p_m]$ and $Q = [q_1, q_2, \ldots, q_n]$.
410 Only: Describe an $O(m + n)$-time algorithm to compare the areas of P and Q assuming both are convex.
584 Only: Describe an algorithm to compare the areas of P and Q assuming both are simple. What is the time complexity of your algorithm?
Hint: Every simple polygon with > 3 vertices has a *diagonal*: A segment between two vertices that lies inside the polygon, such as p1—p5 below.