

**CS 410/586: Quiz 9, 31 May 2011**

**Name:** KEY

**No books or notes. Work individually.**

Question 9A (10 points): Consider the following two similar problems.

$\text{LONG}(G, v, w, k)$ : Is there a simple path of **length at least  $k$**  from node  $v$  to node  $w$  in directed graph  $G$ ?

$\text{LONGEST}(G, v, w, k)$ : Does the **longest** simple path from node  $v$  to node  $w$  in directed graph  $G$  have length  $k$ ?

In both cases, assume length is just the number of edges in the path.

Suppose **A** is a polynomial-time algorithm for the LONG problem. Explain why there then must be a polynomial-time algorithm **B** for the LONGEST problem. Assume for both **A** and **B** that the graph  $G$  is given as a list of nodes plus a list of edges.

*Given algorithm **A**, we can construct **B** using two calls to **A**:*

*$B\text{-Longest}(G, v, w, k)$*

***if**  $A\text{-Long}(G, v, w, k)$  **and not**  $A\text{-Long}(G, v, w, k+1)$  **then return true**  
**else return false***

*The longest  $v-w$  path in  $G$  has length  $k$  if there is a path of length at least  $k$ , but no path of length  $k+1$  or longer. The conversion of input to **B** to inputs to **A** is trivial:  $O(1)$ . Thus we have a polynomial-time reduction to two calls of a polynomial-time algorithm, so **B** runs in polynomial time.*