CS 410/586: Quiz 7, 17 May 2011 Name:___KEY_____

No books or notes. Work individually.

The product of two complex numbers

$$(a + bi)\cdot(c + di)$$

is

$$(ac - bd) + (ad + bc)i$$

where a, b, c and d are real numbers.

Question 7A (2 points): Compute $(8 + 2i)\cdot(1.5 + 4.5i)$.

*(8*1.5 – 2*4.5) + (8*4.5 + 2*1.5)i =*
(12 – 9) + (36 + 3)i =
3 + 39i

Question 7B (8 points) The obvious way of computing a complex product takes 4 multiplications of real numbers. Show how to compute any complex product with just 3 multiplications of real numbers (and any number of additions and subtractions).

Hint: Consider the two quantities

$$P = (a + b)c$$

and

$$Q = -b(c + d)$$

If we expand P we get ac + bc, which has the ac term we need in the real part of product.
Expanding Q gives –bc – bd, which has the –bd term we need, and moreover will cancel the bc term from P.
So P + Q = ac – bd.
However, each of P and Q take one multiplication, so we aren't really ahead yet.

Note that P contains the bc term from the imaginary part of the product. If we can cancel the ac term and add an ad term, we will have what we need. Thus we need
R = ad – ac, which we can compute with one multiplication as a(d – c).
Thus R + P = ad + bc.

Thus we can compute the two terms with three multiplications (and five additions and subtractions).