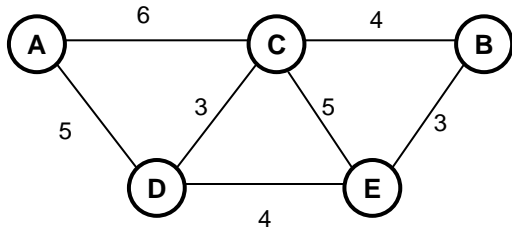


CS 410/586: Quiz 5, 26 April 2011

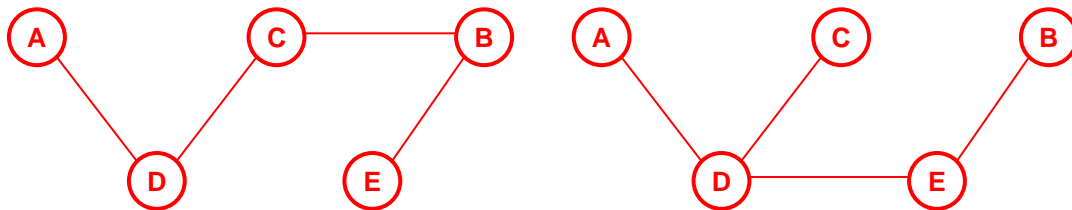
Name: KEY

No books or notes. Work individually.

Questions 5A and 5B concern the labeled, undirected graph $G = (N, E)$ below.



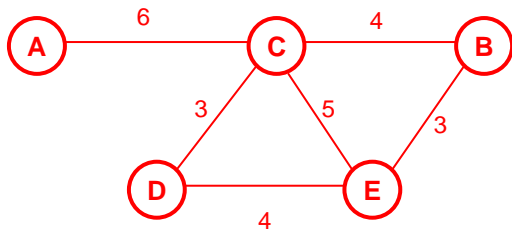
Question 5A (4 points): Give two different minimal spanning trees (MSTs) for G.



Question 5B (3 points): Explain why the edge $\{A, C\}$ cannot be part of any MST for G.

An MST cannot contain all of $\{A, C\}$, $\{A, D\}$ and $\{C, D\}$, since they form a cycle. So at if $\{A, C\}$ were in an MST, then at least one of $\{A, D\}$ and $\{C, D\}$ must be missing. If $\{A, D\}$ were missing, then nodes C and D must be connected by a path in the MST that doesn't go through A. So we could remove $\{A, C\}$ and add $\{A, D\}$ and still have a connected graph, while lowering the cost. If $\{A, D\}$ were present, and $\{C, D\}$ were absent, we could replace $\{A, C\}$ by $\{C, D\}$, and still have connectivity to C. So any spanning tree with $\{A, C\}$ could be transformed to one with lower cost, contradicting minimality.

Question 5C (3 points): Draw a labeled, undirected graph H where the highest-cost edge is part of an MST for H.



Edge $\{A, C\}$ must be selected for connectivity.