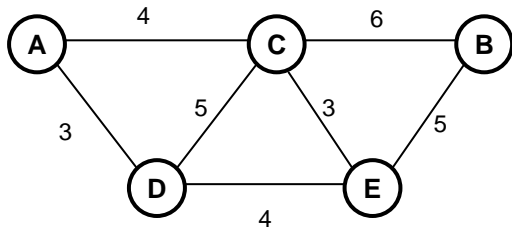


# CS 410/586: Quiz 5, 27 April 2009

Name: \_\_\_\_\_ **KEY** \_\_\_\_\_

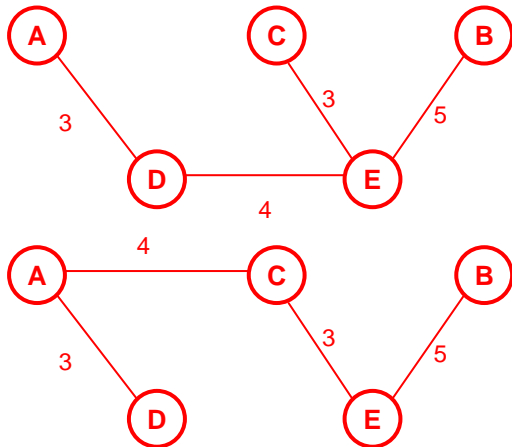
No books or notes. Work individually.

Questions 5A and 5B concern the labeled, undirected graph  $G = (N, E)$  below.



Question 5A (5 points): Give two different minimal spanning trees (MSTs) for  $G$ .

*For Kruskal's algorithm, there comes a choice of two 4-edges, which produce the two MSTs below.*



Question 5B (3 points): Explain why the edge  $\{B, C\}$  cannot be part of any MST for  $G$ .

*Suppose  $\{B, C\}$  were part of an MST, call it  $T$ . Then  $T$  cannot contain both  $\{C, E\}$  and  $\{B, E\}$  as well, or there would be a cycle in  $T$ . If  $\{B, E\}$  is missing, add it, which will create a cycle that includes  $\{B, C\}$ . Remove  $\{B, C\}$  to break the cycle and reduce the cost of  $T$ . If  $\{B, E\}$  is present but  $\{C, E\}$  is missing, we again can add  $\{C, E\}$  and remove  $\{B, C\}$  to reduce the cost of  $T$ .*

Question 5C (2 points): Is it always the case that the highest-cost edge in a graph  $H$  cannot be part of any MST for  $H$ ? Why or why not?

*No, sometimes the highest-cost edge is part of the MST. For example, form a graph  $H$  by removing edge  $\{B, E\}$  from graph  $G$  above. Then any MST of  $H$  needs to include  $\{B, C\}$  to be connected, even though  $\{B, C\}$  has highest cost.*