

CS 410/586: Quiz 3, 13 April 2009 Name: _____ KEY _____

No books or notes. Work individually.

Consider an elevator with maximum capacity M (say, 1000 pounds). We have a collection P of people and we want to know who to put on the elevator, in order to maximize some goal. Consider two different goals, and possible greedy choice steps for them:

Question 1 (5 points): The goal is to put the **most weight** on the elevator without exceeding the maximum load M . Consider the following greedy step:

Step H: Add the **heaviest** person from P to the elevator who won't cause the total weight to go over M .

Will a greedy algorithm based on repeating Step H achieve the goal? Why or why not?

This rule does not have the greedy-choice property. For example, if P has 3 people weighing 600, 500 and 500 pounds, then picking the 600-pound person first means the weight of the load will be 600 pounds. However, there is a better solution picking the two 500-pound persons, for a total load of 1000 pounds.

Question 2 (5 points): The goal is to put the **most people** on the elevator without exceeding the maximum load M . Consider the following greedy step:

Step L: Add the **lightest** person from P to the elevator who won't cause the total weight to go over M .

Will a greedy algorithm based on repeating Step L achieve the goal? Why or why not?

This step does have the greedy choice property. Suppose S is a maximal set of people in P with combined weight within M that does not contain the lightest person p_1 . Let p_2 be any person in S . Then $S - \{p_2\} + \{p_1\}$ has the same number of people, and is still within M . So there is an optimum solution that includes the greedy choice.

[We also need to verify the optimal substructure property. If p_1 is the greedy choice, and S is an optimal solution containing p_1 , then $S' = S - \{p_1\}$ is an optimal solution for $P - \{p_1\}$ with weight bound $(M - \text{weight}(p_1))$. However, I didn't take off any points if you didn't mention this property.]