Homework Assignment #4

CS 410/584 Algorithm Design & Analysis: Spring 2011

This assignment is due Thursday, 12 May, at the beginning of class. You should work alone on this assignment. However, you are free to discuss the problems on the class mailing list, or get help from the TA. (Or you can send me email questions directly; please put "CS 584" at the beginning of the subject line.) Please put "410" or "584" on your paper, depending on which section you are registered in.

Reading: Chapter 26.1-3; Chapter 2.4 (Strassen's Algorithm); Chapter 28.1, 28.2; Chapter 30.

Note: On any homework exercise where you are asked to give an algorithm, you must also provide an English description of how it works and at least one example execution.

4A (15 points): Show the operation of the Edmonds-Karp algorithm on the graph in Figure 26.1a. Show each augmenting path (and indicate the order in which they are used), the final flow, and the value of the final flow. Give a cut whose capacity is equal to the final flow.

4B (10 points): Let A be the adjacency matrix for directed graph G. What does A^2 represent (assuming Boolean matrix multiplication)? Show an example for a graph of at least 5 nodes.

4C (10 points): Explain how you would evaluate $x^{55} + x^{37} + x^6$ at a point using many fewer than 55 multiplications.

4D (584: Do both parts; 410: Do one part – your choice): Let B(n) be the time to multiply two n × n Boolean matrices. Let T(n) be the time to find the transitive closure of an n × n Boolean matrix.
a. (10 points) Show how to use a Boolean matrix-multiplication that runs in B(n) time to compute transitive closure in O(B(n) lg n) time.
b. (10 points) Show how to use a transitive-closure algorithm that runs in T(n) time to multiply Boolean matrices in O(T(n)) time.

4E. (10 points) Show how to use an algorithm that squares an n × n matrix in S(n) time to multiply two matrices in O(S(n)) time.