Homework Assignment #2

CS 410/584 Algorithm Design & Analysis: Spring 2011

This assignment is due Thursday, 14 April, at the beginning of class. You should work alone on this assignment. However, you are free to discuss the problems on the class mailing list or with me. (Or you can send me email questions directly; please put “CS 584” at the beginning of the subject line.)

Reading: 15.1-15.4, 16.1-16.3.

Recall: On any homework exercise where you are asked to give an algorithm, you must also provide an English description of how it works and at least one example execution.

2A (5 points) Explain why a good divide-and-conquer algorithm, such as the Freq algorithm from class, is not a good candidate for memoization.

2B (10 points) Suppose we try to solve the matrix-chain problem in a top-down fashion, by always trying to split the product $M_i \ldots M_j$ at the matrix $M_k$ that minimizes $p_{i-1}p_kp_j$. Show that this method can yield a non-optimal solution.

2C (10 points) A bitonic sequence is a list of numbers that increases then decreases, such as 2 5 11 17 40 51 33 31 6. Give an algorithm to find the longest bitonic subsequence of a list of $n$ distinct integers in $O(n^2)$ time. (Remember that a subsequence does not need to be contiguous in the containing sequence.)

2D (410 only; 20 points) Let $A[1..n]$ be an array of floating-point numbers.


b. Illustrate your method on array $A$ below.

```
  i  1  2  3  4  5  6  7  8
A[i] 3.1 -4.2 6.7 -3.2 7.5 2.0 -15.4 4.7
```

2D (584 only; 20 points) We have $n$ people with integer weights $w_1, w_2, \ldots, w_n$. There are two elevators, and we want to know if we can divide the people into two groups that will put equal weights on each elevator. (For example, with weights 3, 4, 6, 2, 11, 4 we can split them into Group 1 = 4, 11 and Group 2 = 3, 6, 2, 4, each with total weight 15).

a. Describe a method to solve the problem in $O(nW)$ time, where $W$ is the maximum weight of any person.

b. Illustrate your method with the weights 4, 6, 14, 4, 4.
2E (15 points) Suppose we have a set of activities (with start and end times) that we want to schedule into meeting rooms. Give an algorithm that will schedule the activities into the fewest number of rooms. What is the time complexity of your algorithm?

2F (10 points) Construct two different Huffman codes for the set of symbols below, with their relative frequencies.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>2</th>
<th>2</th>
<th>4</th>
<th>3</th>
<th>9</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
</tr>
</tbody>
</table>

Give the encoding for “ACEBAG” in each of your codes.