Emissions Minimization Vehicle Routing Problem

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Abstract

Environmental, social and political pressures to limit the impacts associated with greenhouse gas (GHG) emissions are mounting rapidly. To date there has been no or limited research which seeks to reduce emissions as the primary objective of a routing problem despite the fast growth and high impact of commercial vehicles. In the capacitated vehicle routing problem with time windows (VRPTW), it is traditionally assumed that carriers minimize the number of vehicles as a primary objective and distance travelled as a secondary objective without violating time windows, route durations, or capacity constraints. This research focuses on a different problem, the minimization of emissions and fuel consumption as the primary or secondary objective. This creates a new type of VRP which is denoted the Emissions Vehicle Routing Problem or EVRP. This research presents a formulation and solution approaches for the EVRP. Decision variables and properties are stated and discussed. Results obtained using a proposed EVRP solution approach under different levels of congestion are compared and analyzed.

KEYWORDS: Vehicle routing, time dependent travel time, emissions, urban, congestion.
1. Introduction

The fast rate of commercial vehicle activity growth over recent years and the higher impact of commercial vehicles are increasing preexisting concerns over their cumulative effect in urban areas. In particular, environmental, social and political pressures to limit the impacts associated with greenhouse gas (GHG) emissions and our dependence on fossil fuels is mounting rapidly. A key challenge for public transportation agencies companies is to improve the efficiency of urban freight and commercial vehicle movements while ensuring environmental quality, livable communities, and economic growth.

Private companies are also interested in reducing GHG emission not only for marketing purposes, i.e. the more favorable social perception towards companies that are “greening” their operations, but also for economic reasons. The level of GHG emissions is a proxy for fuel consumption in diesel engines and in the near future it is likely that GHG emissions will have a monetary cost. Under cap and trade emissions system initiatives currently being proposed by the federal United States government (and several state governments), emission costs will have a clear economic value, e.g CO₂ emissions in $/kg.

This research aims to formulate, study, and solve a new vehicle routing problem where the minimization of emissions and fuel consumption is the primary objective or is part of a generalized cost function. In addition, departure times and travel speeds become decision variables. To the best of the author’s knowledge, there is no research or formulation that minimizes vehicle emissions when designing routes in congested environments with time-dependent travel speeds, hard time windows, and capacity constraints. This creates a new type of VRP which is denoted the Emissions Vehicle Routing Problem or EVRP.

This research is organized as follows: Section 2 provides the necessary background and a literature review. Section 3 presents the mathematical formulation of the problem. Section 4 formulates the EVRP. Section 5 describes a solution approach and properties of the emission function. Section 6 introduces an algorithm for the EVRP. Section 7 describes experimental results and Section 8 ends with conclusions.

2. Background and Literature Review

There is an extensive literature related to vehicle emissions and several laboratory and field methods are available to estimate vehicle emissions rates [1]. Research indicates that carbon dioxide (CO₂) is the predominant transportation GHG and is emitted in direct proportion to
fuel consumption, with a variation by type of fuel [2]. For most vehicles, fuel consumption and the rate of CO\textsubscript{2} per mile traveled decreases as vehicle operating speed increases up to approximately 55 or 65 mph and then begins to increase again [2]; hence, the relationship between emission rates and travel speed is not linear.

Congestion has a great impact on vehicle emissions and fuel efficiency. In real driving conditions, there is a rapid non-linear growth in emissions and fuel consumption as travel speeds fall below 30 mph [3]. CO\textsubscript{2} emissions double on a per mile basis when speed drops from 30 mph to 12.5 mph or when speed drops from 12.5 mph to 5 mph. Frequent changes in speed, i.e. stop and go traffic conditions, increases emission rates because fuel consumption is a function of not only speed but also acceleration rates [4]. These results were obtained using an emission model and freeway sensor data in California and weighted on the basis of a typical light-duty fleet mix in 2005. The volume of emissions per mile is a function of the speed profile from the departure time until reaching destination.

In congested urban areas with significant speed changes due to recurrent congestion, e.g. predictable low speeds due to capacity constraints at peak hours, departure time must be considered when designing EVRP routes. The Time Dependent Vehicle Routing Problem (TDVRP) takes into account that links in a network have different costs or speeds during the day. Typically this is used to represent varying traffic conditions. The TDVRP was originally formulated by Mandleraki and Daskin [5]. Time dependent models are significantly more complex and computationally demanding than static VRP models; recent approaches to solve the TDVRP can be found in [6-8]. The reader is referred to [8] for an up-to-date and extensive TDVRP literature review.

TDVRP instances are more data intensive than static VRP instances but their solution is likely to achieve environmental benefits in congested area albeit in an indirect way because emissions are not directly optimized [9]. Other researchers have conducted surveys that indicate that substantial emission reductions can be obtained if companies improve the efficiency of routing operations [10, 11]. Woensel et al. [12] used queuing theory to model the impact of traffic congestion on emissions and recommend that private and public decision makers should take into account the high impact of congestion on emissions. Goodchild and Sandoval [13] discuss the factors that affect emissions in urban areas and potential solution methods, case studies, and public policy applications. However, no formulation, solution methods, or results are provided. To the best of the author’s knowledge, there is no published research that deals with the formulation, properties, or solution approaches for the EVRP.
The EVRP considered in this paper has time windows and capacity constraints as well as time-dependent travel times. The paper deals with a static problem, the dispatcher is assumed to know the impact of recurrent congestion on travel speeds, i.e. morning/evening rush hours. For example, in a practical case, the dispatcher/carrier designs the routes the night before the route is serviced; the carrier is committed to visit a specific group costumers within a predetermined and hard time-window.

3. Notation

Using a traditional flow-arc formulation [14], the EVRP with hard time windows and time dependent speeds studied in this research can be described as follows. Let \( G=(V,A) \) be a graph where \( A = \{(v_i,v_j):i \neq j,i,j \in V\} \) is an arc set and the vertex set is \( V = (v_0,...,v_{n+1}) \). Vertices \( v_0 \) and \( v_{n+1} \) denote the depot at which vehicles of capacity \( q_{\text{max}} \) are based. Each vertex in \( V \) has an associated demand \( q_i \geq 0 \), a service time \( g_i \geq 0 \), and a service time window \([e_i,l_i]\); in particular the depot has \( g_0 = 0 \) and \( q_0 = 0 \). The set of vertices \( C = \{v_1,...,v_n\} \) specifies a set of \( n \) customers. The arrival time of a vehicle at customer \( i \), \( i \in C \) is denoted \( a_i \) and its departure time \( b_i \). Each arc \( (v_i,v_j) \) has an associated constant distance \( d_{ij} \geq 0 \) and a travel time \( t_{ij}(b_i) \geq 0 \) which is a function of the departure time from customer \( i \). The set of available vehicles is denoted \( K \). The cost per unit of route duration is denoted \( c_i \); the cost per unit distance traveled is denoted \( c_d \); the cost per unit of emission generated is denoted \( c_e \); and the cost per vehicle is denoted \( c_k \). The unit costs are finite and non-negative real numbers.

Emission Costs

Emission costs are proportional to the amount of GHG emitted which is a function of travel speed and distance traveled. This assumes a market value for the ton of CO\(_2\); however this approach may have limitations as it is difficult to estimate social, health, and environmental costs [15].

To incorporate recurrent congestion impacts and following a standard practice in TDVRP models, the depot working time \([e_0,l_0]\) is partitioned into \( M \) time periods \( T = T^1,T^2,...,T^M \);
each period $T^m$ has an associated constant travel speed $0 \leq s^m$ in the time interval $T^m = [t^m, \bar{T}^m]$.

For each departure time $b_i$ and each pair of customers $i$ and $j$, a vehicle travels a non-empty set of speed intervals $S^{ij}_y(b_i) = \{s^{m^i}_y(b_i), s^{m^i+1}_y(b_i), ..., s^{m^p+1}_y(b_i)\}$ where $s^{m^i}_y(b_i)$ denotes the speed at departure time, $s^{m^p+1}_y(b_i)$ denotes the speed at arrival time, and $p + 1$ is the number of time intervals utilized. The departure time at speed $s^{m^i}_y(b_i)$ takes place in period $T^m$, the arrival time at speed $s^{m^p}_y(b_i)$ takes place in period $T^{m^p}$, and $1 \leq m \leq m + p \leq M$.

For the sake of notational simplicity the departure time will be dropped even though speed intervals and distance intervals are a function of departure time $b_i$. The corresponding set of distances and times travelled in each time period are denoted $D^{ij}_y(b_i) = \{d^{m^i}_y, d^{m^i+1}_y, ..., d^{m^p+1}_y\}$ and $T^{ij}_y(b_i) = \{t^{m^i}_y, t^{m^i+1}_y, ..., t^{m^p+1}_y\}$ respectively. The following conditions are necessary:

\[
\begin{align*}
t^{m^i}_y(b_i) &= \sum_{i=0}^{i=p} t^{m^i}_y \\
d^{m^i}_y &= \sum_{i=0}^{i=p} d^{m^i}_y \\
d^{m^i+l^i} &= t^{m^i+l^i} \cdot s^{m^i+l^i} \quad \forall \{0, 1, ..., p\}
\end{align*}
\]

\[
\begin{align*}
t^m_y &\leq t^m_y = b_i \leq \bar{t}^m_y \\
t^{m+p}_y &\leq t^{m+p}_y = a_j \leq \bar{t}^{m+p}_y
\end{align*}
\]

For heavy duty vehicles, the Transport Research Laboratory has developed a function that links emissions and travel speeds [16]:

\[
\alpha_0 + \alpha_1 s^{l^i}_y + \alpha_2 (s^{l^i}_y)^3 + \alpha_3 \frac{1}{(s^{l^i}_y)^2} d^{l^i}_y
\]

(1)

The coefficients $\{\alpha_0, \alpha_1, \alpha_2, \alpha_3\} = \{1,576 ; -17.6 ; 0.00117 ; 36,067\}$ are constant parameters for each vehicle type and for other vehicle types there may be other polynomial terms or their inverse [16]. The optimal travel speed that minimizes emissions is assumed to be the speed $s^*$, which for expression (1) the value is $s^* \approx 44$ mph or 71 kmh. Expression (1) outputs
CO2 emissions in Kg/km when the speed is expressed in kmh. As congestion increases, the amount and cost of emissions increases dramatically [3], see Figure 1 which has been produced for light utility vehicles in California highways real-world conditions. The volume of emissions generated by travelling from customer $i$ to customer $j$ and departing at time $b_i$ is denoted $v_y(b_i)$:

$$v_y(b_i) = \sum_{j=0}^{n} (\alpha_0 + \alpha_1 s_y^j + \alpha_2 s_y^j + \alpha_3 \frac{1}{(s_y^j)^2})d_{ij}$$

(2)

Figure 1. CO2 emissions as a function of average speed – source reference [3].

Total emission costs for a departure time $b_i$ is the product $c_y v_y(b_i)$. Expression (2) provides a simple yet good approximation for real-world CO2 emissions vs. travel speed profiles. Finally, in the EVRP, the emission function can be tailored to the travel/path characteristics between any two customers $i$ and $j$.

4. Problem Formulation

Two formulations are presented. The first formulation assumes a multi-objective function that includes the costs of vehicles, distance travelled, route durations, and emissions. The second formulation follows the more traditional hierarchical approach.
Formulations of the VRP have only one type of decision variable, \( x_{ij}^k \). There are two decision variables in the EVRP formulation; \( x_{ij}^k \) is a binary decision variable that indicates whether vehicle \( k \) travels between customers \( i \) and \( j \). The real decision variable \( y_{ij}^k \) indicates service start time if customer \( i \) is served by vehicle \( k \); hence the departure time is given by the customer service start time plus service time \( b_i = y_{ij}^k + g_i \). The real variable \( y_{ij}^k \) allows for waiting at customer \( i \); service start time may not necessarily be the same as arrival time, formally:

\[
a_i + g_i \leq b_i = \sum_{j \in I} \sum_{k \in K} (y_{ij}^k + g_i)x_{ij}^k
\]

(3)

In addition, travel speed is also a decision variable because the amount of emissions is a function of travel speed. However, under some mild assumptions, the EVRP problem can be simplified as analyzed in Section 5. It is assumed that vehicle engines are turned off while serving a customer (service times are \( \frac{1}{2} \) hour or longer); the emissions generated by a stop do not change because the total number of stops (customers) is constant. In this research stopping after leaving a customer is not allowed.

(a) **Total Cost Minimization EVRP**

minimize

\[
\sum_{k \in K} \sum_{j \in C} c_k x_{ij}^k + c_d \sum_{k \in K} \sum_{(i,j) \in V} d_{ij} x_{ij}^k + c_i \sum_{k \in K} \sum_{j \in C} (y_{ij}^k - y_{ij}^0) x_{ij}^k + \sum_{k \in K} \sum_{(i,j) \in V} x_{ij}^k c_v y_{ij}^k (y_{ij}^k + g_i)
\]

subject to:

\[
\sum_{i \in C} \sum_{j \in V} x_{ij}^k \leq q_{\text{max}}, \forall k \in K
\]

(5)

\[
\sum_{k \in K} \sum_{j \in V} x_{ij}^k = 1, \forall i \in C
\]

(6)

\[
\sum_{i \in V} x_{ij}^k - \sum_{i \in V} x_{ij}^k = 0, \forall i \in C, \forall k \in K
\]

(7)

\[
x_{i0}^k = 0, x_{n+1,i}^k = 0, \forall i \in V, \forall k \in K
\]

(8)

\[
\sum_{j \in V} x_{ij}^k = 1, \forall k \in K
\]

(9)
\[
\sum_{j \in V} x_{j,n+1}^k = 1, \quad \forall k \in K
\]  
\[e \sum_{j \in V} x_{i,j}^k \leq y_i^k, \quad \forall i \in V, \forall k \in K \]  
\[
I \sum_{j \in V} x_{i,j}^k \geq y_j^k, \quad \forall i \in V, \forall k \in K
\]  
\[
x_{i,j}^k (y_i^k + g_i + t_{i,j}(y_j^k + g_j)) \leq y_j^k, \quad \forall (i,j) \in A, \forall k \in K
\]  
\[
x_{i,j}^k \in \{0,1\}, \quad \forall (i,j) \in A, \forall k \in K
\]  
\[
y_i^k \in \mathbb{R}, \quad \forall i \in V, \forall k \in K
\]  
The constraints are defined as follows: vehicle capacity cannot be exceeded (5); all customers must be served (6); if a vehicle arrives at a customer it must also depart from that customer (7); routes must start and end at the depot (8); each vehicle leaves from and returns to the depot exactly once, (9) and (10) respectively; service times must satisfy time window start (11) and ending (12) times; and service start time must allow for travel time between customers (13). Decision variables type and domain are indicated in (14) and (15).

(b) Partial Cost Minimization EVRP

In the capacitated vehicle routing problem with time windows (VRPTW) it is traditionally assumed that carriers minimize the number of vehicles as a primary objective and distance travelled as a secondary objective without violating time windows, route durations, or capacity constraints. This second formulation follows the traditional approach and allows for a partial reduction of potential emissions. The primary and secondary objectives are defined by (16) and (17) respectively. The tertiary objective is the minimization of distance traveled and route duration costs.

\[
\text{minimize } \sum_{k \in K} \sum_{j \in C} x_{a,j}^k,
\]  
\[
\text{minimize } \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}} x_{i,j}^k c_{i,j} v_j (y_j^k + g_j),
\]  
\[
\text{minimize } c_{i,j} \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}} d_{i,j} x_{i,j}^k + c_i \sum_{k \in K} \sum_{j \in C} (y_j^k - y_i^k) x_{a,j}^k
\]  
The same constraints (5) to (15) apply to this hierarchical objective function.
5. Solution Approaches for the partial EVRP

The solution approach for the partial EVRP can benefit from the application of existing algorithms for the TDVRP. After finding a solution for the TDVRP there are at least two alternative approaches: (a) the values of the \( x_{ij} \) decision variables are fixed and the volume of emissions is reduced by an algorithm that can only alter the departure times \( b_i = y_i^k + g_i \) and (b) the volume of emissions is reduced by an algorithm that can alter both departure times \( b_i = y_i^k + g_i \) and assignment variables \( x_{ij}^k \) subject to constraint (19):

\[
\sum_{j \in C} \sum_{k \in K} x_{0j}^k \leq K^* \tag{19}
\]

where \( K^* \) is the best fleet size solution obtained with a TDVRP algorithm.

Given that approach (a) is clearly suboptimal, this research focuses on the development of an algorithm that follows approach (b). This section discusses properties of the emission functions (1) and (2). These properties are useful to reduce the computation effort required to evaluate emission levels for the partial EVRP.

Properties of the Emission Function

Waiting at a customer location may be necessary to reduce emission costs, e.g. waiting at a customer location may be beneficial during periods of high congestion and reduced travel speeds. However, waiting may have an impact on future travel times and reduce the capacity to serve subsequent customers in the route. For any given route \( k \) defined by the sequence of customers \((0,1,2,...,i,j,...,q,q+1)\), where 0 and \( q+1 \) denote the depot, it is possible to define \( y_i^k \) and \( \overline{y}_i^k \) for customer \( i \) where \( y_i^k \) and \( \overline{y}_i^k \) are the earliest and latest feasible service times respectively.

**Property 1:** It may be better to wait at a customer location if travel speeds are lower that a certain speed threshold.

**Proof:** Due to the last component of emissions function (1) it always possible to find a speed \( s^m \geq 0 \) such that the cost of emissions is larger than the cost of waiting but departing before or
at $\overline{y}_k^k$. Emissions per unit of distance traveled increases as travel speeds approach zero and there is a speed at time $\overline{y}_k^k$ where costs are reduced only if the vehicle waits at a customer location.

**Property 2:** for speeds below the optimal level $s^*$ and strictly decreasing travel speeds in the union of intervals defined by $S_y(\overline{y}_j^k + g_i)$ and $S_y(\overline{y}_i^k + g_i)$, the departure time that minimizes emissions is given by $b_i = \overline{y}_i^k + g_i$.

**Proof:** travel speeds are below optimal and strictly decreasing if:

$$s > s^m \geq s^{m+1} \forall m \in S_y(\overline{y}_j^k + g_i) \cup S_y(\overline{y}_i^k + g_i)$$

Emission levels increase as speeds decrease from the optimal value. Given that the emissions function (2) has a unique global minimum at $s$, delaying the departure time will only increase emission costs.

**Property 3:** for speeds below the optimal level $s$ and strictly increasing travel speeds in the union of intervals defined by $S_y(\overline{y}_j^k + g_i)$ and $S_y(\overline{y}_i^k + g_i)$, the departure time that minimizes emissions is given by $b_i = \overline{y}_i^k + g_i$.

**Proof:** travel speeds are below optimal and strictly increasing if:

$$s^m < s^{m+1} < s \forall m \in S_y(\overline{y}_j^k + g_i) \cup S_y(\overline{y}_i^k + g_i)$$

Given that the emissions function (2) has a unique global minimum at $s^*$, delaying the departure time will only decrease emission costs.

Similar mirror properties can be derived for speeds above the optimal level and strictly decreasing or increasing travel speeds.

**Property 4:** for arbitrary travel speeds in the union of intervals defined by $S_y(\overline{y}_j^k + g_i) \cup S_y(\overline{y}_i^k + g_i)$ and no stops between customers $i, j$ the departure time that minimizes emissions can be found after comparing a finite number of departure times.
number of comparisons is less or at most equal to two times the number of time intervals that define $S_y(\sum_i^k y^i + g_i) \cup S_y(\prod_i^k y^i + g_i)$.

**Proof:** travel speeds and their intensity of emissions are constant during each time interval. Without stops, for any given departure time, the emissions function is a strictly increasing function of travel time duration and the extremes can be found either for departures that coincide with the beginning of time intervals or for departures that result in arrivals at the end of time intervals. Hence, it is sufficient to check the level of emissions for departure times that coincide with the beginning of each time period or for departure times that arrive to customer $j$ the end of a time period.

**6. An Algorithm for the Partial EVRP**

A strategy to solve the Partial EVRP is to first minimize the number of vehicles using a TDVRP algorithm and then optimize emissions subject to a fleet size constraint. A description of the TDVRP algorithm used in this experiment along with a full problem statement is described in detail in Figliozzi [8]. This approach, also denoted IRCI for Iterated Route Construction and Improvement has also been successfully applied to VRP problems with soft time windows [17]. The IRCI algorithm consists of a route construction phase and a route improvement phase, each utilizing two separate algorithms. During route construction, the auxiliary routing algorithm $H_r$ determines feasible routes with the construction algorithm $H_c$ assigning customers and sequencing the routes utilizing a greedy heuristic to approximate the cost of adding customers to a route. Route improvement is performed with the route improvement algorithm $H_i$ which groups underutilized routes or routes with a low number of customers looking to consolidate customers into a set of improved routes.

**Optimization of Departure Times**

In Section 5 we have derived an algorithm to optimize the departure time between any given customers $i, j$, and an initial condition, i.e. $a_i$ the arrival time at customer $i$. Let $H_d(i, j, a_i)$ be the algorithm that optimizes the departure time for any pair of customer and initial condition.
Before defining $H_k(i, j, a_i)$ it is necessary to define an auxiliary function to calculate backward travel time $bt(y^k_j)$:

Data:

$T$ and $S$ : time intervals and speeds
$v_j, v_j', y_j$: two customers served in this order in route $k$, $y^k_j$ is the current service time at customer $j$

START $bt(y^k_j)$

1. if $y^k_j < l_j$ & $y^k_j < y^k_j'$ then
2. $y^k_j ← min(l_j, y^k_j')$
3. end if
4. find $k, t_k ≤ y^k_j ≤ t_\bar{k}$
5. $b_j ← y^k_j - d_{ji} / s_k$
6. $d ← d_{ji}, t ← y^k_j$
7. while $b_j < t_k$ do
8. $d ← d - (t - t_k) s_k$
9. $t ← t_k$
10. $b_j ← t - d / s_{k+1}$
11. $k ← k + 1$
12. end while
13. $y^k_i ← min(b_i - g_i, l_i)$

Output:

$y^k_j, \bar{y}^k_j$

END $bt(y^k_j)$

The algorithm $H_k(i, j, a_i)$ is defined as follows:

1. for customer $i$ in a route the earliest and latest feasible service times $y_i^{k'}$ and $\bar{y}_i^{k'}$ are found using $y_i^{k'} = min(e_i, a_i)$ and $\bar{y}_i^{k'} = bt(y_j^k)$

2. for customer $i$ define the union of intervals defined by $S_y(y_i^{k'} + g_i) \cup S_y(\bar{y}_i^{k'})$ as the intervals of time needed to cover the periods of time between $(y_i^{k'} + g_i, \bar{y}_i^{k'})$. The times periods that cover the ordered pair of times $[x, y]$ is denoted $\{T(x, y)\}$ and is constructed as follows: for each $T^m \in T$ add a time period to $\{T(x, y)\}$ :
- if \( x \leq \underline{t}, \overline{t} \leq y \), then \( T^m \in \{T(x, y)\} \), or
- if \( \underline{t} < x, y \leq \overline{t} \), then \( \{x, \overline{t}\} \in \{T(x, y)\} \), or
- if \( x \leq \underline{t}, y < \overline{t} \), then \( \{t^m, y\} \in \{T(x, y)\} \).

3. Define: \( \min \leftarrow v_y(b_i = y^k), \ b_i^* = b_i \)
   a. For each period of time \( T^m \in \{T(y^k + g_i, y^k)\} \) calculate \( v_y(b_i = \underline{t}^m) \)
      i. if \( v_y(b_i = \underline{t}^m) \leq \min \)
      then: \( \min \leftarrow v_y(b_i = \underline{t}^m), \ b_i^* = b_i \)
   b. For each period of time \( T^m \in \{T(y^k + g_i, y^k)\} \) calculate \( v_y(b_i = \text{bf}(\overline{t}^m)) \)
      i. if \( v_y(b_i = \text{bf}(\overline{t}^m)) \leq \min \)
      then: \( \min \leftarrow v_y(b_i = \text{bf}(\overline{t}^m)), \ b_i^* = b_i \)

4. Return best departure time \( b_i^* \) and emissions costs “min”

Hence, departure times can be optimized given any pair of feasible customers.

Improvement of Emissions Costs by Changing Routes

In Section 5 we have derived an algorithm to optimize the departure time between any given. Emissions are further reduced adapting a heuristic approach developed by Kontoravdis and Bard [18] using a greedy randomized adaptive search concept (GRASP) for the VRPTW. The improvement approach combines the construction phase proposed by Figliozzi [17] with GRASP.

The psude-code can be summarized as follows:

- Select any two routes and joined the customers into a set \( C' \).
  o Choose from \( C' \) two customers \( i_1 \) and \( i_2 \) that are the most time constrained
  o \( C' \leftarrow C'/\{i_1, i_2\} \)
  o Initialize two routes \( r_1 \) and \( r_2 \) by selecting \( i_1 \) and \( i_2 \) as the first customers respectively.
  o Do until no \( i \in C' \) can be feasibly inserted into \( r_1 \) or \( r_2 \)
For each $i \in C'$

- Find the best feasible insertion location into $r_1$ and $r_2$ (the feasible insertion with minimum emissions cost given by $H_6(i, j, a_i)$).

- Insert the minimum emissions cost customers into $r_1$ or $r_2$
  - Complete the routes using a greedy approach (minimizing emissions), calculate insertion costs using $H_6(i, j, a_i)$

- After a customer is inserted, try to insert any unrouted customer into $r_1$ or $r_2$
  - Evaluate if there is an improvement in the total volume of emissions without exceeding the original number of routes.
  - Tabu the previously selected customers and pick a pair of not yet selected routes.
  - Continue until there are no more unselected pairs.

### 7. Experimental Results

The experimental setting is based on the classical instances of the VRP with time windows proposed by Solomon [19]. The Solomon instances include distinct spatial customer distributions, vehicles’ capacities, customer demands, and customer time windows. These problems have not only been widely studied in the operations research literature but the datasets are readily available.

The well-known 56 Solomon benchmark problems for vehicle routing problems with hard time windows are based on six groups of problem instances with 100 customers. The six problem classes are named C1, C2, R1, R2, RC1, and RC2. Customer locations were randomly generated (problem sets R1 and R2), clustered (problem sets C1 and C2), or mixed with randomly generated and clustered customer locations (problem sets RC1 and RC2). Problem sets R1, C1, and RC1 have a shorter scheduling horizon, tighter time windows, and fewer customers per route than problem sets R2, C2, and RC2 respectively.

This section proposes new test problems that capture the typical speed variations of congested urban settings. The problems are divided into three categories of study: (1) uncongested,
somewhat congested, and (3) congested. In order to provide readily replicable instances, the travel speed distributions apply to ALL arcs among customers, i.e. in the arc set:

$$A = \{(v_i, v_j) : i \neq j \land i, j \in V\}$$

The depot working time $[e_i, l_i]$ is divided into five time periods of equal durations: $[0, 0.2l_i)$; $[0.2l_i, 0.4l_i)$; $[0.4l_i, 0.6l_i)$; $[0.6l_i, 0.8l_i)$; and $[0.8l_i, l_i]$ and the corresponding travel speeds are in the 3 cases as follows:

Uncongested  = $[2.00, 2.00, 2.00, 2.00, 2.00]$,  
Somewhat Congested = $[2.00, 1.25, 2.00, 1.25, 2.00]$,  
Congested = $[2.00, 0.90, 1.20, 0.90, 2.00]$.  

It is assumed that the optimal travel speed, i.e. 44 mph, is equivalent to a speed of 2.0 in the Solomon problems. This assumption ensures that the properties stated in Section 5 are valid and applicable. This is a mild assumption in congested urban areas with low travel speeds and low speed limits. For example, the commercial vehicle maximum travel speed in the urban interstate freeways in Portland, Oregon, is only 55 mph.

The average results per routing class are presented in Table 1 and 2. Table 1 compares the “Somewhat Congested” case against the “Uncongested” case. In all cases, the percentage change taking the uncongested situation as a base. For example, a positive % in the row of routes (or emissions levels) indicates that the average number of needed routes (or emissions levels) has increased.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>R1</th>
<th>R2</th>
<th>C1</th>
<th>C2</th>
<th>RC1</th>
<th>RC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>10%</td>
<td>17%</td>
<td>0%</td>
<td>0%</td>
<td>7%</td>
<td>10%</td>
</tr>
<tr>
<td>Duration</td>
<td>2%</td>
<td>0%</td>
<td>-7%</td>
<td>-4%</td>
<td>0%</td>
<td>-2%</td>
</tr>
<tr>
<td>Emissions</td>
<td>28%</td>
<td>25%</td>
<td>18%</td>
<td>23%</td>
<td>25%</td>
<td>24%</td>
</tr>
</tbody>
</table>

Table 1. Percentage Change, Somewhat Congested vs. Uncongested travel times

It can be observed in Table 1 that emissions do not increase across the board. Types C1 and C2 are constrained by capacity and they register a small change in emissions. In addition, the customers are clustered. On the other hand, types RC1 and RC2 register the greatest changes
in emissions levels as travel speed decreases. The greatest change in average fleet size take place with the R1 and R2 types. As expected, duration or travel time increases across the board.

Table 2 compares the “Congested” case against the “Uncongested” case. In all cases, the percentage change taking the uncongested situation as a base. As expected, duration increases across the board. It can be observed in Table 2 that emissions do not increase across the board. Types C1 and C2 are constrained by capacity and they register a small change in emissions. A similar pattern is observed but RC2 problems do not register an increase in emissions.

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>C1</th>
<th>C2</th>
<th>RC1</th>
<th>RC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle</td>
<td>27%</td>
<td>38%</td>
<td>0%</td>
<td>0%</td>
<td>22%</td>
<td>29%</td>
</tr>
<tr>
<td>Distance</td>
<td>4%</td>
<td>5%</td>
<td>-11%</td>
<td>-10%</td>
<td>3%</td>
<td>0%</td>
</tr>
<tr>
<td>Duration</td>
<td>60%</td>
<td>68%</td>
<td>52%</td>
<td>35%</td>
<td>57%</td>
<td>60%</td>
</tr>
<tr>
<td>Emissions</td>
<td>26%</td>
<td>25%</td>
<td>6%</td>
<td>0%</td>
<td>30%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 2. Percentage Change, Congested vs. Uncongested travel times

These results highlight the fact that emissions are not easy to minimize. It is clear that uncongested travel speeds tend to reduce emissions on average, however, this is not always the case and in some cases the opposite trend can be observed. Further research is needed to explore alternative algorithms to minimize emissions in congested areas.

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>C1</th>
<th>C2</th>
<th>RC1</th>
<th>RC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle</td>
<td>29%</td>
<td>42%</td>
<td>0%</td>
<td>0%</td>
<td>26%</td>
<td>29%</td>
</tr>
<tr>
<td>Distance</td>
<td>12%</td>
<td>24%</td>
<td>0%</td>
<td>7%</td>
<td>7%</td>
<td>17%</td>
</tr>
<tr>
<td>Duration</td>
<td>73%</td>
<td>97%</td>
<td>59%</td>
<td>59%</td>
<td>63%</td>
<td>85%</td>
</tr>
<tr>
<td>Emissions</td>
<td>22%</td>
<td>-27%</td>
<td>4%</td>
<td>-7%</td>
<td>28%</td>
<td>-11%</td>
</tr>
</tbody>
</table>

Table 3. Percentage Change, Congested vs. Uncongested travel times

If the objective function is to minimize emissions and lifting the constraint that restricts any increase in the number of routes, the results are encouraging (see Table 3). It can be observed in Table 3 that relatively small increases in fleet size lead to dramatic reductions in emissions levels, for example R2 problems. Minor reductions are obtained for problems R1 and RC1. In some cases the reductions take place even maintaining the same number of vehicles on average, for example in C1, C2, and RC2 problems. These preliminary results indicate that
there may be significant emissions savings if commercial vehicles are routed taking emissions into consideration.

8. Conclusions

This research introduced a new kind of vehicle routing problem, the emissions Vehicle Routing Problem or EVRP. Two variants of the problem have been formulated. Properties of the emissions formula and optimal departure times are stated. A heuristic is proposed to reduce the level of emissions given a number of feasible routes for the time-dependent VRP.

These preliminary results indicate that there may be significant emissions savings if commercial vehicles are routed taking emissions into consideration. In congested areas, it may be possible to reduce unhealthy or GHG emissions with a minimal or null increase in routing costs. However, these benefits are not to be expected across the board. The results indicate that congestion impacts on emission levels are not uniform. The route characteristics and dominant constraint type seem to play a significant effect on emissions levels.
Acknowledgements

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