

Vehicle technologies and bus fleet replacement optimization: problem properties and sensitivity analysis utilizing real-world data

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Abstract This research presents a bus fleet replacement optimization model to analyze vehicle replacement decisions when there are competing technologies. The focus of the paper is on sensitivity analysis. Model properties that are useful for sensitive analysis are derived and applied utilizing real-world data from King County (Seattle) transit agency. Two distinct technologies, diesel hybrid and conventional diesel vehicles, are studied. Key variables affecting optimal bus type and replacement age are analyzed. Breakeven values and elasticity values are estimated. Results indicate that a government purchase cost subsidy has the highest impact on optimal replacement periods and total net cost. Maintenance costs affect the optimal replacement age but are unlikely to change the optimal vehicle type. Greenhouse gas emissions costs are not significant and affect neither bus type nor replacement age.

Keywords Bus fleet replacement · Optimization model · Model properties · Diesel · Hybrid diesel · Subsidy · Cost elasticity · Breakeven values

JEL Classification R41

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1 Introduction

Transit agencies typically own hundreds or thousands of buses; large transit agencies may have multiple fleets of buses with different types of buses serving different routes. For example, King County Metro Transit (KCMT) (Seattle, WA) operates about 1,300 vehicles with multiple bus drivetrain technologies (electric trolley buses, conventional diesel buses, hybrid diesel buses, etc.), sizes and capacities (60 ft. articulate, 30 or 40 ft. standard) and brands/models (New Flyer, Gillig, etc). Fleet capital, operational and maintenance costs are a significant expense for transit agencies. Due to budget and fiscal constraints, it is ever more imperative for transit agencies to manage their fleets in an optimal way without reducing service quality.

To minimize fleets total costs over a given time horizon fleet managers have to consider two important tradeoffs. First, as buses age, per-mile operating and maintenance (O and M) costs tend to increase; replacing old vehicles with new ones reduces O and M costs but significantly increases capital costs. Therefore, there is an optimal replacement age (lifecycle) that minimizes the total net cost over a planning time horizon. Second, costs associated to vehicle purchases and per-mile operating, maintenance and fuel costs vary across bus types (conventional diesel, hybrid, electric trolley, etc.), bus designs, and operating environments (congested or not congested routes, hilly or flat routes).

In practice, many transit agencies replace their vehicles based on swift yet suboptimal policies derived from rules of thumb, for example every 12 or 20 years or when annual maintenance costs increase above a given threshold. It is possible to formulate and easily solve an integer problem to find the optimal bus type for a specific and constant demand, financial and operating environment. In the case of KCMT, analysts were not interested just in an optimal solution but in obtaining a better understanding of the key variables and factors affecting the relative competitiveness of diesel and hybrid vehicles.

The contributions of this study are to develop and use a fleet replacement model in a way that is suitable for decision makers and sensitivity analysis. More specifically, the contributions of this research are to: (1) present an optimization model to minimize fleet costs that are relevant to decision makers; (2) study properties of the optimization model that can facilitate the sensitivity analysis; (3) apply the model and properties to real-world KCMT data for 60-ft. diesel bus and hybrid diesel bus fleets; and (4) study the impacts of government purchase cost subsidy and other input variables on the optimal replacement decisions.

The reminder of this paper is organized as follows: Sect. 2 briefly reviews bus fleet replacement practices and replacement optimization models. Section 3 presents the model formulation. In Sect. 4 model properties are explored. Section 5 describes KCMT bus fleet data. Section 6 shows baseline scenario results. Section 7 presents sensitivity analysis results. Finally, Sect. 8 wraps up with conclusions.



2 Literature review

Previous studies in the public transport field have shown how fuel efficiency and operating and maintenance costs change when vehicles age; significant differences have been found across bus models, transit agencies and service environments (Lammert 2008; Chandler and Walkowicz 2006; Schiavone 1997). Bus life cycle costs have been previously compared across bus engine types and design models (Clark et al. 2007; Laver et al. 2007; Clark et al. 2009; Kim et al. 2009). The papers referenced in this paragraph focus on vehicle characteristics and lifecycle costs assuming a constant replacement age. Optimal replacement schedules and bus type choice that minimize bus fleet total net cost have not been studied in the previously stated references.

There is a large body of literature dealing with vehicle replacement optimization models in the operations research field. These models can be broken into two categories depending on whether buses in a fleet are homogeneous or heterogeneous. In homogeneous models, the objective is to find the best bus replacement age for a set of identical vehicles, in other words, buses with the same type and age have to be replaced together (also known as the “no cluster splitting rule”). These models are usually solved using a dynamic programming (DP) approach (Bellman 1955; Oakford et al. 1984; Bean et al. 1984; Bean et al. 1994; Hartman 2001; Hartman and Murphy 2006). Dynamic programming has the advantage of allowing the consideration of probabilistic distributions for some state variables such as utilization or operational costs.

Heterogeneous models are more appropriate when multiple bus fleets have to be optimized simultaneously or when budget constraints are needed. For example, the “no cluster splitting rule” cannot be applied when vehicles of the same type and age may be replaced in different years due to budget limitations. These models are able to solve more practical problems but input variables are usually deterministic. Stochastic heterogeneous models are difficult to solve. Most heterogeneous models employ integer programming (IP) formulations (Simms et al. 1984; Karabakal et al. 1994; Hartman 1999, 2000, 2004). With additional assumptions a DP approach can be applied to heterogeneous problems (Jones et al. 1991). None of these theoretical models mentioned in this paragraph deals real world fleet data.

Fan et al. (2012) developed a fleet optimization framework using a DP approach; however the simultaneous optimization of heterogeneous vehicles and sensitivity analysis of input variables were not addressed. Figliozzi et al. (2011), Feng and Figliozzi (2013) adopted IP models to study a fleet of heterogeneous passenger cars and delivery trucks with real world operational data. Impacts of policy, market, utilization, emissions, and technological factors were analyzed using scenario analysis and elasticity analysis. Boudart and Figliozzi (2012) studied how economic and technological factors affect a single bus optimal replacement age.

Summarizing, several papers have described the use of optimization models to solve real world problems. Keles and Hartman (2004) adopted an IP model in a transit fleet replacement problem with multiple types of buses. The optimization model used in this paper is deterministic and it is derived from the models developed by Hartman (2000); Keles and Hartman (2004). However, unlike these



two references, the objective function incorporates emissions costs and all the model parameters used in the paper are based on real-world data; in addition, this paper derives breakeven analysis properties and presents a thorough sensitivity analysis based on key vehicle characteristics, costs and utilization levels. The contribution of this paper is a first step that facilitates a better understanding of the key parameters of the problem. A more complete analysis should include stochastic models but this type of models is beyond the scope of this research.

3 Methodology

In the optimization model, five major cost components are considered: capital (purchase) costs, salvage revenue (represented as a negative cost), energy (fuel) costs, maintenance costs, and emissions costs. The objective function of this model is to minimize the discounted sum of all five costs for all buses over a planning time horizon. The decision variables are when and which buses should be replaced with what type of new buses. Once the optimal solution is found, costs breakdowns and bus utilization statistics can be easily calculated. The optimization approach is depicted in Fig. 1.

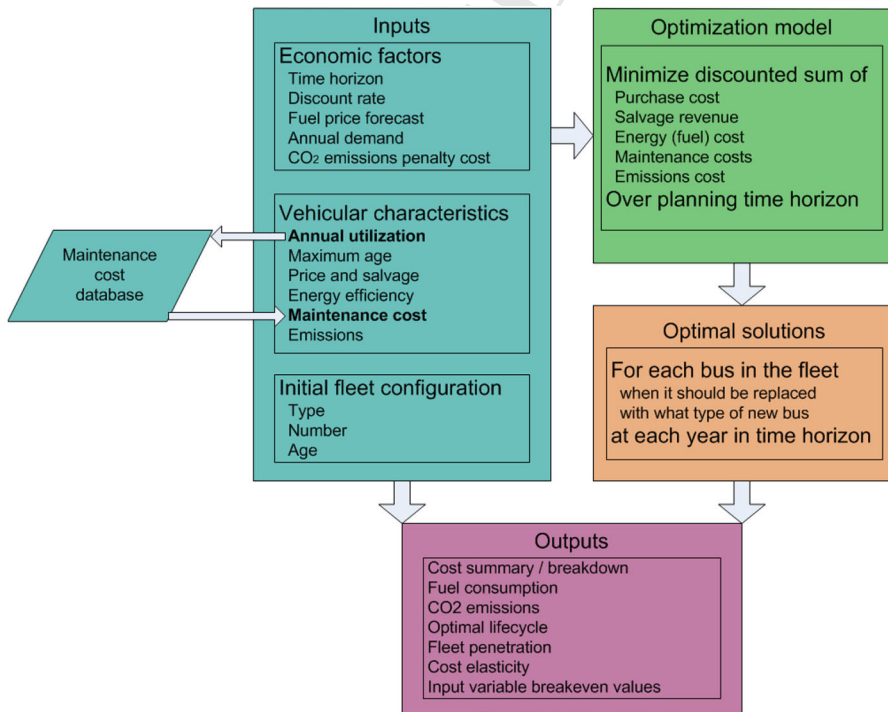


Fig. 1 Bus fleet replacement optimization framework

The optimization model requires three categories of inputs: economic factors, vehicle characteristics, and initial fleet configuration. Economic factors include planning time horizon, annual number of vehicles (demand) or annual miles that must be traveled, discount rate, and forecasted fuel costs. Vehicle factors include types of buses and for each bus type, its maximum physical life, purchase cost and salvage value as a function of age, fuel economy as a function of age, annual utilization (miles traveled) as a function of age, and per-mile maintenance cost as a function of age and annual utilization. Initial fleet configuration includes the type, age, and number of existing buses. Once the inputs are specified, the model can provide optimal replacement policies.

The optimization model is formulated as a deterministic heterogeneous fleet replacement model, which means all input variables are known with certainty.

3.1 Indices

Type of bus: $k \in K = \{1, 2, \dots, K\}$.

Age of a bus type k in years: $i \in A_k = \{0, 1, 2, \dots, A_k\}$,

Time periods: $j \in T = \{0, 1, 2, \dots, T\}$, and

3.2 Decision variables

X_{ijk} = The number of i -year old, k -type buses used in year j ,

Y_{ijk} = The number of i -year old, k -type buses salvaged at the end of year j , and

P_{jk} = The number of k -type buses purchased at the beginning of year j .

3.3 Parameters

3.3.1 (a) Constraints

A_k = Maximum age of bus type k (it must be salvaged when a bus reaches this age),

u_{ik} = Utilization (annual miles traveled by an i -year old, k -type bus),

d_j = Demand (miles traveled by all buses) in year j ,

b_j = Budget (available for purchasing new buses) constraint in year j ,

3.3.2 (b) Cost or revenue

v_k = Purchase cost of a k -type bus,

f_{ik} = Fuel economy (mpg) for an i -year old, k -type bus,

fc_j = Fuel price(\$/gallon) in year j ,

m_{ik} = Per-mile maintenance cost for an i -year old, k -type bus,

s_{ik} = Salvage revenue (negative cost) from selling an i -old, k -type bus,

$ec =$ Emissions cost per ton of GHG,

$\delta =$ Discount rate.

3.3.3 (c) Emissions

$e_{ik} =$ Utilization emissions in GHG equivalent tons per mile for an i -year old, k -type bus, and

3.3.4 (d) Initial conditions

$h_{ik} =$ The number of i -year old, k -type buses available at the beginning.

3.4 Objective function

$$\begin{aligned} \min Z = & \sum_{j=0}^{T-1} \sum_{k=1}^K \left[v_k \cdot P_{jk} + \sum_{i=0}^{A_k-1} \left(\frac{fc_j}{f_{ik}} + m_{ik} + ec \cdot e_{ik} \right) u_{ik} \cdot X_{ijk} \right] \cdot (1 + \delta)^{-j} \\ & + \sum_{j=0}^T \sum_{k=1}^K \sum_{i=1}^{A_k} s_{ik} Y_{ijk} (1 + \delta)^{-j} \end{aligned} \quad (1)$$

3.5 Constraints

$$\sum_{k=1}^K v_{jk} \cdot P_{jk} \leq b_j, \quad \forall j \in \{0, 1, 2, \dots, T-1\} \quad (2)$$

$$\sum_{i=0}^{A_k-1} \sum_{k=1}^K X_{ijk} \cdot u_{ik} \geq d_j, \quad \forall j \in \{0, 1, 2, \dots, T-1\} \quad (3)$$

$$P_{jk} = X_{0jk}, \quad \forall j \in \{1, 2, \dots, T-1\}, \quad \forall k \in \mathbf{K} \quad (4)$$

$$P_{0k} + h_{0k} = X_{00k}, \quad \forall k \in \mathbf{K} \quad (5)$$

$$X_{i0k} + Y_{i0k} = h_{ik} \forall i \in \{1, 2, \dots, A_k\}, \quad \forall k \in \mathbf{K} \quad (6)$$

$$X_{(i-1)(j-1)k} = X_{ijk} + Y_{ijk}, \quad \forall i \in \{1, 2, \dots, A_k\}, \quad \forall j \in \{1, 2, \dots, T\}, \quad \forall k \in \mathbf{K} \quad (7)$$

$$X_{iTk} = 0, \quad \forall i \in \{0, 1, 2, \dots, A_k - 1\}, \quad \forall k \in \mathbf{K} \quad (8)$$

$$X_{A_k j k} = 0, \quad \forall j \in \{0, 1, 2, \dots, T\}, \quad \forall k \in \mathbf{K} \quad (9)$$

$$Y_{0jk} = 0, \quad \forall j \in \{0, 1, 2, \dots, T\}, \quad \forall k \in \mathbf{K} \quad (10)$$

$$P_{jk}, X_{ijk}, Y_{ijk} \in \mathbf{I} = \{0, 1, 2, \dots\} \quad (11)$$

The objective function, expression (1), minimizes the sum of purchasing, energy (fuel), maintenance, salvage, and emissions costs over the period of analysis, i.e.



from time zero (present) to the end of year T ; there is no fixed purchase costs associated to the purchase of vehicles. Purchase costs cannot exceed the annual budget, expression (2). The number of vehicles in the fleet at any time must equal or exceed the minimum needed to cover the demand in terms of annual number of buses or annual miles traveled, expression (3). The number of vehicles purchased must equal the number of new vehicles for each vehicle type and year, except for year zero, expression (4). The number of new vehicles utilized during year zero must equal the sum of existing new vehicles plus purchased vehicles, expression (5). Similarly, expression (6) ensures the conservation of vehicles (i.e. the initial vehicles—not 0-age ones—must be either used or sold). The age of any vehicle in use will increase by 1 year; after each time period vehicles are either used or sold (7). At the end of the last time period, all vehicles will be sold at the corresponding salvage value (8). When a vehicle reaches its maximum age, the vehicle must be sold (9). A newly purchased vehicle should not be sold before being used at least 1 year (10). Finally, the decision variables associated with purchasing, utilization, and salvaging decisions must be integer non-negative numbers, expression (11).

4 Model sensitivity analysis properties: breakeven points

The model described in the previous section can provide the optimal vehicle replacement schedule given a set of deterministic input parameters. However, decision makers usually want to know not only the optimal solution but also the sensitive of the optimal solution. For example, decision makers can be interested in understanding the breakeven values that delimit the competitiveness of each vehicle type (diesel vs. hybrids). In particular, USA transit agency managers are interested in FTA subsidy levels and how they impact the optimal vehicle type. In this case, a purchase cost subsidy breakeven value indicates when the optimal vehicle type changes as function of the subsidy level when holding all other input parameters constant (ceteris paribus). In this section, as in the model presented in Sect. 3, we assume that there are no budget constraints.

We define that there is a breakeven value for a parameter (e.g. government subsidy) for vehicle types k and k' when the value of the objective function (1) is the same when: (a) $P_{0k} = n$, $P_{0k'} = 0$ and (b) $P_{0k} = 0$, $P_{0k'} = n$ where n is an integer number. The definition of the breakeven value indicates that the objective function value is the same when only one type of the vehicle (either k or k' but not both) is purchased in year zero. The definition does not guarantee that (1) there exists at least one breakeven value, (2) that this value is unique, and that (3) there is an efficient procedure to obtain breakeven values. This section defines under what conditions and assumptions it is possible to find unique breakeven values.

For any given feasible replacement policy π given by the sets of decision variables $(P_{jk}, X_{ijk}, Y_{ijk})$, let the subsidy level be denoted by s , with $0 \leq s \leq 1$, and the total replacement policy costs for policy π and subsidy s be denoted by $f(\pi, s)$.

Property 1 *The total net cost $f(\pi, s)$ as a function of the subsidy level s for a given replacement policy π is a linear decreasing function in the interval $0 \leq s \leq 1$.*

262 *Proof* It is possible to write $f(\pi, s)$ as:

$$f(\pi, s) = \sum_{j=0}^{T-1} \sum_{k=1}^K \left[v_k(1-s) \cdot P_{jk} + \sum_{i=0}^{A_k-1} \left(\frac{fc_j}{f_{ik}} + m_{ik} + ec \cdot e_{ik} \right) u_{ik} \cdot X_{ijk} \right] \\ \cdot (1+\delta)^{-j} + \sum_{j=0}^T \sum_{k=1}^K \sum_{i=1}^{A_k} s_{ik} Y_{ijk} (1+\delta)^{-j}$$

264 Then, if $\pi = (P_{jk}, X_{ijk}, Y_{ijk})$ is a replacement policy, then its cost for a certain
265 level of subsidy, $f(\pi, s)$, can be rewritten as the sum of a constant term and a liner
266 decreasing function in the interval $0 \leq s \leq 1$.

$$f(\pi, s) = g(\pi) - s \sum_{j=0}^{T-1} \sum_{k=1}^K (v_k \cdot P_{jk}) \cdot (1+\delta)^{-j}$$

267 where $g(\pi)$ is a constant term that does not depend on the value of s .
268

269

$$g(\pi) = \sum_{j=0}^{T-1} \sum_{k=1}^K \left[v_k \cdot P_{jk} + \sum_{i=0}^{A_k-1} \left(\frac{fc_j}{f_{ik}} + m_{ik} + ec \cdot e_{ik} \right) u_{ik} \cdot X_{ijk} \right] \cdot (1+\delta)^{-j} \\ + \sum_{j=0}^T \sum_{k=1}^K \sum_{i=1}^{A_k} s_{ik} Y_{ijk} (1+\delta)^{-j}$$

271 In this property it is also assumed that purchase prices are positive.

272 **Property 2** For a given vehicle type k , the minimum of the total net cost $f_k(\pi, s)$ as
273 a function of the subsidy level s is a decreasing concave function in the interval
274 $0 \leq s \leq 1$.

275 *Proof* Let Π_k be the set of all feasible replacement policies $(P_{jk}, X_{ijk}, Y_{ijk})$ for a
276 given vehicle type k . Then, let's denote $Z_k(s)$ as the minimum total net cost function
277 for vehicle type k for a given subsidy level $s, 0 \leq s \leq 1$:

$$Z_k(s) = \min_{\pi \in \Pi_k} f(\pi, s) \text{ for } \forall \pi \in \Pi_k$$

278 The function $Z_k(s)$ is a concave and decreasing function in the interval $0 \leq s \leq 1$
279 because the minimum of a set of linear functions is a concave function and also
280 because each one of the $f_k(\pi, s)$ functions is a decreasing function in the interval
281 $0 \leq s \leq 1$.
282

283 **Property 3** Given two different vehicle types k and k' and functions $Z_k(s)$ and
284 $Z_{k'}(s)$ such that $Z_k(s=0) < Z_{k'}(s=0)$ and $Z_k(s=1) > Z_{k'}(s=1)$ then the
285 functions cross at least at one point.
286

287 *Proof* $Z_k(s)$ and $Z_{k'}(s)$ are continuous concave decreasing functions since they are
288 obtained as the minimum of a set of continuous decreasing linear functions. Then,
289 applying the Bernard Bolzano theorem (also known as the intermediate value
290 theorem), there is a point in the interval $0 \leq s \leq 1$. Because the functions are
291 continuous and decreasing, the crossing value will be found in the interval
292



$(Z_k(s = 1), Z_k(s = 0))$.

Because the functions are concave, continuous and decreasing in the interval $[0, 1]$, the breakeven value is unique.

Lemma 1 Given two different vehicle types k and k' and functions $Z_k(s)$ and $Z_{k'}(s)$ such that $Z_k(s = 0) < Z_{k'}(s = 0)$ and $Z_k(s = 1) > Z_{k'}(s = 1)$ then it is possible to apply a bisection method search procedure and obtain a crossing point.

Proof The bisection method converges to a root of a function $h(x)$ if the function is continuous in a given interval $[a, b]$ and $h(a)$ and $h(b)$ have opposite signs. By defining $h(s) = Z_k(s) - Z_{k'}(s)$ the result is a continuous function where $h(0)$ and $h(1)$ have opposite signs.

The importance of the Lemma 1 is that usually when comparing two rival technologies, e.g. hybrid diesel and conventional diesel, costs are such that $Z_k(s = 0) < Z_{k'}(s = 0)$ and $Z_k(s = 1) > Z_{k'}(s = 1)$; where type k is the “cheap” technology with lower capital costs but higher operating costs and k' is the “expensive” but more efficient technology with higher initial capital costs but lower operations/maintenance costs. If $Z_k(s = 0) < Z_{k'}(s = 0)$ and $Z_k(s = 1) < Z_{k'}(s = 1)$ then the choice is trivial since technology k dominates technology k' .

It is possible to implement the IP model formulated in the previous section within a bisection algorithm; given Lemma 1 the algorithm will converge and find a breakeven point. The existence of breakeven points and the certain convergence can be also extended to other parameters such as fuel/energy costs that will generate linear or concave cost functions that are increasing functions in an interval. The properties studied in this section are applied in the sensitivity analysis section.

5 Basic scenarios

There are three types of parameters or inputs: vehicle, economic, and initial fleet configuration parameters. This section describes the parameters, based on real-world KCMT data, used in this research.

5.1 Vehicle parameters

The two vehicle technologies (types) are hybrid diesel and conventional diesel. The vehicles compared are both New Flyers (<http://www.newflyer.com/>), the New Flyer 60 ft. hybrid diesel bus ($k = 1$) and the New Flyer 60 ft. conventional diesel bus ($k = 2$).

$A_1 = A_2 = 20$. The maximum ages are assumed to be 20 years for both buses; most transit agencies in the US replace buses in a 12–16-year cycle (Laver et al. 2007).

$v_1 = \$958,000$, $v_2 = \$737,000$. The purchase costs for the two buses are \$958,000 for hybrid bus and \$737,000 for diesel bus, including ordering costs and communication/data collection equipment. Also, transit agencies can receive purchase cost subsidies from the US Federal Transit Administration (FTA). However FTA adds additional age replacement constraints; for example, if an 80 %

purchase cost subsidy is received the bus must be kept for a minimum of 12 years. This can be added to the model as a new constraint shown in equation.

$s_{ik} = -\$1,000, \forall k \in \mathbf{K}, i \in \mathbf{A}_k$. The salvage values for the two buses are assumed to be \$1,000 regardless of bus type or bus age according to KCMT request.

$f_{i1} = 3.65 \text{ mpg}, f_{i2} = 2.50 \text{ mpg}, \forall i \in \mathbf{A}_k$. The KCMT data indicate that the hybrid bus fuel economy is 3.65 mpg and the diesel bus is 2.50 mpg on average if they were operated in the same existing routes; fuel economy does not significantly vary with age.

$u_{ik} = 33,045 \text{ miles/year}, \forall k \in \mathbf{K}, i \in \mathbf{A}_k$. Because the two competing buses will serve the same bus route, their annual utilizations will be similar. Statistical data shows that the average annual miles traveled per bus is 33,045 miles.

$e_{i1} = 2.504 \text{ kg/mi}, e_{i2} = 3.407 \text{ kg/mi}, \forall i \in \mathbf{A}_k$. Only the tailpipe CO₂ emissions are considered in the model and the generation rates are 2.504 kg/mile for hybrid buses and 3.407 kg/mile for diesel buses regardless of age but a function of fuel economy, see also Clark et al. (2007).

To estimate per-mile maintenance cost as a function of age and utilization a regression model was estimated utilizing three independent variables: vehicle utilization, age, and vehicle type. The general administrative and operating costs (overhead) are constant and independent of the chosen vehicle technology and therefore excluded from the per-mile maintenance costs. In general, bus per-mile maintenance cost increases with both age and cumulative utilization. Though, these two variables are highly correlated in practice; hence, the per-mile maintenance cost is expressed solely as a function of age. In the model age is associated to the additional u_k miles traveled by a bus per year.

According to the baseline annual utilization $u_{ik} = 33,045 \text{ miles/year}, \forall k \in \mathbf{K}, i \in \mathbf{A}_k$, for this utilization, the estimated per-mile maintenance cost functions for hybrid and diesel buses are:

$$m_{i1} = 0.530 + 0.0867 \times i; m_{i2} = 0.372 + 0.0673 \times i, \forall i \in \{0, 1, \dots, A_k - 1\}, \forall k \in \mathbf{K}$$

5.2 Economic parameters

$T = 100$. A long planning time horizon of 100 years was used to abate the effect of the last incomplete vehicle life cycle on the first optimal bus type choice. Because the maximum age is 20 year, there are at least five replacements which ensure that the impact of the planning horizon length is negligible. Furthermore, due to the compound interest and given realistic discount rates of 4 % or higher the discounted annual costs quickly decrease after year ten.

$\delta = 9.55\%$. A 7.0 % annual discount rate (APR) for capital investments and 2.55 % consumer price index (CPI) are assumed throughout the planning time horizon following KCMT policies; these assumptions yield a 9.55 % nominal annual discount rate.

$fc_j = fc_0 \cdot (1 + 2.6\%)^j, \forall j \in \mathbf{T}$. Three fuel price ($fc_0 = \$2.64/\text{gal}, \$3.48/\text{gal}$ and $\$4.46/\text{gal}$) forecast functions are utilized following a recent report (Parametrix and



LTK Engineering Servies (2011) and forecasts based on long-term oil-price projections from US Energy Information Administration (2011).

$ec = \$30/\text{ton}$. CO_2 emissions penalty costs (suggested by KCMT) are used to account for the impact of emissions costs on optimal replacement decision.

$b_j = +\infty, \forall j \in \mathbf{T}$. Transit agencies like KCMT usually purchase a group of buses in certain years instead of purchasing only a few buses annually; in addition there is a limit on the total number of bus types. Having one bus type simplifies maintenance and fleet management (for example, less spare parts are specialized mechanics are needed among other benefits). In this case, budget constraints are not considered, typically federal subsidies are received or a bond pays for the new vehicles.

$d_j = u_{ik}, \forall j \in \mathbf{T}, k \in \mathbf{K}, i \in \mathbf{A}_k$. Because there are no budget constraints, for convenience and to facilitate the analysis of bus type tradeoffs replacement results are presented on a per bus basis.

5.3 Initial fleet parameters

KCMT was interested in analyzing what type of bus will be the best choice for its fleet 60-foot buses that must be replaced by the end of 2013. To simplify spare part inventories and maintenance operations, by limiting the number of bus types/models/brands, all the old buses are replaced by new ones. The question is: what type of new bus, diesel or hybrid? Therefore, for the scenario analysis and sensitivity report we can then assume that all existing vehicles are sold at the end of 2013 and new ones will be purchased at the beginning of the following year ($h_{ik} = 0, \forall k \in \mathbf{K}, i \in \mathbf{A}_k$). The reader is reminded that there is no fixed cost charge per purchase and that all the buses purchased at year zero will be replaced at the same time.

6 Baseline scenario results

Given the data presented in the previous section, the problem thus becomes, which new bus type should KCMT buy in 2014 to minimize future fleet total net costs, the New Flyer 60 ft. hybrid bus or the New Flyer 60 ft. diesel bus? What will be the optimal replacement cycle?

The optimization model was implemented using a Python interface; Cplex 12.4 was used to solve the integer programming model. Results for two baseline scenarios (no subsidy and 80 % subsidy) are shown in Figs. 2, 3.

The net present value (NPV) of the five cost components and their sum (total net cost) are shown for both scenarios. Figure 2 presents the no subsidy scenario where the optimal solution is to choose diesel bus and replace it every 20 years with a total net cost is \$1.546 million; if an alternative replacement is chosen (choose hybrid bus and replace it every 20 years), the total net cost would be \$1.688 million. Therefore, the savings per bus is approximately \$0.142 million. For a fleet with 300 vehicles, almost \$42.6 million can be saved from choosing the optimal replacement solution. In this no subsidy scenario, the purchase cost has the highest percent share (57 %) of

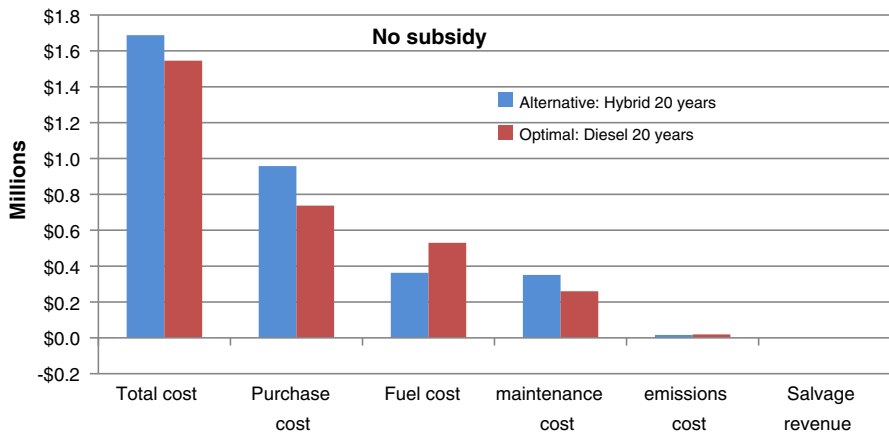


Fig. 2 Total net cost and cost breakdown for no subsidy scenario

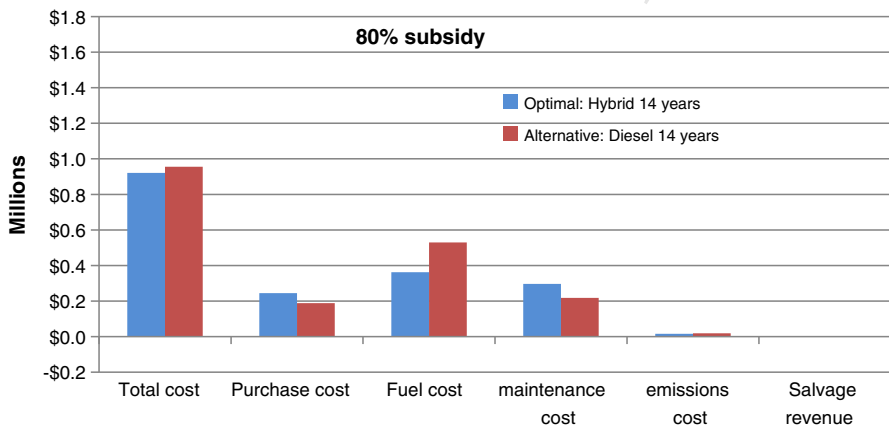


Fig. 3 Total net cost and cost breakeven for 80 % subsidy scenario

the total net costs. Hence, the optimal solution tends to extend bus life cycle as long as possible (the maximal age of 20 years is bounding in this case).

Figure 3 shows the results of the maximum subsidy scenario results (80 % in the USA). In this scenario the optimal bus type switched to hybrid bus and the optimal replacement cycle decreased to 14 years. Because the purchase cost has been reduced significantly, the fuel cost difference becomes the dominant factor. The saving is of approximately \$34,000 per bus or \$10.2 million for a fleet with 300 vehicles. Also, because the initial purchase cost is reduced significantly, the optimal replacement cycle has decreased from over 20 to just 14 years. Results indicate that a 80 % government bus purchase subsidy greatly affects total costs and optimal replacement type and age.

7 Sensitivity analysis

The effects of each input variable on the optimal bus type, replacement cycle, and total net cost are evaluated individually by holding all other input variables in the baseline scenarios constant (i.e. ceteris paribus).

7.1 Impacts of key input parameters on optimal bus type and lifecycle

7.1.1 Fuel price

To investigate the impacts of uncertain fuel prices on the optimal replacement plan, a wide range of potential fuel prices (between \$2.64/gal and \$4.46/gal) are tested with both 0 and 80 % subsidy levels. Results are shown in Table 1.

Results indicate that with a 0 % purchase cost subsidy, the optimal solution is always to choose the diesel bus and replace it every 20 years. In other words, when there is no purchase cost subsidy, fuel price has no impact on the optimal replacement solution within realistic values.

With an 80 % cost subsidy, if the fuel price is very low (less than \$2.78/gal) the optimal solution is to choose diesel bus and replace it every 13 or 12 years. If the fuel price is more than \$2.78/gal the optimal solution is to choose a hybrid bus and replace it every 14 years. Optimal solutions are more sensitive to low fuel prices when there is a high purchase cost subsidy.

7.2 Fuel economy

According to the data provided by KCMT, the 60 ft. New Flyer hybrid bus fuel economy varies slightly between 3.59 mpg and 3.69 mpg and the 60 ft. New Flyer diesel bus fuel economy varies between 2.39 and 2.58 mpg. To investigate the impact of relative fuel economies between diesel and hybrid buses different fuel economies were optimized. Sensitivity results are summarized in Table 2.

The number in the table, “14H” for example, indicates that the optimal solution is to choose a hybrid bus and replace it every 14 years. Table 2 shows how optimal replacement solutions change with varying diesel and hybrid bus fuel economies in both 0 and 80 % purchase cost subsidy scenarios. Without a purchase cost subsidy, the optimal solution remains to choose the diesel bus and replace it every 20 years even if: (a) diesel bus fuel economy decreases to 2.2 mpg holding hybrid bus fuel economy constant as 3.65 mpg, or (b) hybrid bus fuel economy increases to 3.95 mpg holding diesel bus fuel economy constant as 2.5 mpg. This indicates that diesel buses are significantly better than hybrid buses in the 0 % subsidy scenario

Table 1 Impacts of fuel price on optimal replacement plan

Fuel price (\$/gal)	2.64	2.78	2.92	3.06	3.20	3.34	3.48	3.62	3.76	3.90	4.04	4.18	4.32	4.46
0 % subsidy	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D
80 % subsidy	13D	12D	14H	14H	14H	14H	14H	14H	14H	14H	14H	14H	14H	14H

Table 2 Impacts of diesel bus fuel economy on optimal replacement plan

Diesel (mpg) hybrid 3.65 mpg	2.2	2.3	2.4	2.5	2.6	2.7	2.8
0 % subsidy	20D	20D	20D	20D	20D	20D	20D
80 % subsidy	14H	14H	14H	14H	14H	13D	14D
Hybrid (mpg) diesel 2.5 mpg	3.35	3.45	3.55	3.65	3.75	3.85	3.95
0 % subsidy	20D	20D	20D	20D	20D	20D	20D
80 % subsidy	13D	14H	14H	14H	14H	14H	14H

even with high variability in the relative fuel economies between the two bus technologies.

In the 80 % subsidy scenario, the best bus type is a function of the relative fuel economies between the two bus types. When the hybrid bus fuel economy is 35 % higher than the diesel bus fuel economy, hybrid buses are preferred; when the difference is less than 35 %, diesel buses are preferred. The reader should recall that the baseline relative fuel economy between hybrid and diesel bus is approximately 46 % for average fuel economy values (3.65 vs. 2.50 mpg).

7.3 Annual utilization

Historical data provided by KCMT indicated that the average annual utilization ranges between 28,379 and 39,679 miles per bus. Therefore, to investigate whether and how annual utilization affects the optimal replacement solutions, different annual utilizations are tested from 28,379 to 39,679 miles/year/bus. Results are summarized in Table 3. Note that per-mile maintenance cost functions were updated to account for varying annual utilization levels.

Results show that in the 0 % subsidy scenario the optimal solution is always to choose the diesel bus and replace it every 20 years (not affected by the annual utilization within the examined range). However, in the 80 % subsidy scenario, the optimal solution is always to buy hybrid buses but the optimal replacement cycle decreases from 16 to 12 years as the annual utilization increases (per-mile maintenance cost increases faster with age with higher annual utilization).

Table 3 Impacts of annual utilization on optimal bus choice and lifecycle

Annual utilization (miles/year/bus)	28,379	29,509	30,639	31,769	32,899	34,029	35,159	36,289	37,419	38,549	39,679
0 % subsidy	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D
80 % subsidy	16H	15H	15H	14H	14H	13H	13H	12H	12H	12H	12H



Table 4 Impacts of capital purchase cost on optimal replacement plan

Diesel bus price % change (hybrid: \$958,000)	−20 %	−15 %	−10 %	−5 %	0 %	5 %	10 %	15 %	20 %
0 % subsidy	20D	20D	20D	20D	20D	20D	20D	20D	20H
80 % subsidy	12D	14H	14H	14H	14H	14H	14H	14H	14H
Hybrid bus price % change (diesel: \$737,000)	−20 %	−15 %	−10 %	−5 %	0 %	5 %	10 %	15 %	20 %
0 % subsidy	20H	20H	20D	20D	20D	20D	20D	20D	20D
80 % subsidy	12H	12H	13H	13H	14H	14H	14H	12D	13D

7.3.1 Capital purchase cost

Capital costs can vary due to market fluctuations, technology improvements, and purchase quantity. It has also been shown in the baseline scenario results that purchase costs may have a significant share of total life cycle costs. Therefore, it is necessary to evaluate the sensitive the optimal replacement plans as a function of varying capital purchase costs. Up to 20 % under and over the current purchase cost for diesel and hybrid buses are tested and results are shown in Table 4.

Results from Table 4 indicate that if there is no subsidy, the optimal solution is always to choose diesel buses and replace them every 20 years except when the diesel bus price increases 20 % or more holding hybrid bus price constant. Alternatively, when the hybrid bus price decreases 15 % or more holding diesel bus price constant it is better to choose hybrid buses. The reader is reminded that there is no fixed cost charge per purchase and that all the buses purchased at year zero are replaced at the same time.

In the 80 % subsidy scenario, the optimal bus type is always hybrid except for diesel bus price reductions of 20 % or more (holding hybrid bus price constant) or when hybrid bus price increases 15 % or more holding diesel bus price constant. The optimal replacement cycle increases slightly with increasing purchase cost for each optimal bus.

7.3.2 Initial age and bus type

The baseline scenarios assume that there are no existing buses. However, it is interesting to evaluate scenarios with an existing fleet of buses of different ages. Scenarios with different initial fleet configurations (types and ages) are also tested. The initial fleet configurations is assumed to be one bus, hybrid or diesel bus, with any of the following six ages: 3, 6, 9, 12, 15, and 18. Results for the 24 scenarios are shown in Tables 5, 6.

Results indicate that initial age has little impact on replacement age or optimal bus type. In the 80 % subsidy scenario, if the initial bus is a hybrid, the optimal solution will be to keep using the hybrid bus and replace it every 16 years. If the initial bus is diesel, the optimal solution will be to keep using the diesel bus until it

Table 5 Impacts of initial fleet configuration on optimal replacement plan (80 % subsidy)

Diesel FE (mpg)	2.50 mpg						3.32 mpg					
Initial bus age (hybrid)	3	6	9	12	15	18	3	6	9	12	15	18
Hybrid replacement age	16	16	16	16	16	18	16	16	16	16	16	18
Diesel replacement age	–	–	–	–	–	–	–	–	–	–	–	–
Initial bus age (diesel)	3	6	9	12	15	18	3	6	9	12	15	18
Hybrid replacement age	16	16	16	16	16	18	16	16	16	16	16	18
Diesel replacement age	12	12	12	12	15	18	15	15	15	15	15	18

In *italics* a one-time replacement

Table 6 Impacts of initial fleet configuration on optimal replacement plan (0 % subsidy)

Diesel FE (mpg)	2.50 mpg						3.32 mpg					
Initial bus age (hybrid)	3	6	9	12	15	18	3	6	9	12	15	18
Hybrid replacement age	20	20	20	20	20	20	20	20	20	20	20	20
Diesel replacement age	20	20	20	20	20	20	20	20	20	20	20	20
Initial bus age (diesel)	3	6	9	12	15	18	3	6	9	12	15	18
Hybrid replacement age	–	–	–	–	–	–	–	–	–	–	–	–
Diesel replacement age	20	20	20	20	20	20	20	20	20	20	20	20

In *italics* a one-time replacement

reaches age 12 (or age 15 or 18 if the initial diesel bus age is already 15 or 18), and then replace it with a hybrid bus every 16 years in all future years in the time horizon. In the 80 % subsidy case, the optimal bus is the hybrid, even if the initial bus is a diesel there is always a reversion towards the optimal policy. In the 0 % subsidy scenario the opposite takes place.

7.3.3 Subsidy level

Results indicate that the 0 and 80 % subsidy levels lead to different optimal bus type choices and replacement cycle. It is interesting to investigate how the optimal replacement plan changes with subsidy level. Ten subsidy levels (from 0 to 90 % with 10 % interval) were tested and the results are shown in Table 7.

Results indicate that with less than 50 % purchase subsidy, the optimal solution is always to purchase diesel bus and replace it every 20 years; with 60 % purchase subsidy, the optimal solution is still diesel bus but the optimal replacement cycle decreases to 19 years. When purchase subsidy increased to 70 %, the optimal bus type switched to hybrid bus because the additional capital cost of purchasing a hybrid bus is smaller than the benefit (higher fuel efficiency and less fuel cost) of utilizing a hybrid bus, the replacement cycle also decreased to 18 years. And as the purchase subsidy level reaches to high level, the optimal replacement cycle decreased rapidly.



Table 7 Impacts of subsidy level on optimal replacement plan

Subsidy level	0 %	10 %	20 %	30 %	40 %	50 %	60 %	70 %	80 %	90 %	100 %
Optimal solution	20D	20D	20D	20D	20D	20D	19D	18H	14H	9H	1H

7.4 Breakeven analysis

From the initial analysis of Sect. 7.1 it is clear that there is no single dominant technology. The breakeven values indicate to what extent each factor by itself can change optimal vehicle type when holding all other input parameters at their baseline scenario values. All scenarios have consistently shown that, without government subsidy, it is more economical to buy diesel buses. However, with 80 % purchase cost subsidy, the best option is to buy the hybrid bus. Thus, as proofed in Sect. 4, there is a breakeven value that can be found using a bisection method.

The breakeven value for the government purchase subsidy is found to be 63 % for the baseline scenario. Therefore it is more economical to buy a hybrid/diesel bus if the purchase cost subsidy is more/less than 63 %, with all other variables held constant as in the baseline scenario. Similarly, breakeven values for other input variables have been calculated for baseline scenarios in both 0 and 80 % subsidy scenarios. Results are summarized in Table 8.

In the baseline scenarios diesel buses win without government subsidy, hence, the breakeven values in 0 % subsidy column in Table 8 indicate when hybrid buses would win if any of the factors meet the condition. For example, with 0 % subsidy, if the diesel bus fuel economy is less than or equal to 1.98 mpg compared to the

Table 8 Breakeven values for 0 % subsidy 80 % subsidy scenarios

	Scenario Baseline solution Baseline values	0 % subsidy Diesel bus 20 years Breakeven value for hybrid bus	80 % subsidy Hybrid bus 14 years Breakeven value for diesel bus
Vehicle factors			
Diesel bus mpg	2.50	≤ 1.98	≥ 2.67
Hybrid bus mpg	3.65	≥ 5.92	≤ 3.34
Diesel bus purchase cost (\$)	737,000	$\geq 875,934$	$\leq 613,242$
Hybrid bus purchase cost (\$)	958,000	$\leq 819,066$	$\geq 1,093,217$
General factors			
Annual utilization (miles/bus)	33,045	$\geq 128,716$	$\leq 13,760$
Fuel price (\$/gal)	3.48	≥ 6.38	≤ 2.79
Fuel inflation rate	2.6 %	≥ 10.2 %	$\leq \text{inf.}$
CO ₂ penalty cost (\$/ton)	30	≥ 506	$\leq \text{inf.}$
Nominal annual discount rate	9.55 %	$\leq \text{inf.}$	≥ 25.09 %

inf. means infeasible, there is no feasible value of the parameter within assigned range that can change the optimal solution

hybrid bus baseline fuel economy of 3.65 mpg, or if the hybrid bus fuel economy is greater than or equal to 5.92 mpg compared to the diesel bus baseline fuel economy of 2.50 mpg, the optimal solution will choose the hybrid bus.

The breakeven values for diesel and hybrid bus purchase cost are not too far from their baseline values, indicating that the purchase cost difference between the two bus technologies dominates the optimal choice of bus type. However, only when fuel price is higher than \$6.38/gal or fuel price is \$3.48/gal but fuel inflation rate is more than 10.2 % (both somewhat unrealistic in the near term), can hybrid bus be chosen as optimal solution. The breakeven values for annual utilization ($\geq 128,716$ miles/year/bus) and CO₂ emissions penalty cost ($\geq \$506/\text{ton}$) are even more unrealistic. There is no feasible breakeven value for the nominal annual discount rate to make hybrid bus the optimal solution.

On the other hand, since hybrid buses win in the 80 % purchase cost subsidy scenario, the breakeven values indicate when diesel buses would win if any of the condition is met. For example, if the diesel bus fuel economy is greater than or equal to 2.67 mpg compared to the hybrid bus baseline fuel economy of 3.65 mpg, or if the hybrid bus fuel economy is less than or equal to 3.34 mpg compared to the diesel bus baseline fuel economy of 2.50 mpg, the optimal solution will choose the diesel bus. These two breakeven values are very close to baseline values, indicating that the optimal solution is very sensitive to the relative fuel economy between the two bus types.

However, because the 80 % purchase cost subsidy has significantly reduced the purchase cost difference between the two buses, only very large deviations of bus purchase cost from the baseline values will change the optimal choice of bus types. Also, fuel price ($\leq \$2.79/\text{gal}$) and annual utilization ($\leq 13,760$ miles/year/bus) breakeven values are far from the realistic values, indicating that they are not factors that can likely change the optimal solution. Other factors such as fuel inflation rate, CO₂ emissions penalty cost, and discount rate are either impossible or infeasible.

In general, most of the breakeven values for the general factors shown in Table 8 are unrealistic in either the 0 or 80 % subsidy scenario. The purchase cost breakeven values in the 0 % subsidy scenario and relative fuel economy between bus types in the 80 % subsidy scenario are close to realistic values; this indicates these two factors are important when evaluating optimal bus type choice.

7.5 Net cost elasticity

The above two subsections focus on the impacts of fuel price, fuel economy, annual utilization, and capital purchase costs on the optimal replacement solution. It is also necessary to analyze which input variable has the highest impact on the optimal total net cost. Elasticity of total net cost in the first 20 years to each of the above input factors was calculated using the following arc elasticity formula (13), where η_x^c is the elasticity of total net cost in the first 20 years c to parameter x :



Table 9 Elasticity between various input variables and net cost over the first 20 years

Factors	0 % subsidy	80 % subsidy
Vehicle factors		
Diesel bus mpg (2.2–2.8)	−0.34	−0.09
Hybrid bus mpg (3.35–3.95)	0.00	−0.39
Diesel bus price (\$589,600–\$737,000)	0.45	0.05
Hybrid bus price (\$766,400–\$958,000)	0.15	0.27
General factors		
Annual utilization (28,379–39,679 miles/year)	0.63	0.85
CO2 emissions penalty cost (\$0–\$100/ton)	0.01	0.01
Fuel price (\$2.64–\$4.46/gallon)	0.34	0.41
Fuel inflation rate (0–5 %)	0.06	0.07
Nominal annual discount rate (5–15 %)	−0.37	−0.54
Purchase cost subsidy (0–80 %)	−0.25	

$$\eta_x^c = \frac{(x_1 + x_2)/2}{(c_1 + c_2)/2} \cdot \frac{A_c}{A_x} = \frac{(x_1 + x_2)}{(c_1 + c_2)} \cdot \frac{(c_2 - c_1)}{(x_2 - x_1)} \quad (13)$$

Elasticity values and the evaluation range of each factor are summarized in Table 9. For example, with an annual utilization range between 28,379 and 39,679 miles/year/bus, each additional 1 % increase in annual utilization, the total net cost in the first 20 years increases 0.63 % (in 0 % subsidy scenario) or 0.85 % (in 80 % subsidy scenario). Results show that annual utilization has the highest absolute elasticity value, followed by nominal annual discount rate and fuel price, diesel bus purchase cost (0 % subsidy), hybrid (80 % subsidy) and diesel (0 % subsidy) bus fuel economy, and purchase cost subsidy.

8 Conclusions

This research presented a fleet replacement optimization model that can help fleet managers to not only minimize fleet total net cost but also perform sensitivity analysis by readily finding break-even values and elasticities. This research has (a) proofed the existence of unique break-even values and (b) estimated break-even values utilizing an algorithm that combines MIP solvers and a bisection search method.

To exemplify the application of the model to real-world fleet data two competing vehicle technologies—diesel and hybrid buses—were analyzed. The bus purchase cost subsidy has a significant impact on optimal bus type choice and its replacement age. Without a purchase cost subsidy, the optimal solution is to choose diesel buses and replace them every 20 years. Sensitivity analysis and breakeven analysis results indicate that the optimal solution is not sensitive to most of the input or baseline parameters (within realistic ranges). The only exception is when hybrid bus purchase costs are more than 10 % higher.

With the maximum allowable purchase cost subsidy in the USA (80 %), the optimal solution is to choose hybrid buses and replace them every 14 years. The breakeven value of government subsidy indicates that hybrid buses are not optimal unless the subsidy is equal or greater than 63 % ceteris paribus. With higher subsidies the optimal solutions are more sensitive to input parameters. Sensitivity analysis and breakeven value analysis also indicate that: (1) the optimal solution is to purchase diesel buses when the base year fuel price is less than \$2.79/gal or hybrid bus additional fuel economy is lower than 35 %; (2) annual utilization, annual discount rate, fuel inflation rate and CO₂ emissions cost have no impact on the optimal vehicle type within realistic ranges; (3) higher utilizations or hybrid bus purchase cost decreases optimal replacement ages from 15 to 12 years.

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References

- Bean JC, Lohmann JR, Smith RL (1984) A dynamic infinite horizon replacement economy decision model. *Eng Econ* 30(2):99–120
- Bean JC, Lohmann JR, Smith RL (1994) Equipment replacement under technological change. *Nav Res Logist (NRL)* 41(1):117–128
- Bellman R (1955) Equipment replacement policy. *J Soc Ind Appl Math* 3(3):133–136
- Boudart J, Figliozzi M (2012) “A study of the key variables affecting bus replacement age decisions and total costs”. In: The 91st Annual Meeting of Transportation Research Board, Washington, p. 19
- Chandler K, Walkowicz K (2006) King county metro transit hybrid articulated buses: Final evaluation results. Technical report. National renewable energy laboratory
- Figliozzi Miguel, Boudart Jesse, Feng Wei (2011) Economic and environmental optimization of vehicle fleets: impact of policy, market, utilization, and technological factors. *J Transp Res Board* 2252:1–6
- Hartman J (1999) A general procedure for incorporating asset utilization decisions into replacement analysis. *Eng Econ* 44(3):217–238
- Hartman J (2000) The parallel replacement problem with demand and capital budgeting constraints. *Nav Res Logist (NRL)* 47(1):40–56
- Hartman J (2001) An economic replacement model with probabilistic asset utilization. *IIE Trans* 33(9):717–727
- Hartman JC, Murphy A (2006) Finite-horizon equipment replacement analysis. *IIE Trans* 38(5):409–419
- Jones PC, Zydiak JL, Hopp WJ (1991) Parallel machine replacement. *Nav Res Logist (NRL)* 38(3):351–365
- Joseph H (2004) Multiple asset replacement analysis under variable utilization and stochastic demand. *Eur J Oper Res* 159(1):145–165
- Karabakal N, Lohmann JR, Bean JC (1994) Parallel replacement under capital rationing constraints. *Manag Sci* 40(3):305–319
- Keles P, Hartman JC (2004) Case study: bus fleet replacement. *Eng Econ* 49(3):253–278
- Kim D, Porter J, Kriett P, Mbugua W, Wagner T (2009) Fleet replacement modeling. Final report
- Lammert M (2008) Long beach transit: two-year evaluation of gasoline-electric hybrid transit buses. Technical report. National renewable energy laboratory
- Laver R, Schneck D, Skorupski D, Brady S, Cham L (2007). Useful life of transit buses and vans. Final report. Federal transit administration
- Nigel C, Zhen F, Wayne S, Lyons D (2007) Transit bus life cycle cost and year 2007 emissions estimation. Final report. West Virginia University
- Nigel C, Zhen F, Wayne W, Schiavone J, Chambers C, Golub A, Chandler K (2009) Assessment of hybrid-electric transit bus technology. Transportation research board



Oakford RV, Lohmann J, Salazar A (1984) A dynamic replacement economy decision model. *IIE Trans* 16(1):65–72

Parametrix and LTK Engineering Services (2011) King county trolley bus evaluation. King County Metro Transit, Seattle

Schiavone J (1997) Monitoring bus maintenance performance. Transit cooperative research program (TCRP) synthesis. Transportation research board

Simms BW, Lamarre BG, Jardine AKS, Boudreau A (1984) Optimal buy, operate and sell policies for fleets of vehicles. *Eur J Oper Res* 15(2):183–195

US EIA (2011) Annual energy outlook 2011. US energy information administration

Wei F, Figliozzi M (2013) “An economic and technological analysis of the key factors affecting the competitiveness of electric commercial vehicles: a case study from the USA market”. *Transportation research part C: emerging technologies* 26 (0) (January), pp. 135–145

Wei F, Machemehl R, Gemar M, Brown L (2012) “A stochastic dynamic programming approach for the equipment replacement optimization with probabilistic vehicle utilization”. In: the 91st annual meeting of transportation research board, Washington, p. 18

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