Model for Network Assignment Problem of Capacitated Freight with Disruptions

Integer Multicommodity Flow with Recourse

Avinash Unnikrishnan, Mehrdad Shahabi, and Miguel Figliozzi

A modeling framework is proposed for the freight network assignment problem with recourse, network travel time and cost disruptions, and link flow capacity constraints. "Recourse" is defined as the ability of a user to reconsider and adapt his or her routing decisions in response to newly acquired information about network disruption. When real-time tracking and monitoring of network shipments allow companies a routing recourse, network capacity must be simultaneously considered. If a disruption significantly alters network flows, the capacity of alternative or secondary routes may be quickly reached. A new freight mathematical model with a capacitated network and adaptive routing was developed and solved. Results showed that simultaneous consideration of recourse and capacity constraints was superior to benchmark results obtained with nonadaptive deterministic behavior.

Modern supply chains operate with growing availability of real-time information but also with higher exposure to disruptions and uncertain events. Supply chains are increasingly connected and interdependent, which makes them vulnerable to natural or man-made disruptions (*1, 2*). State officials and business managers have become more aware of and sensitive to the vulnerabilities of their transportation and supply chain systems. State agencies and private companies are starting to question how to improve system resilience and mitigate the negative impact of network disruptions (*3*). Freight flow disruptions can be caused by many factors, ranging from weather-related events such as flooding (*4*) to network capacity limitations. Freight network capacity is a growing problem in the United States and has been documented and analyzed extensively (*5, 6*); more than 100 freight bottlenecks have been identified and ranked for severity and economic cost.

In this research, network disruptions are defined as any event that could interrupt freight flows or provoke delays that result in cost increases for the supply chain. Disruption costs are difficult to estimate because one out-of-service factory or vehicle can have a disproportionate ripple effect along connected supply chains. Research has shown great variation in freight value of time across regions, roadway conditions, and carrier types. NCHRP Report 431 recommends the use of travel time values for congested periods that are 2.5 times the value of time estimates during uncongested periods (*7*). Kawamura investigated differences in value of time among operators and trucking industry segments; findings from Kawamura's research show that freight carriers have a value of time that is several times higher than that of passenger vehicles and that there is significant heterogeneity among carriers (*8*). Therefore, for accurate modeling of the impact of disruptions, a freight network assignment model must account for the variation in time value and associated costs under disruptions.

Disruptions can provoke delays that result in longer travel times, higher fuel consumption, and loss of a driver's productivity. Delays and congestion at bottlenecks not only affect a company's bottom line but also increase greenhouse gas emissions and pollution levels (*9*). There is also heterogeneity in vehicle–driver costs. A recent study by the American Transportation Research Institute found that specialized carrier types have the highest cost per mile, followed by less-than-truckload and truckload carrier types (*10*). Hence, an assignment framework for freight networks must account for this variation in disruption and travel costs by commodity and carrier types.

Transportation costs can be up to 10% of the final product costs (*11*). Supply chain costs can greatly exceed transportation-related disruption costs if companies rely heavily on lean practices, such as just-in-time deliveries, as well as increasingly long international transportation chains. Companies use sophisticated logistics systems to meet customer requirements without an overall increase in operating costs (*12*). Continuous real-time monitoring of shipment cost and delivery time is being used by companies to reduce transportation costs while avoiding costly delays or disruptions, high levels of safety stocks, and system redundancies (*13*). Continuous tracking and monitoring of vehicles and containers from origin to destination can increase shipment efficiency and profitability. Real-time information and adaptive routing can help companies maintain high levels of customer service with less cost than comparable asset- or capital-intensive strategies that rely on redundant vehicles or capacity.

This paper develops an integer multicommodity flow with recourse formulation to capture the adaptive routing behavior of freight decision makers (FDM), shippers, and carriers and that incorporates heterogeneity in response to disruptions and valuations of time cost by commodity type. This presents a new modeling framework that builds on the work of Unnikrishnan and Figliozzi (*14*).

The essential difference between this paper and that earlier work is that capacity constraints are applied to every link of the network noticeably; the addition of capacity constraints makes the problem

A. Unnikrishnan and M. Shahabi, Department of Civil and Environmental Engineering, West Virginia University, Room 621 ESB, P.O. Box 6103, Morgantown, WV 26506. M. Figliozzi, Department of Civil and Environmental Engineering, Portland State University, P.O. Box 751, Portland, OR 97207-0751. Corresponding author: A. Unnikrishnan, Avinash.Unnikrishnan@mail.wvu.edu.

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and optimization model much more realistic yet significantly more difficult to solve. The model presented in this work is for regional long-haul freight truckload transportation and may not be applicable to an urban network where multistop tours may be more appropriate. Because truckloads are the basic unit of flow, there is a natural integer flow constraint. However, the presented model and solution algorithm are readily applicable to problems with nonintegral flows.

LITERATURE REVIEW

Little work has been done on freight network models that can account for changes in routing decisions in response to travel time or cost uncertainties caused by network disruptions. With advances in information technology and tracking, FDMs now have access to real-time information about network conditions. Therefore, under network disruptions, FDMs can progressively update their routing strategy by using updated information about network conditions; this is called routing recourse. The actual route chosen will depend on network conditions revealed as shipments traverse the network. Under such conditions, the objective is to determine the best routing policy that describes the routes to be chosen for different disruption scenarios. Such routing problems can be described as stochastic shortest paths with recourse or online shortest path problems. Croucher (*15*) and Andreatta and Romeo (*16*) studied the online shortest path problem with stochastic arc presence and known probability distribution functions. Polychronopoulos and Tsitsiklis solved the online shortest path problem with recourse using dynamic programming (*17*). They assumed that the user would obtain new information while traversing the network and compute the least expected cost for each scenario. Waller and Ziliaskopoulos developed an efficient algorithm for the case in which network uncertainties had specific limited spatial and temporal dependencies (*18*). Hall (*19*), Pretolani (*20*), and Miller-Hooks (*21*) studied the shortest path problem with recourse in an uncertain and time-varying environment. Harks et al. solved the online version of the multicommodity flow problem by developing greedy algorithms that route the commodities sequentially in the network, but they did not consider capacity restriction in their formulation (*22*). Recently, Unnikrishnan and Waller studied the problem of user equilibrium traffic network assignment with recourse as a nonlinear convex optimization problem (*23*). Unnikrishnan and Waller focused on an equilibrium formulation in which travel costs are a function of flow, the impact of hard capacities is not modeled, and there are no integrality restrictions on flow (*23*).

The only formulation that considers the impact of disruption and recourse routing in a dynamic freight network is that of Unnikrishnan and Figliozzi (*14*); however, that work did not consider capacity constraints. When capacity constraints are added to the formulation, the resulting problem becomes an integer multicommodity problem with recourse. This paper represents the first work that formulates and solves the integer multicommodity flow problem with recourse (MCF-R).

The multicommodity network flow problem is well known and has been solved by many researchers (*24–26*). Ahuja et al. made a comprehensive study of the multicommodity flow problem (*24*). Methods that can be used to solve multicommodity flow problems include Lagrangian relaxation and column generation heuristics. Multicommodity flow problems can be classified into two types, depending on whether there are integrality flow constraints. Barnhart et al. used branch-and-price-and-cut to solve the integer formulation type (*27*). The solution method proposed in this paper is a Lagrangian relaxation heuristic that has been applied to other multicommodity flow problems (*28, 29*). Two types of Lagrangian relaxation have been applied in the literature: (*a*) relaxing of link flow capacity constraints and (*b*) relaxing of link flow and flow conservation constraints (*30, 31*). In this paper, a Lagrangian relaxation is applied to the capacity constraints, and the algorithm presented by Unnikrishnan and Figliozzi is used to solve the resulting subproblem (*14*).

MODEL FORMULATION

This section presents an integer optimization formulation for solving the capacitated online freight network assignment problem. The model is formulated as an integer MCF-R.

Consider a network $G = (N, A)$, where N denotes the set of nodes and A the set of arcs. An FDM, which can be a shipper, a carrier, or a third-party logistics agent, needs to transport a set of commodities K^{rs} from a node in the set of origins R to a node in the set of destinations S. The network can exist in multiple states that correspond to various disruption scenarios. The occurrence of a disruption is independent of prior disruption occurrences. Let Ω represent the set of all potential network disruption scenarios. A discrete probability distribution P characterizes the relationship between any scenario ω and Ω; $p^{\omega} \in P$ and represents the probability of scenario $\omega \in \Omega$. In the presented model, for simplicity, every specific commodity must be sent from a single distinct origin to a single distinct destination. Therefore, the decision variable is the amount of flow of the commodity $k \in K^{rs}$ for each arc between origin $r \in R$ and destination $s \in S$ in system state $\omega \in \Omega$.

The generalized cost of transporting commodities in the network is considered to be a weighted linear combination of three costs:

1. Transportation cost $c_a^{k\omega}$, the unit cost of transporting commodity $k \in K^{rs}$ on arc $a \in A$ in system state $\omega \in \Omega$;
2. Travel time cost $t_a^{k\omega}$, the unit travel time for transporting commodity $k \in K^{rs}$ on arc $a \in A$ in system state $\omega \in \Omega$; and
3. Unreliability parameter $u_a^{k\omega}$, which is based on the experience of the FDM in using a particular carrier or mode to transport goods.

The weight of each cost parameter, denoted $w_{trs}^{k\omega}$, $w_{crs}^{k\omega}$, $w_{urs}^{k\omega}$, is chosen according to the relative importance of each component of the generalized cost. Therefore, the generalized cost for commodity $k \in K^{rs}$ on arc $a \in A$ in system state $\omega \in \Omega$ between origin $r \in R$ and destination $s \in S$ can be written as

$$C_{ars}^{k\omega} = w_{trs}^{k\omega} t_a^{k\omega} + w_{crs}^{k\omega} c_a^{k\omega} + w_{urs}^{k\omega} u_a^{k\omega}$$

As the commodities traverse the network, the FDM receives new information and updates about network disruption scenarios. When a node is reached, the FDM can use the information received until that moment to eliminate a set of potential network disruption scenarios in the set Ω. The FDM can use this knowledge to update the travel cost parameters, probability distributions, and routes. Under such conditions, the FDM's routing strategy is described by a routing policy. A routing policy uses the information received to provide the routes or paths from each node to the destination; the routing policy thus captures the FDM's response to information about network disruptions. Let H_k^{rs} represent the set of all routing policies between origin $r \in R$ and destination $s \in S$ for commodity $k \in K^{rs}$, and let f_k^h denote the flow of commodity $k \in K^{rs}$ on routing policy $h \in H_k^{rs}$.

Let D represent the origin–destination vector comprising all elements d_k^{rs} denoting the demand of commodity k between origin $r \in R$ and destination $s \in S$. Every arc has a flow capacity denoted by V_a, $\forall a \in A$. The capacity constraint either can represent the constraints on number of trucks or rail cars available to travel on that arc or can represent flow restrictions caused by environmental considerations (*32*).

The formulation and solution algorithm can be trivially extended to the case in which the capacity of the arc depends on the system state. Let δ_{ah}^{ω} be the incidence variable where $\delta_{ah}^{\omega} = 1$ if routing policy $h \in H_k^{rs}$ uses arc $a \in A$ in system state $\omega \in \Omega$ and $\delta_{ah}^{\omega} = 0$ otherwise. The incidence variable relates the flow of the commodity $k \in K$ for an arc $a \in A$ between origin $r \in R$ and destination $s \in S$ in system state $\omega \in \Omega$, $x_{ars}^{k\omega}$, and the flow on routing policy $h \in H_k^{rs}$ as follows:

$$x_{ars}^{k\omega} = \sum_{h \in H_{rs}} \delta_{ah}^{\omega} f_k^h \qquad \forall a \in A,\ h \in H_k^{rs},\ \omega \in \Omega,\ r \in R,\ s \in S$$

The notation used in the formulation is given in Table 1.

Given the preceding definitions and notation, the optimization formulation for the capacitated freight network assignment problem under disruption and recourse is given next. In network optimization terminology, the formulation can be described as an integer MCF-R or an online integer multicommodity flow problem (Equation 1).

$$\min Z = \sum_{a \in A} \sum_{s \in S} \sum_{r \in R} \sum_{k \in K^{rs}} \sum_{\omega \in \Omega} \left(w_{trs}^{k\omega} t_a^{k\omega} + w_{crs}^{k\omega} c_a^{k\omega} + w_{urs}^{k\omega} u_a^{k\omega} \right) x_{ars}^{k\omega} p^{\omega} \tag{1}$$

subject to

$$x_{ars}^{k\omega} = \sum_{h \in H_{rs}} \delta_{ah}^{\omega} f_k^h \qquad \forall a \in A,\ h \in H_k^{rs},\ \omega \in \Omega,\ r \in R,\ s \in S \tag{2}$$

$$\sum_{h \in H_{rs}} f_k^h = d_k^{rs} \qquad r \in R,\ s \in S,\ k \in K^{rs} \tag{3}$$

$$\sum_{k \in K} \sum_{r \in R} \sum_{s \in S} x_{ars}^{k\omega} \le V_a \qquad \forall a \in A,\ \omega \in \Omega \tag{4}$$

$$x_{ars}^{k\omega} \ge 0 \qquad \forall a \in A,\ \omega \in \Omega,\ r \in R,\ s \in S,\ k \in K^{rs} \text{ and integer} \tag{5}$$

$$f_k^h \ge 0 \qquad \forall h \in H_k^{rs} \text{ and integer} \tag{6}$$

The first constraint relates the flow on an arc of a commodity between specific origin–destination pairs under a specific scenario to the flow on routing policies that uses the incidence variable. The second constraint states that for a specific origin–destination pair and commodity, the demand is equal to the sum of the flows on all routing policies. The third constraint restricts the flow on each arc to its capacity. Constraints 4 and 5 impose the nonnegativity and integrality constraints on the decision variables.

SOLUTION METHODOLOGY

Equation 1 is an integer MCF-R. As shown in the literature, solving integer multicommodity flow problems caused by capacity constraint on arc flows is relatively difficult. In this research, a Lagrangian heuristic method has been adopted for solving the MCF-R.

TABLE 1 Notation in Formulation

Term	Definition
N	Set of nodes
A	Set of arcs indexed by a
R	Set of origins indexed by r
S	Set of destinations indexes by s
Ω	Set of system scenarios indexed by ω
P	Discrete probability distribution system scenarios
p^{ω}	Probability of scenario ω
K^{rs}	Set of commodities to be transported from origin r to destination s indexed by k
$c_a^{k\omega}$	Unit cost of transporting commodity k on arc a in system state ω
$t_a^{k\omega}$	Unit travel time for transporting commodity k on arc a in system state ω
$u_a^{k\omega}$	Unreliability parameter associated with transporting commodity k on arc a in system state ω
$C_{ars}^{k\omega}$	Generalized cost for commodity k on arc a in system state ω between origin r and destination $s \in S$
$w_{trs}^{k\omega}$, $w_{crs}^{k\omega}$, $w_{urs}^{k\omega}$	Weights that express the relative importance attached by FDM toward travel time, travel costs, and unreliability parameter for commodity k in system state ω between origin r and destination $s \in S$
H_k^{rs}	Set of all routing policies between origin r destination s for commodity k indexed by h
f_k^h	Flow of commodity k on routing policy h
d_k^{rs}	Demand of commodity k between origin r and destination s
D	Origin–destination vector
V_a	Capacity of arc a
δ_{ah}^{ω}	Incidence variable that is equal to 11 if routing policy h uses arc a between origin r and destination s in system state ω and 0 otherwise
$\chi_{ars}^{k\omega}$	Flow of commodity k on arc a between origin r and destination s in system state ω and 0 otherwise

In the Lagrangian relaxation (LR) heuristic-based solution method, Constraint 3 in Equation 1 is relaxed and moved with nonnegative Lagrange multiplier λ_a^{ω} to the objective function, as shown in Equation 7:

$$\min \mathrm{LR}(\lambda) = \sum_{a\in A}\sum_{s\in S}\sum_{r\in R}\sum_{k\in K^{rs}}\sum_{\omega\in\Omega}\left(w_{trs}^{k\omega}t_a^{k\omega} + w_{crs}^{k\omega}c_a^{k\omega} + w_{urs}^{k\omega}u_a^{k\omega}\right)x_{ars}^{k\omega}p^{\omega} + \sum_{a\in A}\sum_{\omega\in\Omega}\lambda_a^{\omega}\left(\sum_{s\in S}\sum_{r\in R}\sum_{k\in K^{rs}} x_{ars}^{k\omega} - V_a\right) \qquad (7)$$

subject to Constraints 1, 2, 4, and 5.

Given a set of values of $\lambda = (\lambda_a^{\omega}, \forall a \in A, \omega \in \Omega)$, the objective function of the preceding formulation can be rewritten as

$$\sum_{a\in A}\sum_{s\in S}\sum_{r\in R}\sum_{k\in K^{rs}}\sum_{\omega\in\Omega}\left(w_{trs}^{k\omega}t_a^{k\omega} + w_{crs}^{k\omega}c_a^{k\omega} + w_{urs}^{k\omega}u_a^{k\omega} + \frac{\lambda_a^{\omega}}{p^{\omega}}\right)x_{ars}^{k\omega}p^{\omega} - \sum_{a\in A}\sum_{\omega\in\Omega}\sum_{s\in S}\sum_{r\in R}\sum_{k\in K^{rs}} V_a\lambda_a^{\omega}$$

The term $\lambda_a^{\omega}V_a$ can be removed because it is constant given $\lambda = (\lambda_a^{\omega}, \forall a \in A, \omega \in \Omega)$; then the Lagrangian relaxed model can be rewritten as Equation 8:

$$\min \mathrm{LR}(\lambda) = \sum_{a\in A}\sum_{s\in S}\sum_{r\in R}\sum_{k\in K^{rs}}\sum_{\omega\in\Omega}\left(w_{trs}^{k\omega}t_a^{k\omega} + w_{crs}^{k\omega}c_a^{k\omega} + w_{urs}^{k\omega}u_a^{k\omega} + \frac{\lambda_a^{\omega}}{p^{\omega}}\right)x_{ars}^{k\omega}p^{\omega} \qquad (8)$$

subject to Constraints 1, 2, 4, and 5. Figure 1 is a flowchart of the relaxation heuristic.

Equation 8 decomposes into $\Sigma_{r\in R}\Sigma_{s\in S}|K^{rs}|$ stochastic shortest path with recourse subproblems, which can be solved with the algorithm presented by Unnikrishnan and Figliozzi (*14*) by appropriately modifying the cost function. The solution for Equation 8 may not satisfy the capacity constraints in Equation 1. Therefore, the solution obtained from solving Equation 8 may be infeasible for the original problem. The solution to Equation 8 can be used to calculate the lower bound to the objective function. In such cases, the primal heuristic presented by Holmberg et al. is adapted and applied (*28*).

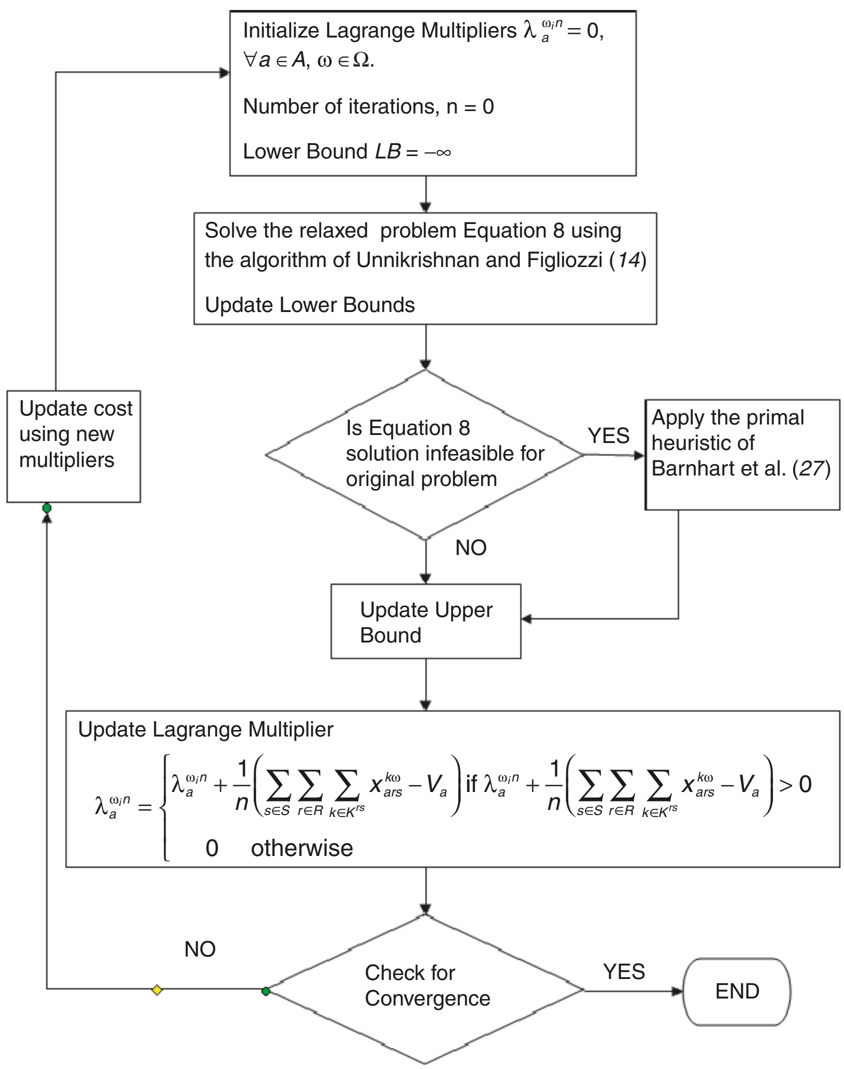

FIGURE 1 Flowchart for Lagrangian relaxation heuristic.

PRIMAL HEURISTIC

In the primal heuristic algorithm, the commodities are assumed to enter the network sequentially. The algorithm is solved for the first commodity, and then every arc is checked for violation of arc capacity. If an arc capacity has been violated, that arc will be closed for the next commodity assignment. This step must be repeated for all commodities until a feasible solution is created. Although the primal heuristic generates feasible solutions in most cases, when the capacity of the network is too low, a good feasible solution may not occur.

The steps of the solution algorithm are as follows:

Step 0. Initialization. Set the Lagrange multiplier $\lambda_a^{\omega,n} = 0$, $\forall a \in A$, $\omega \in \Omega$. The number of iterations is $n = 0$. Initialize the lower bound LB $= -\infty$.

Step 1. Solving the relaxed problem. Solve the relaxed model, Equation 8, and update the lower bound solution as the maximum of the new solution and previous lower bound. The relaxed model can be solved by applying the algorithm presented by Unnikrishnan and Figliozzi (*14*), $\Sigma_{r\in R}\Sigma_{s\in S}|K^{rs}|$ times.

Step 2. Coping with infeasibility and generating feasible upper bounds. In some cases, the solution for Equation 8 may not satisfy Constraint 4 of the original problem. To obtain a feasible solution, the primal heuristic method suggested by Holmberg et al. is applied (*28*). Update the new best upper bound to be the minimum of the existing best upper bound and the upper bound calculated in this iteration.

Step 3. Updating the Lagrange multiplier. Update the Lagrange multiplier $\lambda_a^{\omega,n}$ by subgradient optimization as

$$\lambda_a^{\omega,n} = \begin{cases} \lambda_a^{\omega,n} + \dfrac{1}{n}\left(\displaystyle\sum_{s\in S}\sum_{r\in R}\sum_{k\in K^{rs}} x_{ars}^{k\omega} - V_a\right) \\ \qquad \text{if } \lambda_a^{\omega,n} + \dfrac{1}{n}\left(\displaystyle\sum_{s\in S}\sum_{r\in R}\sum_{k\in K^{rs}} x_{ars}^{k\omega} - V_a\right) > 0 \\ 0 \qquad \text{otherwise} \end{cases}$$

Step 4. Termination criteria. The algorithm will stop if one of the following three conditions is satisfied: (*a*) the difference between the feasible solution and the lower bound solution is less than a predetermined value; (*b*) $\lambda_a^{\omega,n} = 0$, $\forall a \in A$, $\omega \in \Omega$; or (*c*) the maximum number of iterations (κ) is reached. If termination criteria are satisfied, stop; else $n = n + 1$ and go to Step 2. Criterion (*c*) ensures that if criteria (*a*) and (*b*) are not satisfied, then the algorithm will stop after κ iterations. Ahuja et al. adopted similar criteria for multicommodity flow problems (*24*).

NUMERICAL ANALYSIS

Computational experiments are used to estimate the potential savings obtained by adopting a capacitated adaptive online routing approach versus a simpler deterministic nonadaptive routing approach. A numerical study based on a real-world collaborative freight network is described next.

Parameter Setting

The network corresponds to a national-level food distributor that ships three types of products—frozen, dried, and refrigerated—from Idaho, Washington, Nevada, and California to the Portland, Oregon, area. Unnikrishnan and Figliozzi give a map of the network showing the locations of distribution centers (*14*). The network has 32 nodes and 96 arcs with 20 origins and one destination, which is Portland. Table 2 demonstrates the demand for each origin–destination pair in weekly truckloads per type of commodity. There are three types of arcs: (*a*) arcs lying on regular shipping routes, (*b*) arcs lying on alternate routes, and (*c*) supplementary arcs that generally are not used. Supplementary arcs or paths are not used in normal operating condition because their travel time and transport costs are relatively higher than regular and alternate arcs and paths (25% penalty on time and 100% penalty on cost). However, in the case of disruptions, supplementary arcs or routes may become viable, that is, may be a cheaper option than the alternate or normal arcs and

TABLE 2 Demand to Portland in Scale of Truckloads

Origin	Commodity or Product	Weekly Truckloads	Origin	Commodity or Product	Weekly Truckloads
Burlington, Wash.	Refrigerated	1	Riverside	Refrigerated	1
Spokane, Wash.	Dry	1	Ontario, Calif.	Dry	2
Vancouver, Wash.	Dry	1	Ontario	Frozen	2
Tacoma, Wash.	Dry	1	Ontario	Refrigerated	2
Idaho Falls, Idaho	Dry	1	Los Angeles, Calif.	Dry	4
Aberdeen, Idaho	Frozen	1	Los Angeles	Frozen	9
Burley, Idaho	Frozen	2	Los Angeles	Refrigerated	2
Boise, Idaho	Refrigerated	1	Fresno, Calif.	Dry	4
Fruitland, Idaho	Frozen	1	Fresno	Refrigerated	2
Las Vegas, Nev.	Refrigerated	1	Salinas, Calif.	Refrigerated	1
Reno, Nev.	Dry	1	Stockton, Calif.	Dry	9
Long Beach, Calif.	Frozen	1	Oakland, Calif.	Dry	1
Riverside, Calif.	Dry	1	San Francisco, Calif.	Frozen	1
Riverside	Frozen	1			

routes. The supplementary arcs are assumed to have infinite capacity because disruption takes place on the heavily used regular links (*a*) and (*b*). To get 15% of the regular shipping route arcs over capacity, an arbitrary link capacity constraint of three weekly truckload units was set. As a result of the capacity constraints, regular shipping routes around the Portland area become a bottleneck because capacity is exceeded. The objective is to analyze the impact of freight bottlenecks on routing strategies. (The constraint is arbitrary and could apply to a different mode, for example, number of trains limited to three per day, in another context.)

Google Maps is used to compute travel times between nodes. Operating travel costs that cover fuel and other per-mile costs are \$1.45, \$1.65, and \$1.75 per mile per truck for dry, refrigerated, and frozen commodities, respectively. The weight of the parameters for each of the three cost types is considered to be different in normal conditions and during disruption; the parameter value changes are consistent with the values obtained from stated preference surveys (*33*). The values of the weight parameters are as follows:

1. 0.0975, 0.0029, and 0.1219 for travel time, costs, and unreliability parameters, respectively, under normal operating conditions and
2. 0.4579, 0.0029, and 0.3657 for travel time, costs, and unreliability parameters under disruption.

Under disruption, travel time and unreliability parameters do increase significantly; however, cost per mile to operate the vehicle remains constant, but this cost would increase if rerouting takes place on a longer path.

Modeling Disruption

The various disruption scenarios are randomly generated, and the transportation costs, travel times, and unreliability levels are scaled up according to Equation 9. Three parameters are considered for modeling disruption in the network:

1. Severity factor Ψ. Two disruption levels are considered: low and high. In the case of low disruption, the severity factor is equal to 1. In the case of high disruption, the severity factor is set equal to the cardinality of the node states of the upstream arc.
2. Scaling factor (SF). The SF is used to further scale up the disruption within each disruption severity level.
3. Random parameter ε. The random parameter is a realization of a uniform random number between 0 and 1.

Travel costs, travel time, and unreliability parameter, the three parts of the generalized cost, are scaled up to model the two cases of low and high disruption. The random cost function for generating cost for each scenario is

$$C_a^{k\omega} = C_a^{k1}\left(1 + \Psi \mathrm{SF} \varepsilon\right) \tag{9}$$

The network is assumed to operate under normal conditions in Scenario 1 and C_a^{k1} is the cost under normal operating conditions.

The proposed algorithm is tested on the network in case of low and high disruption for different amounts of severity factors. The performance metric used is the cost savings obtained by online routing over deterministic routing. Two types of deterministic routing are used, the base case and the expected cost case. In the base case, the goods will be shipped using the costs under nondisruption normal scenarios; in the expected cost case, routing will be done based on expected costs on each arc. The deterministic problem is solved with a Lagrangian relaxation algorithm given by Ahuja et al. (*24*). The total savings *S* when adaptive online routing is used is calculated with the formula

$$S = \left(\frac{Z_{\mathrm{DET}} - Z_{\mathrm{AD}}}{Z_{\mathrm{DET}}}\right) \times 100$$

where Z_{DET} is the total expected cost of the system under deterministic routing and Z_{AD} is the total cost of the system under adaptive online routing.

The effect of randomness in the results was reduced by using the average of 30 generated network disruption samples to calculate the savings:

$$\overline{S} = \sum_{i=1}^{30} \frac{S^i}{30}$$

RESULTS OF EXPERIMENTS

Four types of experiments are conducted to demonstrate the savings of capacitated adaptive online routing over deterministic routings.

Disruption Severity Levels

The first set of computational runs, shown in Tables 3 and 4, compares the savings for different disruption severity levels. The numbers given in the base case columns correspond to the savings over base case deterministic routing, and the numbers given in the expected value columns correspond to savings over expected value deterministic routing. The number of scenarios is considered to be four in this test, and the experiments are done for two cases of probability distributions for each scenario: (*a*) equally likely probability distribution and (*b*) asymmetric probability distribution. In the case of asymmetric probability distribution, the probability of Scenario 1 is assumed to be the highest among all scenarios and is equal to .70; the probability of the remaining scenarios is the same and equal to .10. In the case of equally likely probability distribution, all the probabilities are the same and equal to .25.

As expected, the benefit of adaptive routing is greater in the high-disruption scenario than in the low-disruption scenario. Within each disruption severity level, as the SFs increase, the savings obtained by adaptive routing over deterministic routing increases significantly. Planning for normal operation conditions would return a considerably worse solution than would planning for expected value. When the disruption scenarios are equally likely, the savings are higher than when the scenarios are asymmetric. The gain obtained by adaptive routing is found to be highly dependent on the commodity type.

Number of Uncertain Scenarios

The impact of different numbers of scenarios on the savings through online routing is tested in this experiment. The SF is assumed to be 1 for the cases of low and high disruption. The results are shown in Table 5. The numbers in the base case columns correspond to the savings over base case deterministic routing, and the numbers in the expected value columns correspond to savings over expected value

TABLE 3 Disruption Severity Levels: Percentage Gain Under High Disruption

	Savings, Base Case Routing (%)			Savings, Expected Value Routing (%)		
SF	Dry	Frozen	Refrigerated	Dry	Frozen	Refrigerated
Equally Distributed						
0.5	5.68	2.82	6.42	0.95	2.13	1.67
1	20.63	10.19	21.33	0.86	2.08	1.40
1.5	36.73	18.09	37.04	0.74	1.75	1.19
2	53.06	26.1	52.91	0.66	1.53	1.08
2.5	69.47	34.13	68.83	0.60	1.40	1.01
3	85.91	42.19	84.76	0.57	1.31	0.97
Asymmetrically Distributed						
0.5	4.14	2.02	4.71	1.59	3.35	3.24
1	15.26	7.39	15.69	2.32	5.18	4.1
1.5	27.17	13.11	27.27	2.35	5.31	4.11
2	39.29	18.92	38.97	2.34	5.23	4.07
2.5	51.47	24.76	50.7	2.31	5.14	4.02
3	63.67	30.62	62.45	2.28	5.07	3.99

deterministic routing. The savings are found to increase significantly with an increase in the number of scenarios. However, the increase in savings is marginal for low-disruption scenarios. The product type has significant influence on the savings under high disruption compared with the low-disruption scenarios. The results show that as uncertainty in the network increases, there is significantly more benefit for FDMs to invest in technology that will allow them to adopt and implement the adaptive recourse routing strategies.

Impact of Scenario Probabilities

The effects of different values of scenario probability over the savings in costs through online routing are tested. Table 6 shows the savings for low and high disruption. The numbers in the base case columns correspond to the savings over base case deterministic routing, and the numbers in the expected value columns correspond to savings over expected value deterministic routing. The SF is considered to be 1, the number of scenarios is four, and the probability of Scenario 1 increases from .25 to .7 in increments of .05. The probability of the remaining scenarios is assumed to be equally likely and equal to $(1 - p_1)/3$.

In the high-disruption case, as the probability of Scenario 1 increases, the savings obtained by adaptive routing over deterministic base case routing decrease, whereas the savings obtained by adaptive routing over expected value routing increase. The change in savings is relatively marginal for low disruption compared with high disruption. However, the increase is relatively marginal compared with the high-disruption scenario. In the low-disruption case, the savings over the expected value routing is relatively stable with an increase in probability of Scenario 1.

The computational times for solving the algorithm were found to depend on the total number of uncertain scenarios. The computational time for two, three, four, five, and six uncertain scenarios was found to be 36.375, 53.994, 93.19, 150.26, and 275.52 s, respectively.

TABLE 4 Disruption Severity Levels: Percentage Gain Under Low Disruption

	Savings, Base Case Routing (%)			Savings, Expected Value Routing (%)		
SF	Dry	Frozen	Refrigerated	Dry	Frozen	Refrigerated
Equally Distributed						
0.5	0.31	0.11	0.10	0.07	0.16	0.08
1	2.06	1.18	2.10	0.69	1.13	1.11
1.5	6.25	3.54	6.10	1.09	2.16	1.41
2	11.49	6.46	10.59	1.18	2.48	1.34
2.5	17.16	9.63	15.32	1.17	2.43	1.3
3	23.02	12.91	20.12	1.11	2.27	1.21
Asymmetrically Distributed						
0.5	0.23	0.08	0.07	0.08	0.15	0.07
1	1.52	0.85	1.53	0.82	1.31	1.45
1.5	4.63	2.58	4.47	1.85	3.3	2.93
2	8.53	4.71	7.77	2.48	4.75	3.38
2.5	12.76	7.03	11.24	2.82	5.54	3.56
3	17.13	9.42	14.76	3.00	5.9	3.65

TABLE 5 Uncertain Scenarios

	Savings, Base Case Routing (%)			Savings, Expected Value Routing (%)		
Scenario	Dry	Frozen	Refrigerated	Dry	Frozen	Refrigerated
Equally Distributed: High Disruption						
2	9.01	4.38	9.21	1.75	0.83	1.48
3	12.54	6.14	12.71	2.28	0.98	1.64
4	20.63	10.19	21.33	2.08	0.86	1.4
5	25.11	12.41	25.39	1.87	0.79	1.21
6	32.18	15.95	33.13	1.61	0.67	1.11
Equally Distributed: Low Disruption						
2	1.61	0.9	1.6	0.64	0.42	0.75
3	1.42	0.79	1.33	1	0.6	1.02
4	1.67	0.88	1.68	1	0.59	1.02
5	1.86	1.06	1.88	1.36	0.83	1.4
6	2.16	1.25	2.28	1.49	0.93	1.56

The savings with the use of adaptive routing is expected to increase with an increase in incident severity and with the number of potential disruption scenarios. The numerical experiments conducted in the study show that, depending on the commodity and the scenarios, the savings can be as high as 85%. An FDM can use this model to evaluate the benefits obtained by developing adaptive routing strategies and decide whether to invest in technologies that can help implement such a system.

CONCLUSIONS

FDMs now have the ability to optimize their routing processes by using real-time information provided by monitoring and tracking technologies. This research developed an assignment model for freight networks that captures the adaptive routing behavior of an FDM in response to network disruptions. The model takes into account capacity constraints on flows. To the authors' knowledge,

TABLE 6 Impact of Scenario Probabilities

	Savings, Base Case Routing (%)			Savings, Expected Value Routing (%)		
Probability of Scenario 1	Dry	Frozen	Refrigerated	Dry	Frozen	Refrigerated
High Disruption						
0.25	20.63	10.19	21.33	2.08	0.86	1.4
0.3	20.27	10	20.94	2.3	0.96	1.59
0.35	19.94	9.83	20.6	2.5	1.06	1.76
0.4	19.49	9.59	20.12	2.77	1.19	1.99
0.45	18.96	9.31	19.57	3.08	1.34	2.26
0.5	18.51	9.07	19.11	3.35	1.46	2.48
0.55	17.83	8.72	18.39	3.75	1.65	2.83
0.6	17	8.28	17.53	4.22	1.87	3.24
0.65	16.03	7.79	16.48	4.75	2.12	3.73
0.7	15.26	7.39	15.69	5.18	2.32	4.1
Low Disruption						
0.25	2.06	1.18	2.1	1.13	0.69	1.11
0.3	2.03	1.15	2.06	1.03	0.62	1.07
0.35	1.99	1.13	2.02	1.2	0.74	1.22
0.4	1.95	1.11	1.98	1.24	0.77	1.28
0.45	1.9	1.07	1.92	1.29	0.81	1.35
0.5	1.85	1.05	1.87	1.32	0.83	1.39
0.55	1.78	1.01	1.8	1.35	0.85	1.45
0.6	1.7	0.96	1.72	1.36	0.85	1.48
0.65	1.63	0.92	1.65	1.35	0.84	1.47
0.7	1.52	0.85	1.53	1.31	0.82	1.45

this is the first model formulation of an integer multicommodity flow problem with recourse and capacity constraints. A Lagrangian relaxation-based heuristic for solving the formulation was provided. The model can be applied to quantify the value of adaptive routing in response to network disruption over deterministic solutions and to consider the effects of disruption on shipper behavior. Numerical analysis was conducted on a real-world truckload network. Wherever actual data, such as actual travel times, were not available because of privacy concerns, realistic assumptions were made. The numerical analysis shows that adaptive capacitated routing can lead to significant savings over nonadaptive deterministic routing behavior. During high-severity disruptions, FDMs can benefit significantly by investing in technologies that enable implementation of an adaptive routing process. The model can be used by FDMs to evaluate the value of developing adaptive routing strategies that account for early information about disruption and avoid formation of freight bottlenecks.

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