Online Freight Network Assignment Model with Transportation Disruptions and Recourse

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Continuous real-time monitoring of shipment cost and delivery time is increasingly used by companies to reduce transportation costs while avoiding costly delays or disruptions. Recourse refers to the ability of the shipper to update a routing strategy on the basis of information obtained about the state of the network disruptions. An adaptive routing policy is proposed to help shippers and carriers save costs by reacting to information updates. Public transportation agencies can use the formulation to predict the behavior of shippers under disruptions in multimodal transportation networks. A mathematical model is formulated and analyzed; the model accounts for a new type of freight network assignment problem with recourse defined in a dynamic environment and in the presence of probable network disruptions or significant delays. The mathematical model is intended (*a*) to capture shipper behavior better in the presence of network disruptions and rerouting and (*b*) to include the heterogeneity in shipper routing behavior as a result of commodity or product, mode, and logistics system characteristics. Results indicate that models that do not account for the likelihood of disruption can severely misrepresent freight system performance and flows over the network, especially in situations in which freight is continuously monitored and disruptions are either significant or frequent.

The development of cost-efficient supply chain transportation networks has significantly contributed to the outsourcing of manufacturing activities to low-cost suppliers and the dispersion of economic activity (*1*). In addition, ubiquitous and inexpensive tracking and communication systems have allowed companies to heavily focus on lean practices, such as just-in-time deliveries, as well as increasingly longer transportation chains. At the same time, expectations for high levels of customer service have grown. Thus, companies are using increasingly sophisticated logistics systems to meet customer requirements without an overall increase in operating costs (*2*).

Balancing customer service and transportation costs is complex and delicate because of the high cost of disruptions or delays. Delays or disruptions in the transportation system can be caused by recurrent congestion or peak-season demand and any other nonrecurrent system perturbation due to natural or manmade causes, such as meteorological events and labor strikes. Companies can significantly reduce the negative impact of delays on customer service level by keeping redundant inventory or safety stock. Although safety stock protects companies from disruptions or delays, this comes at a cost because higher inventory levels tie up working capital and increase inventory management costs, such as warehousing and damage rates. In addition, the higher inventory level increases the risks associated with product obsolescence and perishability (*3*). Building supplier and capacity redundancies in the supply chain can also reduce the impact of disruptions; however, redundancies also increase fixed costs and reduce competitiveness (*4*).

Continuous real-time monitoring of shipment cost and delivery time is increasingly used by companies to reduce transportation costs while avoiding costly delays or disruptions, high levels of safety stocks, and system redundancies (*5*). For example, real-time monitoring and decision support systems are especially relevant to containerized multistage transportation chains for which alternative modes or carriers are available at each stage. The principal focus of this paper is the formulation and analysis of a mathematical model for a new type of freight network assignment problem with recourse defined in a dynamic environment and in the presence of probable network disruptions or significant delays. Recourse refers to the ability of the shipper to update a routing strategy based on information obtained about the state of the network disruptions. The adaptive routing policy will help the shipper save costs. The paper develops a mathematical model (*a*) to better capture shipper behavior in the presence of network disruptions and rerouting and (*b*) to include the heterogeneity in shipper routing behavior as a result of commodity or product, mode, and logistics system characteristics. The model presented in this paper is more suitable for national and regional freight transportation and may not be applicable to urban freight flows, which are dominated by multistop tours starting and ending at depots.

LITERATURE REVIEW

Three strands of related research are reviewed: freight value of time, freight network models, and online routing problems with recourse.

Although the evaluation of shippers' value of time has not received as much attention as the evaluation of passengers' value of time, the literature has clearly confirmed that freight value of time significantly changes in the presence of delays or disruptions. The quantification of freight value of time and reliability has been successfully undertaken with econometric methods. It is widely accepted that shippers and carriers have a higher valuation of time and reliability in congested

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situations or during disruptions. For example, one report recommends that under congested conditions, the value of travel time should be increased 2.5 times (*6*). A study by Cohen and Southworth indicates that the value of time of freight trucks under congested conditions can be between two and six times higher than the value of travel time under normal operating conditions (*7*). The impact of commodity or product type on the hourly cost of delay in truck transportation can also be significant (*8*). The containerized transportation chains survey results have shown that logistics managers identify freight rates, transit time, and reliability as the key transportation performance indicators (*9*). Stated-choice surveys of logistics managers also indicate that, on average, shippers' willingness to pay for travel time reductions have a fivefold increase when a transportation disruption takes place. De Jong (*10*) and Massiani (*11*) discuss theoretical and practical issues related to the evaluation of travel time savings in freight transportation. Freight value of time and reliability under congested conditions are explicitly incorporated in the proposed model, as the shipper or carrier updates his or her knowledge of the state of the freight network.

Freight network models have steadily evolved in the past three decades. Several proposed models have been based on the spatial price equilibrium concept using mathematical programming and freight network assignment-based formulations (*12–16*). Most of the mentioned works use deterministic models and do not focus on the effect of parameter uncertainty on freight decision making. Friesz et al. (*17*) and Unnikrishnan et al. (*18*) have developed models that capture the impact of demand uncertainty on shipper pricing and carrier network design decisions. However, the authors are not aware of any work focused on modeling the effect of network disruptions and shipper routing decisions and resulting commodity or product flows.

Under stochastic network conditions, the cost realizations may be revealed to the shipper when his commodities traverse the network. For example, once a shipper's commodity or product reaches a node, the shipper may learn the cost values associated with all arcs emanating from that node. Using the observed values, the shipper can update his knowledge and make inferences about the rest of the network and then choose a routing policy for his goods. The routing policy will determine the routes on which the shipper will send his goods for different disruption scenarios. Similarly, a carrier can update routes as a vehicle arrives at a terminal or network node. Such problems are called online routing problems or routing problems with recourse. The model and discussion presented in this research apply to the freight decision maker (FDM) agent (shippers, carriers, or third-party logistic providers) that can alter routing policies for shipments or vehicles as a function of information updates regarding the state of the network.

Several works in the network optimization area focusing on online routing and assignment problems under stochastic conditions are relevant to the research presented in this paper. Croucher (*19*) and Andreatta and Romeo (*20*) studied the online shortest path problem on a network with stochastic topologies where the existence of arcs was random and described by a probability distribution. Polychronopoulos and Tsitsiklis provided an exact algorithm and several heuristics for a more general variation of the problem with stochastic arc costs described by using discrete probability distributions (*21*). Cheung (*22*), Provan (*23*), and Waller and Ziliaskopoulos (*24*) have developed efficient algorithms for time-invariant versions of the online shortest path problems with different assumptions on the characterization of cost uncertainties. Several other researchers have studied variations of the online routing problem in time-varying networks where the information learned by the user and thus the decision taken depend on the time of arrival at a node (*25–28*).

Unnikrishnan and Waller formulated the traffic assignment problem with recourse as a convex mathematical program (*29*) and solved it by using the algorithms developed by Polychronopoulos and Tsitsiklis (*21*) as a subproblem in a successive linearization framework. The authors are not aware of any work capturing the impact of FDM recourse decisions in the event of network disruptions and determining the resulting network flows.

PROBLEM DESCRIPTION

Consider a network $G = (N, A)$, where N denotes the set of all nodes in the network and A represents the set of all arcs in the network. A finite set of FDMs wants to transport goods from a set of origins R to a set of destinations S. Every FDM is assumed to transport K commodity or product types. For easing up on the notation, a distinct FDM is assumed to be associated with every origin–destination pair. This assumption is not restrictive, and the methodology presented in this paper is applicable even if this assumption is relaxed. Let D denote the origin–destination matrix. Every element of D is represented as d^k_{rs}, which represents the amount of good of type $k \in K$ to be transported from origin $r \in R$ and destination $s \in S$. The arcs are assumed to be under the control of carriers; different arcs can correspond to different modes used by carriers. It is assumed that on every arc, a carrier or mode has enough capacity to handle all of FDM's goods either on its own or by subcontracting to other carriers.

The cost experienced by an FDM in transporting a commodity or product on an arc (which corresponds to a specific carrier) is assumed to be a linear function of three attributes—unit transportation cost of transporting a commodity or product, unit travel time to traverse the arc for commodity or product, and the unreliability parameter. The unreliability parameter is based on the FDM's experience in transporting goods by using a particular carrier or mode. The reliability parameter is assumed to capture all attributes other than travel time and travel cost that affect FDM routing decisions.

The system is assumed to exist in multiple states or realizations Ω corresponding to various network disruption scenarios. A discrete probability distribution P is used to describe the system states. Each system state corresponds to a vector of arcs attributes. For any system realization $\omega \in \Omega$, let $c^{k\omega}_a$ and $t^{k\omega}_a$ denote the unit cost and unit travel time for transporting commodity or product $k \in K$ on arc $a \in A$. In the numerical example and without loss of generality, the reliability parameter u^k_a is assumed to be a property of the arc and to not change with the system states.

Information Updates

An FDM progressively gets information about the system state by observing or learning the state of the arcs when its goods traverse the arcs. In this paper, it is assumed that whenever the goods reach a node $i \in N$, the FDM learns the states of all outgoing arcs from that node. Consider an FDM that wants to transport goods between origin–destination pairs ($r \in R$, $s \in S$). The FDM observes the arcs emanating from node r. The FDM learns that the travel time and travel cost of an arc a' emanating from node r to be $t^k_{a'}$ and $c^k_{a'}$, respectively. Now based on this information, the FDM can eliminate all scenarios from Ω where the travel time and travel cost on arc a' are not $t^k_{a'}$ and $c^k_{a'}$. The FDM will then update its probability, travel cost, and travel time matrices. Given the updated knowledge about the possible system states, assume that the carrier sends all of its commodities on arc a'. Once the commodities reach the tail node of a' (assume it to be node $i' \in N$), the FDM will observe the states of all arcs emanating from node i' and update the probability, travel cost, and travel time matrices.

At every node, the FDMs routing strategy, also known as routing policy, involves choosing different arcs to transport goods depending on the state of the outgoing arcs. For example, consider a node $i \in N$ and let Γi represent the set of all outgoing arcs and $\Psi(\Gamma i)$ represent the states of all outgoing arcs. For every possible state $\psi \in \Psi(\Gamma i)$, the FDM's routing policy provides the next arc on which the FDM will route its goods. A routing policy thus represents the set of paths on which an FDM can possibly route the goods based on the information received while traversing the network. There is a probability associated with each path of the routing policy, which depends on the probability distribution of the system states. Therefore, an expected cost is associated with every routing policy.

Generalized Costs

Because the arcs used by the FDM depend on the system states, the flow of a commodity or product on an arc also depends on the system states. Let $x^{k\omega}_{ars}$ represent the flow of commodity or product $k \in K$ between origin $r \in R$ and destination $s \in S$ on arc $a \in A$ in scenario $\omega \in \Omega$. For any carrier corresponding to a unique origin–destination pair ($r \in R$, $s \in S$), the cost of transporting commodity or product $k \in K$ on $a \in A$ in scenario $\omega \in \Omega$, $C^{k\omega}_{ars}$ is assumed to be a weighted linear combination of the transportation costs $c^{k\omega}_a$, travel time $t^{k\omega}_a$, and reliability parameter u^k_a. The transportation costs $c^{k\omega}_a$ denotes the unit cost for transporting commodity or product $k \in K$ and incorporates besides freight rates any FDM or product-related characteristics such as volume of sales or use of a just-in-time system. The term $t^{k\omega}_a$ denotes the unit cost and unit travel for realization $\omega \in \Omega$; that is, for a delay or disruption of a given magnitude there is an associated cost increase that reflects the new valuation of the FDM's time. The reliability parameter u^k_a is assumed to be a property of the carrier or mode that is obtained from historical data or past performance. The weights used to capture FDMs' heterogeneity for the trade-offs among travel costs, travel time, and unreliability for various products can be obtained from surveys or discrete choice freight models (*10, 11, 30*).

$$C^{k\omega}_{ars}\left(x^{k\omega}_{ars}\right) = \left(w^k_{trs}t^{k\omega}_a + w^k_{trs}c^{k\omega}_a + w^k_{urs}u^k_a\right)x^{k\omega}_{ars} \qquad \forall a \in A,\ k \in K,\ r \in R,\ s \in S,\ \omega \in \Omega$$

where w^k_{trs}, w^k_{trs}, and w^k_{urs} represent the importance or weight associated with travel time, travel cost, and reliability for the FDM corresponding to the origin–destination pair ($r \in R$, $s \in S$) for commodity or product $k \in K$. In this way it is possible to map product, supply chain, and company characteristics that have been found to significantly affect FDMs' preferences and valuations of freight transportation alternatives.

FORMULATION

The flow of commodity or product $k \in K$ between origin–destination pair $r \in R$, $s \in S$ on arc $a \in A$ under system realization $\omega \in \Omega$, $x^{k\omega}_{ars}$ is equal to the sum of flow on all routing policies connecting origin–destination pair $r \in R$, $s \in S$ and using arc $a \in A$ under system realization $\omega \in \Omega$. Let H_{rs} denote the set of routing policies connecting origin–destination pair $r \in R$, $s \in S$ and let f^k_h represent the flow of commodity or product $k \in K$ on a routing policy $h \in H_{rs}$. Let δ^ω_{ah} be an incidence matrix such that

$$\delta^\omega_{ah} = \begin{cases} 1 & \text{if policy } h \text{ uses arc } a \text{ in system state } \omega \\ 0 & \text{otherwise} \end{cases}$$

$$x^{k\omega}_{ars} = \sum_{h \in H_{rs}} \delta^\omega_{ah} f^k_h \qquad \forall a \in A,\ k \in K,\ r \in R,\ s \in S,\ \omega \in \Omega \tag{1}$$

Given $x^{k\omega}_{ars}$, the cost of transporting commodity or product $k \in K$ on $a \in A$ in scenario $\omega \in \Omega$ can be written as

$$C^{k\omega}_{ars}\left(x^{k\omega}_{ars}\right) = \left(w^k_{trs}t^{k\omega}_a + w^k_{crs}c^{k\omega}_a + w^k_{urs}u^k_a\right)x^{k\omega}_{ars} \qquad \forall a \in A,\ k \in K,\ r \in R,\ s \in S,\ \omega \in \Omega \tag{2}$$

The contribution of link a to the expected cost of routing policy $h \in H$, C^h_a can be calculated as

$$C^h_a = \sum_{\omega \in \Omega} \delta^\omega_{ah} P^\omega C^{k\omega}_{ars}\left(x^{k\omega}_{ars}\right) \qquad \forall a \in A,\ h \in H_{rs} \tag{3}$$

Thus the expected cost of a routing policy $h \in H$, $E[C(h)]$ can be calculated as

$$E\left[C\left(h\right)\right] = \sum_{a \in A}\sum_{\omega \in \Omega} \delta^\omega_{ah} P^\omega C^{k\omega}_{ars}\left(x^{k\omega}_{ars}\right) \tag{4}$$

Since the unit costs are assumed not to vary with flow, the system objective coincides with every FDM's objective. Therefore, minimizing expected total cost will correspond to minimizing every FDM's individual costs for every commodity or product. Thus, the mathematical programming formulation can be written as

$$\text{minimize } Z = \sum_{a \in A}\sum_{k \in K}\sum_{r \in R, s \in S}\sum_{\omega \in \Omega}\left(w^k_{trs}t^{k\omega}_a + w^k_{crs}c^{k\omega}_a + w^k_{urs}u^k_a\right)x^{k\omega}_{ars}p^\omega \tag{5}$$

subject to

$$x^{k\omega}_{ars} = \sum_{h \in H_{rs}} \delta^\omega_{ah} f^k_h \qquad \forall a \in A,\ k \in K,\ r \in R,\ s \in S,\ \omega \in \Omega$$

$$\sum_{h \in H_{rs}} f^k_h = d^k_{rs} \qquad \forall k \in K,\ r \in R,\ s \in S \tag{6}$$

$$x^{k\omega}_{ars} \geq 0 \tag{7}$$

Equation 6 constrains the sum of flow on all routing policies for commodity or product $k \in K$ connecting origin–destination pair *r-s* to be equal to the demand for commodity k between that specific origin–destination pair.

SOLUTION ALGORITHM

The preceding formulation can be solved by assigning flows to the shortest expected cost routing policy for every commodity or product and for every origin–destination pair. The shortest expected cost policy can be calculated with the R-SSPR algorithm developed by Polychronopoulos and Tsitsiklis for the cost structure as defined in this paper (*21*). The procedure is as follows:

Step 1. Cost calculation. Calculate the unit cost for every arc for every possible system realization as

$$C_{\text{ars}}^{k\omega} = \left(w_{\text{urs}}^{k} t_{a}^{k\omega} + w_{\text{crs}}^{k} c_{a}^{k\omega} + w_{\text{urs}}^{k} u_{a}^{k} \right) \quad \forall a \in A,\ k \in K,\ r \in R,\ s \in S,\ \omega \in \Omega$$

Step 2. Determining feasible information sets. As the goods traverse the network, the FDM learns the set of possible system realizations by eliminating the realizations that are not consistent with the observed costs and travel time. The set of possible system realizations is defined as the information set. If the cardinality of the set of system states is $|\Omega|$, then there are $2^{|\Omega|} - 1$ information sets. For example, if $\Omega = \{1, 2, 3\}$, then the set of feasible information sets $\Xi = \{1, 2, 3, 12, 23, 13, 123\}$. Let Ξ^m represent set of subsets of Ξ with cardinality *m*. For example, $\Xi^1 = \{1, 2, 3\}$, $\Xi^2 = \{12, 23, 13\}$, and $\Xi^3 = \{123\}$.

Step 3. Calculate minimum expected cost routing policy using the R-SSPR (*21*). The extra notation used in the expected cost routing policy algorithm is as follows:

Notation

- ξ = information set corresponding to carrier's knowledge about system conditions;
- ψ = node state observed by the carrier when a node is reached;
- $V[i/\xi, \psi]$ = expected cost of routing policy for transporting goods from node *i* to destination *s* given that the carrier has knowledge of information set ξ when the goods arrive at node *i* and learns the node state ψ after reaching node *i*;
- pathptr$[i/\xi, \psi]$ = next node to be chosen by the same carrier;
- $E[i/\xi]$ = expected cost of routing policy for transporting goods from node *i* to destination *s* given that the carrier has knowledge of information set ξ when the goods arrive at node *i*;
- SEL = scan eligible list;
- temp$V[i/\xi, \psi]$ = variable defined to store temporary values of $V[i/\xi, \psi]$;
- temp$E[i/\xi]$ = variable defined to store temporary values of $E[i/\xi]$;
- $p[\psi/\xi]$ = probability of a carrier observing state ψ given the information set ξ; and
- $C_{(v,w)rs}^{k\omega(\psi)}$ = cost of transporting commodity/product $k \in K$ on $(v, w) \in A$ in system state $\omega(\psi) \in \Omega$ corresponding to node states $\psi \in \Gamma(v)$.

The basic premise behind the algorithm is that when the goods arrive at a node $v \in N$, the FDM's knowledge of the system will be any element of the information set $\xi \in \Xi$. Given the carrier's current knowledge of the system states, he then observes $\psi \in \Psi(\Gamma v)$ the states of all outgoing arcs from node *v*. Given the state he observes, ψ, he then updates his information set ξ to ξ'. Information set ξ' will be a subset of ξ as the FDM will learn new information about the state of the system, eliminate states that are not possible, or in the worst case, not learn anything and retain the information set ξ.

For each node $w \in W$ that can be reached from node *v*, the temporary label for the expected cost, temp$V[v/\xi, \psi]$, can be calculated as the cost of arc (v, w) under the observed system state plus the cost of traveling from node *w* to the destination given that the information set the carrier has while arriving at *w* is ξ'. If temp$V[v/\xi, \psi]$ is less than the current label $V[v/\xi, \psi]$, then the current label is updated to be equal to the temporary label. Once the labels $V[v/\xi, \psi]$ are calculated for each possible node state $\Psi \in \Psi(\Gamma v)$, one can calculate the temporary label for the expected cost to travel from node *v* to the destination given the information set ξ, temp$E[v/\xi]$, by multiplying temp$V[v/\xi, \psi]$ with the respective conditional probabilities of observing node state ψ given the information set ξ and summing up across all possible node states. If temp$E[v/\xi]$ is less than the current label $V[v/\xi]$, the current label is updated to be equal to the temporary label. Then node *v* is added to the scan eligible list.

An important property exploited by the algorithm is that to calculate the expected cost of routing policy from every node to the destination for information sets of cardinality *k*, Ξ^k, one needs the expected cost of routing policy from every node to the destination for information sets of cardinality less than *k*. Thus the optimal routing costs are calculated for information sets in increasing order of cardinality. That is, first calculate the costs to travel from each node to the destination assuming that information sets of cardinality 1 is possible (which can be trivially calculated with Djikstra's algorithm). Then calculate expected cost for routing policies for information sets of cardinality 2 at each node to the destination. The process can be repeated until the information set of cardinality $|\Omega|$ is reached.

The current example assumes that the FDM gets information about all outgoing arcs. The preceding algorithm can be easily modified to consider the case in which the FDM at each node gets information about a set of arcs through intelligent transportation system devices by modifying the way the current information set is updated to get a new information set. The pseudocode of the RSSPR algorithm follows. Polychronopoulos and Tsitsiklis provide details on efficient heuristics to speed up the process (*21*).

For $r \in R,\ s \in S,\ k \in K$

  For $n = 1$ to m

  For $\xi \in \Xi^n$

    $E[s/\xi, 1] = 0$: pathptr$[s/\xi, 1] = 0$

    For $i \in N - s, \psi \in \Psi(\Gamma i)$

      Set $E[i/\zeta, \psi] = \infty$: pathptr$[i/\zeta, \psi]$ = undefined

    SEL := *s*

    While SEL :$\neq \phi$

      Remove node *u* from SEL with minimum $E[u/\zeta]$

      For each $v \in N$ such that $(v, u) \in A$

        For each $\psi \in \Psi(\Gamma v)$

          Update ξ based on ψ to obtain ξ'

          For each $w \in N$ such that $(v, w) \in A$

            temp$V[v/\xi, \psi] = C_{(v,w)rs}^{k\omega(\psi)} + V[w/\xi', \psi]$

            If temp$V[v/\xi, \psi] < V[v/\xi, \psi]$

              $V[v/\xi, \psi]$ = temp$V[v/\xi, \psi]$; pathptr$[v/\xi, \psi] = w$

          End of loop: For each $w \in N$ such that $(v, w) \in A$

        End of loop: For each $\psi \in \Psi(\Gamma v)$

$$\text{temp}E[v/\xi] = \sum_{\psi \in \Psi(\Gamma v)} p[\psi/\xi]\, \text{temp}V[v/\xi, \psi]$$

If $\text{temp}E[v/\xi] < E[v/\xi]$,

$E[v/\xi] = \text{temp}E[v/\xi]$

$\text{SEL} := \text{SEL} \cup \{v\}$

End of loop: For each $v \in N$ such that $(v, u) \in A$

End of loop: While $\text{SEL} :\neq \phi$

End of loop: For $\xi \in \Xi^n$

End of loop: For $n = 1$ to m

End of loop: For $r \in R,\ s \in S,\ k \in K$

Step 4. Flow assignment. For every origin–destination pair and every commodity or product, use the algorithm presented in Step 3 to determine the shortest expected cost routing. Set the flow on the shortest routing policy for every origin–destination pair for every commodity or product equal to the demand for that commodity or product between the origin–destination pair. From the shortest expected cost policy, determine the link policy incidence matrix for every scenario. Use Equations 1, 2, and 4 to determine the flow and cost of commodities for every arc and the resulting expected cost of routing policy for every commodity or product for every origin–destination pair.

CASE STUDY

This section demonstrates the advantage of the adaptive strategy with recourse against two deterministic planning models on a realistic freight network. The authors obtained data from a freight network involving shipments from a major distribution center to national-level food chain stores in the Portland, Oregon, area. For simplicity, some details are omitted; however, the network attempts to mimic key aspects of actual operating conditions. In the network, 32 nodes represent distribution centers in Idaho, Washington, Nevada, and California (Figure 1); the 32 nodes have 20 origins and one destination (Portland). Three types of products or commodities are being shipped: dry, frozen, and refrigerated. The demand data provided in Table 1 represent truckload shipments. Travel costs for fuel and operating expenses are \$1.45, \$1.65, and \$1.75 per mile per truck for the dry, refrigerated, and frozen products, respectively. For simplicity, in the example the costs on the arcs do not vary with origin–destination pairs.

The network has three types of arcs: arcs lying on normal shipping routes, arcs lying on alternate shipping routes, and supplementary arcs that are not generally used. The travel times are assumed to be similar to the ones provided by Google Maps. Supplementary routes are created by introducing a 25% time penalty and a 100% cost penalty on the shortest cost routes during normal conditions. These additional cost represent the cost of business during disruptions and when using different carriers, modes, or routes. However, under disruption of the normal or alternate shipping arcs or routes, these supplementary arcs and routes may become cheaper. The network has a total of 55 arcs, and it is assumed that every arc has a base reliability of 10.

Scenario 1 is assumed to be the base case (normal operating conditions). Network disruptions are generated stochastically, and travel costs, travel times, and reliability values are scaled up correspondingly. Both moderate and extreme disruption levels are tested. For example, under a extreme disruption for any arc $a \in A$,

FIGURE 1 Distribution center locations [red, green, and blue denote origin, destination (Portland), and additional nodes, respectively].

$$c_a^{k\omega} = c_a^{k1}\left(1 + |\psi|\text{SF}\epsilon\right) \tag{8}$$

where

ϵ = uniform random number between 0 and 1,
SF = scaling factor within that severity level, and
$|\psi|$ = factor used to scale up the impact of disruption based on the cardinality of the node states of the upstream arcs. Under moderate disruption $|\psi| = 1$ is assumed.

Different scaling factors, SF, are tested for both types of disruptions. Similarly, the travel times and reliability parameters were scaled up. The value of the weighting parameters used to calculate generalized cost (Equation 2) assumed for this study are

- 0.0957, 0.0029, and –0.1219 for travel time, costs, and reliability parameters, respectively, under normal operating conditions and
- 0.4579, 0.0029, and –0.3657 for travel time, costs, and reliability parameters, respectively, under disruptions.

Hence, the travel time weight increases five times, the reliability weight increases three times, and the cost parameter remains

TABLE 1 Truck Load Demands to Portland

Origin	Commodity or Product	Weekly Truck Load
Burlington, Ore.	Refrigerated	1
Spokane, Wash.	Dry	1
Vancouver, Wash.	Dry	1
Tacoma, Wash.	Dry	1
Idaho Falls, Idaho	Dry	1
Aberdeen, Idaho	Frozen	1
Burley, Idaho	Frozen	2
Boise, Idaho	Refrigerated	1
Fruitland, Idaho	Frozen	1
Las Vegas, Nev.	Refrigerated	1
Reno, Nev.	Dry	1
Long Beach, Calif.	Frozen	1
Riverside, Calif.	Dry	1
	Frozen	1
	Refrigerated	1
Ontario, Calif.	Dry	2
	Frozen	2
	Refrigerated	2
Los Angeles, Calif.	Dry	4
	Frozen	9
	Refrigerated	2
Fresno, Calif.	Dry	4
	Refrigerated	2
Salinas, Calif.	Refrigerated	1
Stockton, Calif.	Dry	9
Oakland, Calif.	Dry	1
San Francisco, Calif.	Frozen	1

unchanged under a disruption scenario. Higher reliability is desirable, hence the negative sign associated with this parameter and the positive sign associated with time or cost parameters. These changes in the weight parameters under disruptions are consistent with findings obtained in the literature and stated preference carrier or shipper surveys (*9*).

The performance metric used was the percent gain obtained by the adaptive strategy over the nonadaptive strategy:

$$G = \left(\frac{Z_{\text{nonadaptive}} - Z_{\text{adaptive}}}{Z_{\text{nonadaptive}}} \right) 100$$

where $Z_{\text{nonadaptive}}$ is the system cost when the FDM routes the goods on the normal routes even under disruptions. Two cases are possible:

1. In the base case, the FDM uses the costs under nondisruption scenarios to route the goods; the FDM does not recognize that there may be changes in the state of the network, that is, as if the FDM assumes $|\psi| = 0$. The gain in this case is denoted G_{base}.
2. In the expected cost case, the FDM uses the expected costs considering the base case and the disruptions. The gain in this case is denoted G_{EV}.

In the stochastic optimization literature (*31*)

$$Z_{\text{nonadaptive}} - Z_{\text{adaptive}} = \text{value of stochastic solution}$$

This is the efficiency obtained by accounting for uncertainty in the model over the deterministic scenario. The routing costs under both nonadaptive scenarios are deterministic, and no rerouting is involved. All the results presented are average gains obtained from 30 runs generated with random seeds.

$$\bar{G} = \sum_{i=1}^{30} \frac{G^{\omega^i}}{30}$$

ANALYSIS OF CASE STUDY RESULTS

Three experiments are run to study the effect of (*a*) disruption severity levels, (*b*) number of uncertain scenarios, (*c*) distribution of scenario probabilities, and (*d*) flows on supplementary arcs.

Disruption Severity Levels

In the disruption severity levels experiment, the cost parameters under high disruption are scaled up by the ordinality of the node state, the scaling factor, and a random number between 0 and 1; the cost parameters under low disruption are scaled up only by a random number between 0 and 1, and the scaling factor was used to scale up the cost factors. Four scenarios are assumed in the network, that is, $|\Omega| = 4$ (each arc could take up to four values). In the high-disruption scenario, the number of scenarios and the number of outgoing arcs affect the ordinality of node states $|\psi|$ in Expression 8. The number of scenarios also affects the cardinality of the information sets (15 in this case).

In the equally likely case, all scenarios were assumed to have equal probability; in the asymmetric case, Scenario 1 (normal conditions) has the highest probability of $\Omega_1 = 0.7$, and the other three scenarios were assumed to have a probability of $\Omega_2 = \Omega_3 = \Omega_4 = 0.1$.

It is worth observing that the value of updated information is higher under large disruptions (Table 2). The effect of the initial cost differences to transport the three products is relatively minor compared with the changes caused by the disruption scale. Planning for base case normal operating conditions yielded significantly worse solutions compared with planning for the expected value case. When planning was done with the expected values, the gains appeared to stabilize with an increase in scaling factors. The savings of the adaptive strategy over the expected value case were higher in the asymmetric case compared with the equally likely case. Hence the value of information and updates is higher when the disruption scenarios are not equally likely.

Number of Uncertain Scenarios

The second set of experiments studied the effect of number of scenarios on the average gain (Table 3). The severity scaling factor was set to be equal to 1. In this experiment, a changing number of scenarios affects the cardinality of the information set and the value of $|\psi|$. The solution or running time increases with the number of scenarios, and thus the gains are obtained at higher running times.

For almost all the cases, the gains over expected value strategy were found to increase when the number of scenarios increased to five and then to stabilize or decrease. In the high-disruption case, the gains over the base case routing strategy were found to increase with an increase in number of scenarios. In the low-disruption case, the

TABLE 2 Percentage Gain

SF	Base Dry	Base Frozen	Base Ref.	EV Dry	EV Frozen	EV Ref.
Equally Likely—Low Disruption						
0.5	0.30	0.53	0.21	0.24	0.31	0.16
1	1.14	1.41	0.69	0.78	0.86	0.45
1.5	3.25	3.90	2.64	2.30	2.75	2.12
2	9.75	10.83	9.09	5.05	5.44	4.88
2.5	18.94	20.27	18.08	5.69	6.00	5.67
3	29.63	31.15	28.60	5.47	5.75	5.54
Asymmetric—Low Disruption						
0.5	0.23	0.40	0.16	0.22	0.28	0.15
1	0.89	1.10	0.54	0.74	0.77	0.43
1.5	2.62	3.12	2.12	2.22	2.55	1.87
2	7.95	8.79	7.39	7.08	7.73	6.94
2.5	15.52	16.53	14.79	12.65	13.15	12.44
3	24.34	25.48	23.46	16.37	16.59	16.05
Equally Likely—High Disruption						
0.5	4.32	5.15	3.86	3.33	3.86	3.31
1	29.02	31.00	28.53	5.7	6.22	6.06
1.5	62.92	65.73	62.45	4.49	5.03	5.04
2	98.61	101.87	97.79	4.1	4.48	4.51
2.5	134.60	138.26	133.41	3.97	4.18	4.26
3	170.62	174.71	169.09	3.84	4	4.11
Asymmetric—High Disruption						
0.5	3.47	4.11	3.09	3.05	3.5	2.83
1	23.80	25.32	23.36	15.8	16.48	16.05
1.5	51.80	53.89	51.33	19.21	19.57	19.69
2	81.26	83.62	80.47	19.5	19.92	19.64
2.5	110.97	113.57	109.86	19.48	19.48	19.8
3	140.71	143.58	139.30	19.43	19.31	19.66

NOTE: SF = scaling factor; EV = estimated value; ref. = refrigerated.

gains over the base case strategy were found to stabilize and reduce when the number of scenarios increased from five to six. Thus the gains obtained need not always increase with the increase in the number of uncertain states, which validates the need for such a model to evaluate the actual gain. The gain is also highly dependent on the type of product in some situations, such as under low disruption. However, high disruptions overshadow any influence that product type may have.

Distribution of Scenario Probabilities

The third set of experiments studies the effect of various values of probabilities on the gains (Table 4). There are four scenarios, and the scaling factor is set to be equal to 1. Scenario 1, which is the normal base operating conditions, is assumed to have a probability of p_1 varying in increments of 0.05 from 0.25 to 0.7. The other three scenarios are assumed to be equally likely and have a probability distribution of

$$\frac{(1-p_1)}{3}$$

TABLE 3 Number of Scenarios

Number of Scenarios	Base Dry	Base Frozen	Base Ref.	EV Dry	EV Frozen	EV Ref.
High Disruption						
2	7.03	8.78	6.80	2.83	3.44	3.34
3	18.92	20.14	18.14	7.77	8.30	8.35
4	29.02	31.00	28.53	5.70	6.22	6.06
5	39.30	41.55	39.09	7.44	7.34	7.67
6	52.78	55.54	53.30	5.35	5.49	5.70
Low Disruption						
2	0.85	1.08	0.48	0.18	0.13	0.08
3	1.15	1.43	0.79	0.57	0.77	0.38
4	1.14	1.41	0.69	0.78	0.86	0.45
5	1.16	1.40	0.67	0.91	0.97	0.51
6	1.01	1.30	0.59	0.87	0.96	0.51

As the probability of Scenario 1 increases, the gain obtained over the base case strategy decreases. In the low-disruption case, as the probability values vary, the gain obtained over expected value strategy remains constant and stable. However, in the high-disruption strategy, the gain over the expected value strategy was found to increase with the increase in probability of Scenario 1. This is an interesting result because it states that as the probability increases for the conditions

TABLE 4 Probabilities of Scenario 1

Probability of Scenario 1	Base Dry	Base Frozen	Base Ref.	EV Dry	EV Frozen	EV Ref.
High Disruption						
0.25	29.02	31.00	28.53	5.70	6.22	6.06
0.3	28.98	30.96	28.52	6.40	6.93	6.76
0.35	28.44	30.37	27.96	7.22	7.75	7.58
0.4	28.00	29.89	27.52	8.13	8.66	8.48
0.45	27.59	29.44	27.11	9.15	9.68	9.50
0.5	27.11	28.92	26.65	10.28	10.79	10.59
0.55	26.44	28.19	25.98	11.54	12.03	11.79
0.6	25.77	27.46	25.31	12.96	13.42	13.16
0.65	24.96	26.57	24.51	14.43	14.95	14.63
0.7	23.80	25.32	23.36	15.89	16.48	16.05
Low Disruption						
0.25	1.14	1.41	0.69	0.78	0.86	0.45
0.3	1.13	1.39	0.68	0.78	0.86	0.45
0.35	1.11	1.37	0.67	0.78	0.85	0.45
0.4	1.09	1.35	0.66	0.78	0.85	0.45
0.45	1.07	1.32	0.65	0.78	0.84	0.45
0.5	1.04	1.29	0.63	0.78	0.83	0.45
0.55	1.02	1.26	0.62	0.78	0.82	0.45
0.6	0.98	1.21	0.59	0.77	0.81	0.44
0.65	0.94	1.16	0.57	0.76	0.79	0.44
0.7	0.89	1.10	0.54	0.74	0.77	0.43

being normal, the performance of the expected value strategy in relation to the adaptive strategy may decrease.

Again, the gain is highly dependent on the type of product in some situations, for example, under low disruption. However, high disruptions overshadow any influence that product type may have.

Flows on Supplementary Arcs

A final set of experiments aims at comparing the percentage of flows on supplementary arcs (corresponding to other modes or other carriers that the FDM will not use during normal operating conditions) to percentage of flows on normal shipping routes. The performance metric used was the ratio of flows on supplementary arcs and normal shipping routes expressed as a percentage. With A_n and A_s the set of arcs used under normal operating conditions and the set of supplementary arcs, respectively, where $A_n \cap A_s = \{\}$, $A_n \cup A_s = A$, then for adaptive routing, the flow ratio for commodity k is calculated as follows:

$$\text{FR}^k = \frac{\sum_{r \in R, s \in S} \sum_{a \in A_s} \sum_{\omega \in \Omega} p^{\omega} x_{\text{ars}}^{k\omega}}{\sum_{r \in R, s \in S} \sum_{a \in A_n} \sum_{\omega \in \Omega} p^{\omega} x_{\text{ars}}^{k\omega}}$$

The ratio was calculated for both the high- and low-disruption scenarios for each commodity and for total flows for six scaling factors ranging from 0.5 to 3.0. The ratio was calculated under online adaptive routing and when routing was conducted with the expected cost scenario. The ratio of flow on supplementary arcs to normal shipping route arcs increases with an increase in the magnitude of disruptions (Figure 2). The ratio of flow on supplementary arcs to flow normal arcs is higher in the high-disruption scenario. When the FDM makes routing decisions by using the expected cost scenario, the flow ratio is nearly half the ratio under adaptive routing conditions. Considering the significant cost benefit achieved by the FDM, the adaptive routing strategy leads to more efficient use of other modes or other carriers that the FDM will not use during normal operating conditions (Figure 2).

The use of additional arcs also has implications for transportation agencies. As more supplementary routes are used, the impact of disruption can be distributed more uniformly across the network.

Some of the trends obtained cannot be generalized to all networks. The actual gain obtained by the adaptive strategy may depend on the nature of disruption, network topology (number of rerouting options available), cost parameters on the alternate options, and so forth. However, this section demonstrates the usefulness of the adaptive model in a realistic freight network. Various FDMs (carriers, shippers, and freight operators) can apply this model and estimate the gain obtained by using an adaptive strategy. Depending on the estimates of the gain, they can then decide if it will be worthwhile to invest in communication and sensor technologies that provide up-to-date real-time information on the conditions. In addition, transportation agencies may invest in information systems to provide real-time information to carriers, because this may create a substantial economic benefit by reducing logistics costs and spread out utilization of the network when disruptions do take place.

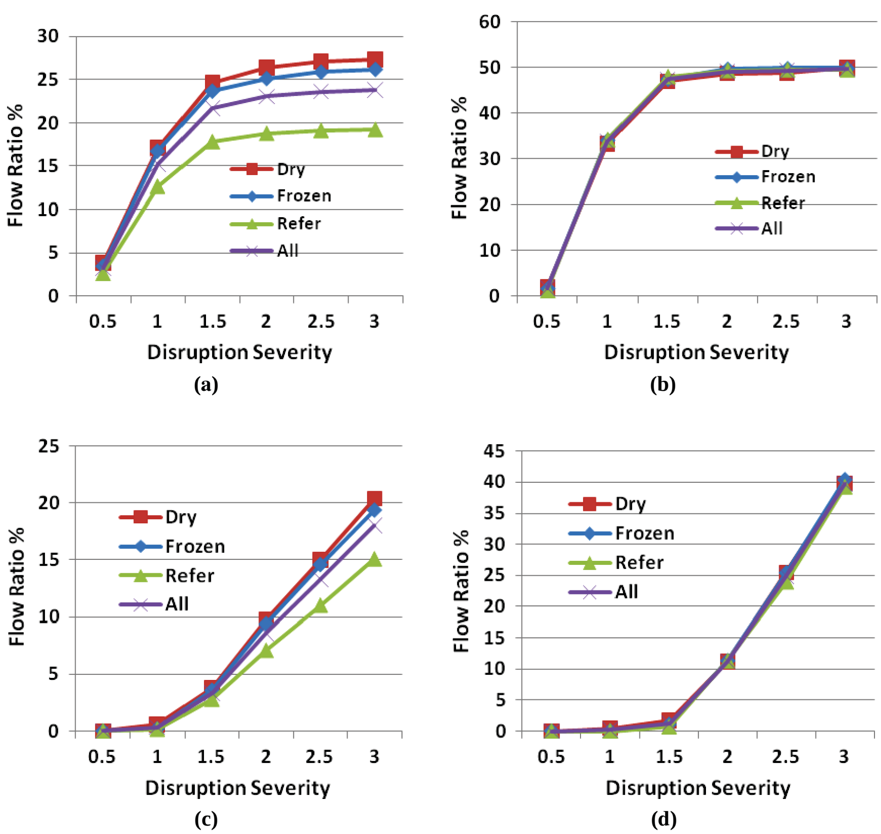

FIGURE 2 Ratio of arc flows in artificial arcs to normal arcs: (*a*) high disruption and expected scenario routing, (*b*) high disruption and online routing, (*c*) low disruption and expected scenario routing, and (*d*) low disruption and online routing.

CONCLUSIONS

This research provides an online freight network assignment model with transportation disruptions and recourse. The model can better reflect the use of real-time tracking technology. Public agencies can use this model to predict the behavior of FDMs (carriers or shippers) when significant disruptions take place. The presented methodology can be used to estimate network flows in situations in which disruptions are either significant or frequent. Given the large difference between freight agents' preferences under normal and disruption conditions, models that do not account for the likelihood of disruption can severely misrepresent freight system performance and flows over the network. Private companies can also apply the proposed model. Freight agents and decision makers can significantly benefit from adopting an adaptive strategy as the severity of the disruption increases.

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