An On-line Freight Network Assignment Model with Transportation Disruptions and Recourse

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ABSTRACT

Continuous real-time monitoring of shipment cost and delivery time is increasingly used by companies to reduce transportation costs while avoiding costly delays or disruptions. Recourse refers to the ability of the shipper to update his/her routing strategy depending on information obtained about the state of the network disruptions. This research proposes an adaptive routing policy that will help shippers and carriers to save costs by reacting to information updates. Public transportation agencies can also utilize this formulation to predict the behavior of shippers under disruptions in multimodal transportation networks. The principal focus of this paper is the formulation and analysis of a mathematical model that accounts for a new type of freight network assignment problem with recourse defined in a dynamic environment and in the presence of probable network disruptions or significant delays. The primary contributions of this work are to develop a mathematical model: (i) to better capture shipper behavior in the presence of network disruptions and re-routing, and (ii) to include the heterogeneity in shipper routing behavior as a result of commodity/product, mode, and logistics system characteristics. Results indicate that models that do not account for the likelihood of disruption can be severely misrepresent freight system performance and flows over the network. Specially in situations where freight is continuously monitored and disruptions are either significant or frequent.

KEYWORDS: Freight, Disruptions, Information technology, Online, Recourse, Assignment
INTRODUCTION

The development of cost efficient supply chain transportation networks has significantly contributed to the outsourcing of manufacturing activities to low cost suppliers and the dispersion of economic activity (1). In addition, ubiquitous and inexpensive tracking and communication systems have allowed companies to heavily focus on “lean” practices such as just-in-time deliveries as well as increasingly longer transportation chains. At the same time, expectations for high “customer service” levels have also increased. Thus, companies are using increasingly sophisticated logistics systems to meet customer requirements without an overall increase in operating costs (2).

Balancing customer service and transportation costs is a complex and delicate balancing act due to the high cost of disruptions or delays. Delays or disruptions in the transportation system can be caused by recurrent congestion or peak-season demand and any other non-recurrent system perturbation due to natural or manmade causes such as meteorological events or labor strikes. Companies can significantly reduce the negative impacts of delays on customer service level by keeping redundant inventory or safety stock. Although safety stock protects companies from disruptions or delays, this comes at a cost since higher inventory levels tie up working capital and increases inventory management costs such as warehousing and damage rates. In addition, the higher inventory level increases the risks associated with product obsolescence and perishability (3). Building supplier and capacity redundancies in the supply chain can also reduce the impact of disruptions, however, redundancies also increases fixed costs and reduces competitiveness (4).

Continuous real-time monitoring of shipment cost and delivery time is increasingly used by companies to reduce transportation costs while avoiding costly delays or disruptions, high levels of safety stocks and system redundancies (5). For example, real-time monitoring and decision support systems are especially relevant to containerized multi-stage transportation chains where alternative modes or carriers are available at each stage. The principal focus of this paper is the formulation and analysis of a mathematical model that for a new type of freight network assignment problem with recourse defined in a dynamic environment and in the presence of probable network disruptions or significant delays. Recourse refers to the ability of the shipper to update his routing strategy depending on information obtained about the state of the network disruptions. The adaptive routing policy will help the shipper to save costs by reacting to the information. The paper is intended to develop a mathematical model: (i) to better capture shipper behavior in the presence of network disruptions and re-routing, and (ii) to include the heterogeneity in shipper routing behavior as a result of commodity/product, mode, and logistics system characteristics. The model presented in this paper is more suitable for national and regional level freight transportation and may not be applicable...
for urban freight flows which is dominated by multi-stop tours starting and ending at depots.

This research is organized as follows: the next section provides a literature review of the relevant literature. The mathematical notation and problem description is then provided followed by the formal problem formulation and the solution algorithm. A case study derived from a real-world problem is then introduced; the case study results are analyzed and the salient results discussed followed by the conclusions.

LITERATURE REVIEW
This section reviews three strands of related research: (a) freight value of time, (b) freight network models, and (c) online routing problems with recourse.

Although the evaluation of shippers’ value of time has not received as much attention as the evaluation of passengers’ value of time, the literature has clearly confirmed that freight value of time significantly changes in the presence of delays or disruptions. The quantification of freight value of time and reliability has been successfully undertaken using econometric methods. It is widely accepted that shippers and carriers have a higher valuation of time and reliability in congested situations or during disruptions. For example, one report recommends that under congested conditions the value of travel time should be increased 2.5 times (6). Another study by Cohen and Southworth (7) indicates that the value of time of freight trucks under congested conditions can be between two and six times higher than the value of travel time under normal operating conditions. The impact of commodity/product type on the hourly cost of delay in truck transportation can also be significant (8). The containerized transportation chains survey results have shown that logistics managers indentify freight rates, transit time, and reliability as the key transportation performance indicators (9). Stated choice surveys of logistics managers also indicate that, on average, shippers’ willingness to pay for travel time reductions have a fivefold increase when a transportation disruption takes place. De Jong (10) and Massiani (11) are general references that discuss theoretical and practical issues related to the evaluation of travel time savings in freight transportation. Freight value of time and reliability under congested conditions are explicitlyincorporated in the proposed model, as the shipper or carrier updates his or her knowledge of the state of the freight network.

Freight network models have steadily evolved in the last three decades. Several models have been proposed based on the spatial price equilibrium concept using, mathematical programming and freight network assignment based formulations (12, 13, 14, 15, 16). Most of the above mentioned works use deterministic models and do not focus on the impact of parameter uncertainty on freight decision making. Recently, Friesz, Mookherjee, Holguin-Veras and Rigdon (17) and Unnikrishnan, Valsaraj and Waller (18) have developed models, which capture the impact of demand uncertainty on
shipper pricing and carrier network design decisions. However, the authors are not aware of any work focused on modeling the impact of network disruptions and shipper routing decisions and resulting commodity/product flows.

Under stochastic network conditions, the cost realizations may be revealed to the shipper when his commodities traverse the network. For example, once a shipper’s commodity/product reaches a node, the shipper may learn the cost values associated with all arcs emanating from that node. Based on the observed values, the shipper can update his knowledge and make inferences about the rest of the network and then choose a routing policy for his goods. The routing policy will determine the routes on which the shipper will send his goods for different disruption scenarios. Similarly, a carrier can update routes as a vehicle arrives to a terminal or network node. Such problems are called online routing problems (ORP) or routing problems with recourse. The model and discussion presented in this research applies to the freight decision maker (FDM) agent (shippers, carriers, or third-party logistic providers) that can alter routing policies for shipments or vehicles as a function of information updates regarding the state of the network.

Several works in the network optimization area focusing on online routing and assignment problems under stochastic conditions are relevant to the research presented in this paper. Croucher (19), Andreatta and Romeo (20) studied the online shortest path problem on a network with stochastic topologies where the existence of arcs was random and described by a probability distribution. Polychronopoulos and Tsitsiklis (21) provided an exact algorithm and several heuristics for a more general variation of the above problem with stochastic arc costs described using discrete probability distributions. Cheung (22), Provan (23), and Waller and Zilaskopoulos (24) have developed efficient algorithms for different time-invariant versions of the online shortest path problems with different assumptions on the characterization of cost uncertainties. Several other researchers have studied variations of the online routing problem in time varying networks where the information learned by the user and thus the decision taken depends on the time of arrival at a node (25, 26, 27).

Unnikrishnan and Waller (28) formulated the traffic assignment problem with recourse as a convex mathematical program and solved it using the algorithms developed by Polychronopoulos and Tsitsiklis (21) as sub-problem in a successive linearization framework.

To date, the authors are not aware of any work capturing the impact of FDM recourse decisions in the event of network disruptions and the determining the resulting network flows.
PROBLEM DESCRIPTION
Consider a network $G = (N, A)$ where $N$ denotes the set of all nodes in the network and $A$ represents the set of all arcs in the network. A finite set of FDMs wants to transport goods from a set of origins $R$ to a set of destinations $S$. Every FDM is assumed to transport $K$ commodity/product types. For easing up on the notation, we assume a distinct FDM is associated with every origin destination pair. This assumption is not restrictive and the methodology presented in this paper is applicable even if this assumption is relaxed. Let $D$ denote the origin destination matrix. Every element of $D$ is represented as $d_{rs}^k$ which represents the amount of good of type $k \in K$ to be transported from origin $r \in R$ and destination $s \in S$. The arcs are assumed to be under the control of carriers; different arcs can also correspond to different modes used by carriers. We assume that on every arc, a carrier or mode has enough capacity to handle all of FDM’s goods either on his own or by sub-contracting to other carriers.

The cost experienced by a FDM in transporting a commodity/product on an arc (which corresponds to a specific carrier) is assumed to be a linear function of three attributes - unit transportation cost of transporting a commodity/product, unit travel time to traverse the arc for commodity/product, and the unreliability parameter. The unreliability parameter is based on the past experience of the FDM in transporting goods using a particular carrier or mode. The reliability parameter is assumed to capture all other attributes other than travel time and travel cost which affects FDMs routing decisions.

The system is assumed to exist in multiple states or realizations $\Omega$ corresponding to different network disruption scenarios. The system states are described using a discrete probability distribution $P$. Each system state corresponds to a vector of arcs attributes. For any system realization $\omega \in \Omega$, let $c_{a}^{k\omega}$ and $t_{a}^{k\omega}$ denote the unit cost and unit travel time for transporting commodity/product $k \in K$ on arc $a \in A$. In the numerical example and without loss of generality, the reliability parameter $u_{a}^{k\omega}$ is assumed to be a property of the arc and not change with the system states.

Information Updates
The FDMs progressively get information about the system state by observing or learning the state of the arcs when his goods traverse the arcs. In this paper, we assume that whenever the goods reach a node $i \in N$, the FDM learns the states of all outgoing arcs from that node. Let us consider a FDM who wants to transport goods between origin destination pairs $(r \in R, s \in S)$. The FDM observes the arcs emanating from node $r$. Let us say he learns that the travel time and travel cost of an arc $a'$ emanating from node $r$ to be $t_{a'}^k$ and $c_{a'}^k$, respectively. Now based on this information, the FDM can eliminate all scenarios from $\Omega$ where the travel time and travel cost on arc $a'$ are not $t_{a'}^k$ and $c_{a'}^k$. The
FDM will then update his probability, travel cost and travel time matrices. Given the updated knowledge about the possible system states, let us assume that the carrier sends all of his commodities on arc \( a' \). Once the commodities reach the tail node of \( a' \) (assume it to be node \( i' \in N \)), the FDM will now observe the states of all arcs emanating from node \( i' \) and update the probability, travel cost and travel time matrices.

At every node, the FDMs routing strategy, also known as routing policy involves choosing different arcs to transport goods depending on the state of the outgoing arcs. For example, consider a node \( i \in N \) and let \( \Gamma_i \) represent the set of all outgoing arcs and \( \Psi(\Gamma_i) \) represent the states of all outgoing arcs. For every possible state \( \psi \in \Psi(\Gamma_i) \), the FDMs routing policy provides the next arc on which the FDM will route his goods. A routing policy thus represents the set of paths on which a FDM can possibly route the goods based on the information received while traversing the network. There is a probability associated with each path of the routing policy, which depends on the probability distribution of the system states. Therefore an expected cost is associated with every routing policy. The next two sections describe the process of calculating the expected costs for each routing policy.

**Generalized costs**

As the arcs used by the FDM depends on the system states, the flow of a commodity/product on an arc also depends on the system states. Let \( x_{ars}^{k_o} \) represent the flow of commodity/product \( k \in K \) between origin \( r \in R \) and destination \( s \in S \) on arc \( a \in A \) in scenario \( \omega \in \Omega \). For any carrier corresponding to a unique origin destination pair \((r \in R, s \in S)\), the cost of transporting commodity/product \( k \in K \) on \( a \in A \) in scenario \( \omega \in \Omega \), \( C_{ars}^{k_o} \) is assumed to be a weighted linear combination of the transportation costs \( c_a^{k_o} \), travel time \( t_a^{k_o} \) and reliability parameter \( u_a^k \). The transportation costs \( c_a^{k_o} \) denotes the unit cost for transporting commodity/product \( k \in K \) and incorporates besides freight rates any FDM or product related characteristics such as volume of sales or usage of a JIT system. The term \( t_a^{k_o} \) denotes the unit cost and unit travel for realization \( \omega \in \Omega \), i.e. for a delay or disruption of a given magnitude there is an associated cost increase that reflects the new valuation of the FDM’s time. The reliability parameter \( u_a^k \) is assumed to be a property of the carrier or mode that is obtained from historical data or past performance. The weights used to capture FDMs’ heterogeneity with respect to the tradeoff between travel costs, travel time and unreliability for different products can be obtained from surveys or discreet choice freight models \((10, 11, 30)\).

\[
C_{ars}^{k_o} (x_{ars}^{k_o}) = (w_{ars}^k t_a^{k_o} + w_{ars}^k c_a^{k_o} + w_{ars}^k u_a^k) x_{ars}^{k_o} \quad \forall a \in A, k \in K, r \in R, s \in S, \omega \in \Omega
\]

In the above equation \( w_{ars}^k \), \( w_{ars}^k \) and \( w_{ars}^k \) represents the importance or weight associated with travel time, travel cost and reliability for the FDM corresponding to the
origin destination pair \((r \in R, s \in S)\) for commodity/product \(k \in K\). In this way it is possible to map product, supply chain, and company characteristics that have been found to significantly impact FDMs’ preferences and valuations of freight transportation alternatives.

**FORMULATION**

The flow of commodity/product \(k \in K\) between origin destination pair \(r \in R, s \in S\) on arc \(a \in A\) under system realization \(\omega \in \Omega\), \(x_{ars}^{k\omega}\), is equal to the sum of flow on all routing policies connecting origin destination pair \(r \in R, s \in S\) and using arc \(a \in A\) under system realization \(\omega \in \Omega\). Let \(H_{rs}\) denote the set of routing policies connecting origin destination pair \(r \in R, s \in S\) and \(f_{i}^{k}\) represent the flow of commodity/product \(k \in K\) on a routing policy \(h \in H_{rs}\). Let \(\delta_{ah}^{\omega}\) be an incidence matrix such that:

\[
\delta_{ah}^{\omega} = \begin{cases} 
1 & \text{if policy } h \text{ uses arc } a \text{ in system state } \omega \\
0 & \text{otherwise}
\end{cases}
\]

\[
x_{ars}^{k\omega} = \sum_{h \in H_{rs}} \delta_{ah}^{\omega} f_{i}^{k} \quad \forall a \in A, k \in K, r \in R, s \in S, \omega \in \Omega
\]

Given \(x_{ars}^{k\omega}\) the cost of transporting commodity/product \(k \in K\) on arc \(a \in A\) in scenario \(\omega \in \Omega\) can be written as:

\[
C_{ars}^{k\omega}(x_{ars}^{k\omega}) = (w_{trs}^{k} + w_{crs}^{k} + w_{ars}^{k})x_{ars}^{k\omega}
\]

\[
\forall a \in A, k \in K, r \in R, s \in S, \omega \in \Omega
\]

The contribution of link \(a\) to the expected cost of routing policy \(h \in H_{rs}\), \(C_{a}^{h}\) can be calculated as:

\[
C_{a}^{h} = \sum_{\omega \in \Omega} \delta_{ah}^{\omega} P_{a}^{\omega} C_{ars}^{k\omega}(x_{ars}^{k\omega}) \forall a \in A, h \in H_{rs}
\]

Thus the expected cost of a routing policy \(h \in H_{rs}\), \(E[C(h)]\) can be calculated as:

\[
E[C(h)] = \sum_{a \in A} \sum_{\omega \in \Omega} \delta_{ah}^{\omega} P_{a}^{\omega} C_{ars}^{k\omega}(x_{ars}^{k\omega})
\]

Since the unit costs are assumed not to vary with flow, the system objective coincides with every FDM’s objective. Therefore minimizing expected total cost will correspond to minimizing every FDM’s individual costs for every commodity/product. Therefore, the mathematical programming formulation can be written as:
Minimize \[ Z = \sum_{a \in A} \sum_{k \in K} \sum_{r \in R} \sum_{s \in S} \sum_{\omega \in \Omega} \left( w_{trs}^k t_{k\omega}^a + w_{crs}^k c_{k\omega}^a + w_{ars}^k u_{ars}^k \right) x_{ars}^{k\omega} p^\omega \] (5)

Subject to:
\[ x_{ars}^{k\omega} = \sum_{h \in H_{rs}} \delta_{ah} f_h^k \quad \forall a \in A, k \in K, r \in R, s \in S, \omega \in \Omega \]
\[ \sum_{h \in H_{rs}} f_h^k = d_{rs}^k \quad \forall k \in K, r \in R, s \in S \] (6)
\[ x_{ars}^{k\omega} \geq 0 \] (7)

Note that equation (6) constrains the sum of flow on all routing policies for commodity/product \( k \in K \) connecting origin destination pair \( r-s \) to be equal to the demand for commodity \( k \) between that specific origin-destination pair.

**SOLUTION ALGORITHM**

The above formulation can be solved by assigning flows to the shortest expected cost routing policy for every commodity/product and for every O-D pair. The shortest expected cost policy can be calculated using the R-SSPR algorithm developed by Polychronopoulos and Tsitsiklis (25) for the cost structure as defined in this paper. The procedure is illustrated below.

**Step 0 Cost Calculation:** Calculate the unit cost for every arc for every possible system realization as
\[ C_{ars}^{k\omega} = \left( w_{trs}^k t_{k\omega}^a + w_{crs}^k c_{k\omega}^a + w_{ars}^k u_{ars}^k \right) \quad \forall a \in A, k \in K, r \in R, s \in S, \omega \in \Omega \]

**Step 1 Determining feasible information Sets:** As the goods traverse the network the FDM learns the set of possible system realizations by eliminating the realizations which are not consistent with the observed costs and travel time. The set of possible system realizations is defined as the information set. If the cardinality of the set of system states is \( |\Omega| \) then there are \( 2^{|\Omega|} - 1 \) information sets. For example if \( \Omega = \{1,2,3\} \) then the set of feasible information sets \( \Xi = \{1,2,3,12,23,13,123\} \). Let \( \Xi^m \) represent set of subsets of \( \Xi \) with cardinality \( m \). For example \( \Xi^1 = \{1,2,3\} \), \( \Xi^2 = \{12,23,13\} \) and \( \Xi^3 = \{123\} \).

**Step 3 Calculate Minimum Expected Cost Routing Policy using the R-SSPR (21):**
The extra notation used in the expected cost routing policy algorithm is given below.
Notation:

\( \xi \)  
Information set corresponding to carrier’s knowledge about system conditions

\( \psi \)  
Node state observed by the carrier when a node is reached.

\( V[i / \xi, \psi] \)  
Expected cost of routing policy for transporting goods from node \( i \) to destination \( s \) given that the carrier has knowledge of information set \( \xi \) when the goods arrive at node \( i \), and learns the node state \( \psi \) after reaching node \( i \).

\( pathptr [i / \xi, \psi] \)  
Next node to be chosen by the same carrier.

\( E[i / \xi] \)  
Expected cost of routing policy for transporting goods from node \( i \) to destination \( s \) given that the carrier has knowledge of information set \( \xi \) when the goods arrive at node \( i \)

\( SEL \)  
Scan eligible list

\( tempV[i / \xi, \psi] \)  
Variable defined to store temporary values of \( V[i / \xi, \psi] \)

\( tempE[i / \xi] \)  
Variable defined to store temporary values of \( E[i / \xi] \)

\( p[\psi / \xi] \)  
Probability of a carrier observing state \( \psi \) given the information set \( \xi \)

\( C_{k,v,w}^{k(v)} \)  
Cost of transporting commodity/product \( k \in K \) on \((v,w) \in A \) in system state \( \omega(\psi) \in \Omega \) corresponding to node states \( \psi \in \Gamma(v) \)

The basic premise behind the algorithm is that when the goods arrive at a node \( v \in N \) the FDM’s knowledge of the system will be any element of the information set \( \xi \in \Xi \). Given the carrier’s current knowledge of the system states, he then observes, \( \psi \in \Psi(\Gamma v) \) the states of all outgoing arcs from node \( v \). Given the state he observes, \( \psi \) he then updates his information set \( \xi \) to \( \xi’ \). Information set \( \xi’ \) will be a subset of \( \xi \) as the FDM will either learn new information about the state of the system, eliminate states which are not possible or in the worst case not learn anything and retain the information set \( \xi \). Thus is we know the optimal routing policy for

For each node \( w \in W \) which can be reached from node \( v \), the temporary label for the expected cost , \( tempV[v / \xi, \psi] \) can be calculated as the cost of arc \((v,w) \) under the observed system state  plus the cost of traveling from node \( w \) to the destination given that the information set he has while arriving at \( w \) is \( \xi’ \). If \( tempV[v / \xi, \psi] \) is less than the current label \( V[v / \xi, \psi] \), then we update the current label to be equal to the temporary label. Once the labels \( V[v / \xi, \psi] \) are calculated for each possible node state \( \psi \in \Psi(\Gamma v) \),
we can calculate the temporary label for the expected cost to travel from node $v$ to the
destination given the information set $\xi$, $\text{tempE}[v/\xi]$, by multiplying $\text{tempV}[v/\xi, \psi]$ with the respective conditional probabilities of observing node state $\psi$
given the information set $\xi$ and summing up across all possible node states. If $\text{tempE}[v/\xi]$ is lesser than the current label $V[v/\xi]$, then update the current label to be
equal to the temporary label. Then node $v$ is added to the scan eligible list.

One important property which is exploited by the algorithm is that in order to
calculate the expected cost of routing policy from every node to the destination for
information sets of cardinality $k, \Xi^k$, we need the expected cost of routing policy from
every node to the destination for information sets of cardinality less than $k$. Thus the
optimal routing costs are calculated for information sets in increasing order of cardinality,
i.e. first calculate the costs to travel from each node to the destination assuming that
information sets of cardinality 1 is possible (which can be trivially calculated using
Dijkstra's algorithm). Then calculate expected cost for routing policies for information
sets of cardinality 2 at each node to the destination. Similarly, the process can be
repeated until we reach information set of cardinality $|\Omega|$.

Note that in the current example we assume that the FDM gets information about
all outgoing arcs. The above algorithm can be easily modified to consider the case where
the FDM at each node gets information about a set of arcs through ITS devices by
modifying the way the current information set is updated to get a new information set.
The pseudo code of the RSSPR algorithm is given below. For more details on efficient
heuristics to speed up the above process, the readers are referred to Polychronopoulos and
Tsitsiklis (21).

```plaintext
For $r \in R, s \in S, k \in K$
   For $n = 1$ to $m$
      For $\xi \in \Xi^n$
         $E[s/\xi,1] = 0$ : $\text{pathptr}[s/\xi,1] = 0$
      For $i \in N - s, \psi \in \Psi(\Gamma i)$
         Set $E[i/\zeta, \psi] = \infty$ : $\text{pathptr}[i/\zeta, \psi] = \text{undefined}$
      $SEL := s$
      While $SEL \neq \emptyset$
         Remove node $u$ from $SEL$ with minimum $E[u/\zeta]$
      For each $v \in N$ such that $(v, u) \in A$
         For each $\psi \in \Psi(\Gamma v)$
```
Update $\xi$ based on $\psi$ to obtain $\xi'$. 

For each $w \in N$ such that $(v, w) \in A$

$$tempV[v / \xi, \psi] = C_{\psi(v)}^{\text{hot}(\psi)} + V[w / \xi', \psi]$$

If $tempV[v / \xi, \psi] < V[v / \xi, \psi]$ 

$$V[v / \xi, \psi] = tempV[v / \xi, \psi]; \text{pathptr}[v / \xi, \psi] = w$$

End of loop: For each $w \in N$ such that $(v, w) \in A$

End of loop: For each $\psi \in \Psi(\Gamma v)$

$$tempE[v / \xi] = \sum_{\psi \in \Psi(\Gamma v)} p[\psi / \xi] \cdot tempV[v / \xi, \psi]$$

If $tempE[v / \xi] < E[v / \xi]$, 

$$E[v / \xi] = tempE[v / \xi]$$

$$SEL := SEL \cup \{v\}$$

End of loop: For each $v \in N$ such that $(v, u) \in A$

End of loop: While $SEL \neq \phi$

End of loop: For $\xi \in \Xi^n$

End of loop: For $n = 1$ to $m$

End of loop: For $r \in R, s \in S, k \in K$

**Step 4: Flow Assignment**

For every origin destination pair and every commodity/product use the algorithm presented in step 3 to determine the shortest expected cost routing. Set the flow on the shortest routing policy for every origin destination pair for every commodity/product equal to the demand for that commodity/product between the origin destination pair. From the shortest expected cost policy, determine the link policy incidence matrix for every scenario. Use equation (1), (2) and (4) to determine the flow and cost of commodities for every arc and the resulting expected cost of routing policy for every commodity/product for every origin destination pair.

**DESCRIPTION OF A CASE STUDY**

This section demonstrates the advantage of the adaptive strategy with recourse against two deterministic planning models on a realistic freight network. The authors obtained data from a freight network involving shipments from a major distribution center to national level food chain stores in the Portland area. For the sake of simplicity some
details are omitted, however, the network attempts to mimic key aspects of actual operating conditions. In the presented network, there are 32 nodes in representing distribution centers in the states of Idaho, Washington, Nevada and California (see Figure 1), the network has 32 nodes with 20 origins and one destination (Portland). Three types of products/commodities are being shipped: dry, frozen, and refrigerated. The demand data provided in Table 1 represents truckload shipments. Travel costs accounting for fuel and operating costs is $1.45, $1.65, and $1.75 per mile per truck for the dry, refrigerated, and frozen products respectively. For the sake of simplicity, in our example the costs on the arcs do not vary with origin-destination pairs.

**INSERT FIGURE 1**

The network has three types of arcs: (i) arcs lying on normal shipping routes, (ii) arcs lying on alternate shipping routes, and (iii) supplementary arcs that are not generally utilized. The travel times are assumed to be similar to the ones provided by Google Maps®. Supplementary routes are created by creating a 25% time penalty and a 100% cost penalty on the shortest cost routes during normal conditions. These additional costs represent the cost of business during disruptions and by utilizing different carriers/modes or routes. However, under disruption of the normal or alternate shipping arcs/routes these supplementary arcs/routes may become cheaper. The network has a total of 55 arcs and for the sake of simplicity we assume that every arc had a base reliability of 10.

**INSERT TABLE 1**

Scenario 1 is assumed to be the base case (normal operating conditions). Network disruptions are generated stochastically and travel costs, travel times and reliability values are scaled up correspondingly. Both moderate and extreme disruption levels are tested. For example, under a extreme disruption for any arc $a \in A$:

$$c^{k \omega}_a = c^{k \omega}_a (1 + |\psi| SF \varepsilon)$$

where:

- $\varepsilon$ = uniform random number between 0 and 1,
- $SF$ = scaling factor within that severity level, and
- $|\psi|$ = factor used to scale up the impact of disruption based on the cardinality of the node states of the upstream arcs. Under moderate disruption we assume that $|\psi|=1$.

We test different scaling factors, $SF$, for both types of disruptions. Similarly, the travel times and reliability parameters were also scaled up. The value of the weighting parameters used to calculate generalized cost (see equation 2) assumed for this study are:
(a) 0.0957, 0.0029 and -0.1219 for travel time, costs and reliability parameters respectively under normal operating conditions; and
(b) 0.4579, 0.0029, and -0.3657 for the travel time, costs, and reliability parameters respectively under disruptions.

Hence, the travel time weight increases five times, the reliability weight increases three times and the cost parameter remains unchanged under a disruption scenario; higher reliability is desirable hence the negative sign associated to this parameter and the positive sign associated to time or cost parameters. These changes in the weight parameters under disruptions are consistent with findings obtained in the literature and stated preference carrier or shipper surveys (9).

The performance metric used was the % gain obtained by the adaptive strategy over the non adaptive strategy:

\[
G = \left( \frac{Z_{\text{NonAdaptive}} - Z_{\text{Adaptive}}}{Z_{\text{NonAdaptive}}} \right) \times 100
\]

where:

\[Z_{\text{NonAdaptive}} = \text{system cost when the FDM routes the goods on the normal routes even under disruptions, two cases are possible:}\]

(i) Base case: where the FDM uses the costs under non disruption scenarios to routes the goods, the FDM does not recognize that there may be changes in the state of the network, i.e. as if the FDM assumes \(|\psi| = 0\). The gain in this case is denoted as \(G_{\text{Base}}\).

(ii) Expected cost case: where the FDM uses the expected costs considering the based case and the disruptions. The gain in this case is denoted as \(G_{\text{EV}}\).

In the stochastic optimization literature:

\[Z_{\text{NonAdaptive}} - Z_{\text{Adaptive}} = \text{value of stochastic solution (31)}\]

This is the efficiency obtained by accounting for uncertainty in the model over the deterministic scenario. Note that the routing costs under both Non Adaptive scenarios are deterministic and there is no re-routing involved. All the results presented herein are average gains obtained over 30 different runs generated using different random seeds.

\[
\bar{G} = \frac{1}{30} \sum_{i=1}^{30} G_{\omega i}
\]
CASE STUDY RESULTS ANALYSIS

Three different experiments are run to study the impact of: (i) disruption severity levels, (ii) number of uncertain scenarios, (iii) distribution of scenario probabilities, and (iv) disruptions on supplementary arcs.

Disruption severity levels
In this experiment the cost parameters under high disruption are scaled up by the ordinality of the node state, the scaling factor, and a random number between 0 and 1; the cost parameters under low disruption are scaled up only by random number between 0 and 1 and the scaling factor was used to scale up the cost factors. We assume that there are four scenarios in the network, i.e. \(|\Omega| = 4\) (each arc could take up to four different values). Note that in high disruption scenario, the number of scenarios and the number of outgoing arcs affect the ordinality of node states \(|\psi|\) in expression (8). The number of scenarios also affect the cardinality of the information sets (15 in this case).

In the equally likely case, all scenarios were assumed to have equal probability; in the asymmetric case scenario 1 (normal conditions) has the highest probability of

\[ \Omega_1 = 0.7 \]  
and the other three scenarios were assumed to have a probability of:

\[ \Omega_2 = \Omega_3 = \Omega_4 = 0.1. \]

It is worth observing that the value of updated information is higher under large disruptions, see Table 2. In addition, the impact of the initial cost differences to transport the three different products is relatively minor when compared to the changes due to disruption scale. Planning for base case normal operating conditions yielded significantly worse solutions compared to planning for expected value case. When planning was done using the expected values, the gains appear to stabilize with increase in scaling factors. The savings of adaptive strategy over expected value case were higher in the Asymmetric case when compared to the equally likely case. Hence, the value of information and updates is higher when the disruption scenarios are not equally likely.

INSERT TABLE 2

Number of Uncertain Scenarios

The second set of experiments studied the impact of number of scenarios on the average gain (see Table 3). The severity scaling factor was set to be equal to 1. In this experiment a changing number of scenarios affect the cardinality of the information set and the value of \(|\psi|\). It must be observed that the solution or running time increases with the number of scenarios and thus the gains are obtained at higher running times.
For almost all the cases, the gains over expected value strategy were found to increase when the number of scenarios increased to 5 and then stabilize or decrease. In the high disruption case, the gains over the base case routing strategy were found to increase with increase in number of scenarios. In the low disruption case, the gains over the base case strategy were found to stabilize and reduce when the number of scenarios increased from 5 to 6. Thus the gains obtained need not always necessarily increase with the increase in the number of uncertain states thus validating the need for such a model to evaluate the actual gain.

The gain is also highly dependent on the type of product in some situations, for example, under low disruption. On the other hand, high disruptions overshadow any influence that product type may have.

**INSERT TABLE 3**

**Scenario Probabilities**
The third set of experiments study the impact of different values of probabilities on the gains. The number of scenarios is set to be equal to 4 and the scaling factor is set to be equal to 1. Scenario 1, which is the normal base operating conditions, is assumed to have a probability of \( p_1 \) varying in increments of 0.05 from 0.25 to 0.7. The other three scenarios are assumed to equally likely and have a probability distribution of:

\[
\frac{(1-p_1)}{3}.
\]

**INSERT TABLE 4**

As the probability of scenario 1 increases, the gain obtained over the base case strategy decreases. In the low disruption case, as the probability values vary the gain obtained over expected value strategy remains constant and stable. However in the high disruption strategy, the gain over the expected value strategy was found to increase with the increase in probability of scenario1. This is an interesting result as it states that as the probability of the conditions being normal increases, the performance of the expected value strategy in relation to the adaptive strategy may decrease.

Again, the gain is also highly dependent on the type of product in some situations, for example, under low disruption. On the other hand, high disruptions overshadow any influence that product type may have.

**Flows on Supplementary Arcs**
A final set of experiments aim at comparing the percentage of flows on supplementary arcs (corresponding to other modes or other carriers which the FDM will not use during normal operating conditions) to percentage of flows on normal shipping routes. The performance metric used was the ratio of flows on supplementary arcs and normal shipping routes expressed as a percentage. Denoting $A_n$ and $A_s$ the set of arcs utilized under normal operating conditions and the set of supplementary arcs respectively, where $A_n \cap A_s = \emptyset$, $A_n \cup A_s = A$ then for adaptive routing, the flow ratio for commodity $k$, is calculated as follows:

$$FR^k = \frac{\sum_{\omega \in \Omega} \sum_{\alpha \in A_n} p^{\omega \alpha x_{k\omega}}_{ars}}{\sum_{\omega \in \Omega} \sum_{\alpha \in A_h} p^{\omega \alpha x_{k\omega}}_{ars}}$$

The ratio was calculated for both high disruption and low disruption scenario for each commodity and for total flows for six different scaling factors ranging from 0.5 to 3.0. The ratio was calculated under online adaptive routing and when routing was conducted using expected cost scenario. The ratio of flow on supplementary arcs to normal shipping route arcs increases with the increase in the magnitude of disruptions (see Figure 2). The ratio of flow on supplementary arcs to flow normal arcs is higher in the high disruption scenario. When the FDM makes routing decisions using the expected cost scenario, the flow ratio is nearly half the ratio under adaptive routing conditions. Considering the significant cost benefit achieved by the FDM, the adaptive routing strategy leads to more efficient usage of other modes or other carriers which the FDM will not use during normal operating conditions (see Figure 2).

**INSERT FIGURE 2**

The utilization of additional arcs also has implications for transportation agencies. As more supplementary routes get used, the impact of disruption can be distributed across the network in a more uniformly manner.

The authors do understand that some of the trends obtained here cannot be generalized to all networks. The actual gain obtained by the adaptive strategy maybe dependent on the nature of disruption, network topology (number of re-routing options available), cost parameters on the alternate options etc. However, the purpose of this section is to demonstrate the usefulness of the adaptive model in a realistic freight network. Different FDMs (carriers, shippers and freight operators) can apply this model and estimate the gain obtained by adopting an adaptive strategy. Depending on the estimates of the gain, they can then decide if it will be worthwhile to invest in communication and sensor technologies which provide up-to-date real-time information on the conditions. In addition, transportation agencies may invest in information systems
to provide real-time information to carriers as this may: (a) create a substantial economic benefit by reducing logistics costs, and (b) spread out the utilization of the network when disruptions do take place.

CONCLUSIONS

This research provides an online freight network assignment model with transportation disruptions and recourse. This model can better reflect the utilization of real-time tracking technology. Public agencies can use this model to predict the behavior of freight decision makers (carriers or shippers) when significant disruptions take place. The methodology presented in this research can be used to estimate network flows in situations where disruptions are either significant or frequent. Given the large difference between freight agents’ preferences under normal and disruption conditions, models that do not account for the likelihood of disruption can be severely misrepresent freight system performance and flows over the network. Private companies can also apply the proposed model. It is clear that freight agents and decision makers can significantly benefit from the adopting of an adaptive strategy as the severity of the disruption increases.
ACKNOWLEDGMENTS

The authors would like to acknowledge the support of Varunraj Valsaraj and other members of the logistics team of Arrowstream who helped in developing the real world freight example and answered numerous questions and doubts with respect to choosing of the parameters.

REFERENCES


LIST OF TABLES AND FIGURES

FIGURE 1  Distribution centers location (Red, Green, and Blue denote origin, destination (Portland), and additional nodes respectively).

TABLE 1  Trucks load demands to Portland

TABLE 2  Percentage Gain

TABLE 3  Number of Scenarios

TABLE 4  Probabilities of Scenario 1

FIGURE 2  Ratio of arc flows in artificial arcs to normal arcs (expressed as a percentage)
**FIGURE 1:** Distribution centers location (Red, Green, and Blue denote origin, destination (Portland), and additional nodes respectively).
**TABLE 1:** Trucks load demands to Portland

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### TABLE 4: Probabilities of Scenario 1

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(a) high disruptions

(b) Low disruptions

FIGURE 2: Ratio of arc flows in artificial arcs to normal arcs (expressed as a percentage)