A Model and Case Study of the Impacts of Stochastic Capacity on Freeway Traffic Flow Benefits and Costs

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ABSTRACT

When freeway traffic flow approaches capacity, minor disturbances or perturbations can cause unstable traffic streams to break down into queued or bottleneck conditions with the accompanying heavy congestion costs. As the traffic volume at which flow breaks down is uncertain, this research utilizes a stochastic capacity model to estimate congestion costs in terms of delays, fuel, and emissions. We apply this stochastic model to a congested freeway corridor in Portland, Oregon in order to demonstrate the impact of various traffic parameters on the net social benefits of traffic flow. Travel time is the dominant cost, followed by fuel costs. For a given value per trip (in $/mile), the traffic flow volume that maximizes social benefits decreases as travel time reliability decreases. Traffic flows near capacity levels are justified by trip values that are 50% higher if the impacts of stochastic freeway capacity are considered. Comparing macroscopic peak-period traffic characteristics among urban areas of varying size and density, we see that for a given peak-period trip value denser urban areas will have higher optimal flow rates. This comparison demonstrates a fundamental cost trade-off for traffic networks between shorter trip lengths and higher traffic intensity from increased urban density.

INTRODUCTION

Traffic congestion is increasing around the world, particularly on urban freeway facilities (European Conference of Ministers of Transport (ECMT) 2007; Schrank and Lomax 2009). Congestion has enormous social and financial impacts related to traveler time, air pollution, fuel consumption, freight costs, and safety costs, among others (Goodwin 2004; HDR 2009; Kriger et al. 2007; Weisbrod, Vary, and Treyz 2001). In terms of traveler time and freight costs, we also now appreciate that it is not only average conditions but unreliable/variable conditions that increase total costs of congestion (Brownstone and Small 2005; Danielis, Marcucci, and Rotaris 2005).

One of the causes of unreliability is instability in a traffic stream as it nears some maximal throughput capacity (Kerner 1999; May 1989). Minor disturbances or perturbations can cause unstable traffic streams to break down into queued or bottleneck conditions with the accompanying heavy congestion costs. But the traffic volume at which flow breaks down is not a certainty, motivating past research to model networks as having uncertain traffic volume capacity (Boyles, Kockelman, and Travis Waller 2010; Lam, Shao, and Sumalee 2008; Lo, Luo, and Siu 2006; Chen et al. 2002). Lo and Tung (2003) modeled link capacities as uniformly distributed random variables, while Brilon (2005) modeled traffic capacity as a stochastic variable following a Weibull distribution. Since traffic flow breakdown is a stochastic event, it should also be treated as such in traffic management. In effort to better utilize existing roadway capacity, many metropolitan areas have established advanced traffic management systems (ATMS). These systems employ various traffic control techniques such as ramp metering, variable speed limits, dynamic congestion pricing, and dynamic traveler guidance (U.S. Department of Transportation n.d.).

The proliferation of ATMS provides opportunities to better manage traffic flows. We propose that traffic flows can be better managed if the impacts of stochastic freeway capacity on delays, fuel consumption, and emissions are properly modeled. We apply a stochastic freeway capacity model of traffic flow social benefits and costs to a congested freeway corridor in Portland, Oregon using archived traffic data. Finally, we make comparisons across a diverse set of urban areas to illustrate relationships between urban density, trip length, and congestion levels.

METHODOLOGY

We contrast the models that result from assuming (a) constant freeway capacity and (b) a stochastic freeway capacity model.

(a) Constant Freeway Capacity

Notation.

λ: traffic demand or vehicle arrival rate [vehicles/hour]

λ_c: roadway capacity [vehicles/hour]

$t(\lambda)$: travel rate, time to travel one mile with demand rate λ [hour/mile]

$v(\lambda) = 1/t(\lambda)$: average travel speed [miles/hour]

$e(\lambda)$: marginal emissions rate, with demand rate λ [kg/veh-mile]

$f(\lambda)$: fuel consumption rate, with demand rate λ [gallons/veh-mile]

l: length of the freeway section under study [miles]

c_e: cost of emissions (of pollutant in $e(\lambda)$) [\$/kg]

c_t: value of time [$/hour]
c_f: cost of fuel [$/gallon]

Travel Rate. Using the well-known Bureau of Public Roads (BPR) volume-travel time function (1964), the travel rate as a function of arrival rate λ (assumed constant over the analysis period) is

$$t(\lambda) = t_o\left(1 + a\left(\lambda/_{\lambda_c}\right)^b\right) = t_o + t_o a\left(\lambda/_{\lambda_c}\right)^b \tag{1}$$

where t_o is the free-flow travel rate and a and b are parameters. For $a > 0$ and $b > 0$, $t(\lambda)$ is an increasing convex function.

Emissions and Fuel Rates. For emissions we estimate a function of λ *per vehicle, per mile,*$e(\lambda)$. The CO_2 emissions estimates used for fitting in this study are from Bigazzi and Figliozzi (2011) based on MOVES 2010 emissions model with a 2010 mixed light/heavy-duty fleet in Portland Oregon. In the present study we estimate only CO_2, but other pollutants can be similarly modeled. For the base emissions rates we use $t(\lambda)$ as above (the BPR model) to relate λ to average speed – which is the input for the average-speed emissions model – with $t_o = 60$mph, $\lambda_c = 2{,}200$vphpl, $a = 0.15$ and $b = 7$.

We then apply a new emissions formulation which is similar to $t(\lambda)$, using four positive fitted parameters: $\alpha_0, \alpha_1, \alpha_2, n$, and nominal capacity λ_c:

$$e(\lambda) = \alpha_0 + \alpha_1\left(\lambda/_{\lambda_c}\right) + \alpha_2\left(\lambda/_{\lambda_c}\right)^n \tag{2}$$

The fitted parameters are estimated by minimizing square error of emissions rates with respect to base rates using λ as the independent variable, from 0 to 3,630 veh/hr/ln. This range of λ covers an average speed range from 10 to 60mph. Fitting this curve we find n to be about 10. This fitting gives us an R^2 of 0.996. Similar fitting is obtained for another emission model proposed by Barth and Boriboonsomsin (2008). The fitted parameters are shown in Table 1, and the fits are illustrated in Figure 1 The difference in magnitude of modeled emissions is to be expected, as the MOVES model includes heavy vehicles while the Barth model does not. Still, the $e(\lambda)$ formulation fits well for each modeled fleet.

Table 1. Fitted CO_2 Emissions Equation Parameters

Parameter	MOVES Estimate	Barth Estimate
α_0	0.4043	0.3253
α_1	0.02793	0.0000
α_2	0.003650	0.002641
n	9.993	9.992

Assuming CO_2 emissions are directly proportional to fuel consumption, we can use the CO_2 emissions formulation to estimate both CO_2 emissions and fuel consumption. Using a fuel carbon intensity F of 10kgCO_2/gallon fuel (U.S. Environmental Protection Agency 2009a), we have the fuel consumption $f(\lambda)$, in gallons/veh-mi,

$$f(\lambda) = e(\lambda)/F = \left[\alpha_0 + \alpha_1\left(\lambda/_{\lambda_c}\right) + \alpha_2\left(\lambda/_{\lambda_c}\right)^n\right]/F \tag{3}$$

Alternatively, fuel consumption could be modeled as a function of average travel speed and fit with a new set of parameters, as was done for emissions above.

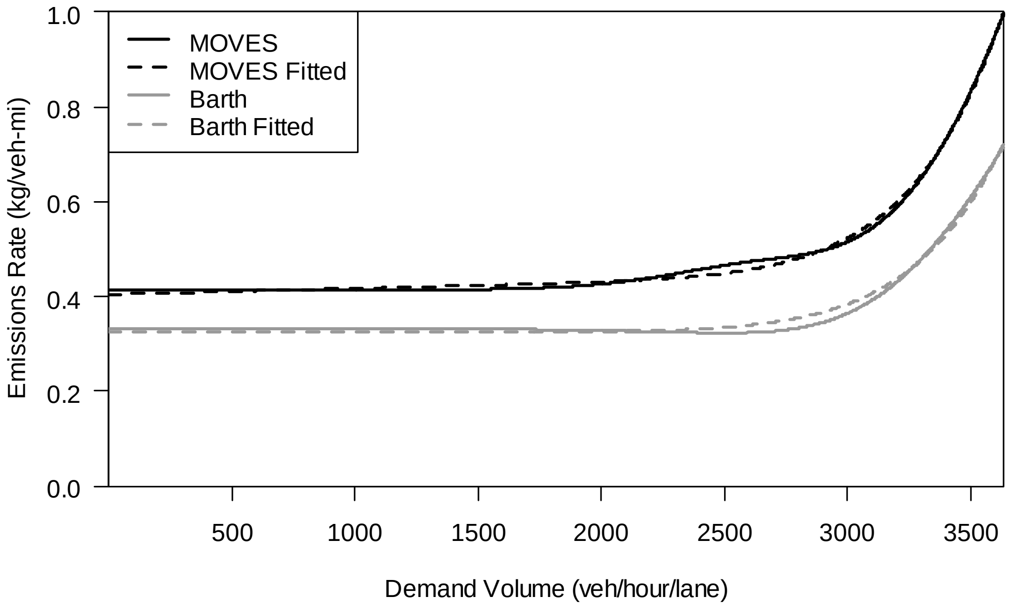


Figure 1. Emissions fit for MOVES (black) and Barth (grey) models

Total Costs and Benefits. Accounting for time, fuel, and emissions costs, the total costs per unit of analysis time of flow rate λ are, in \$/hour,

$$TC(\lambda) = c_t t(\lambda) l\lambda + c_f f(\lambda) l\lambda + c_e e(\lambda) l\lambda \tag{4}$$

Introducing an inelastic demand function such that each trip on the segment has an associated benefit βl [\$/vehicle] where $\beta > 0$ [\$/vehicle-mile], we can calculate the net social benefit per unit time of flow rate λ as

$$NB(\lambda) = \beta l\lambda - c_t t(\lambda) l\lambda - c_f f(\lambda) l\lambda - c_e e(\lambda) l\lambda \tag{5}$$

in \$/hour. If we define a modified emissions cost coefficient $c_\varepsilon = c_e + c_f/F$, we can notate the net benefits simply as $NB(\lambda) = \beta l\lambda - c_t t(\lambda) l\lambda - c_\varepsilon e(\lambda) l\lambda$. The cost coefficient c_ε could also be further modified to take into consideration the cost of local pollutants (assuming their emissions are roughly proportional to CO_2 emissions).

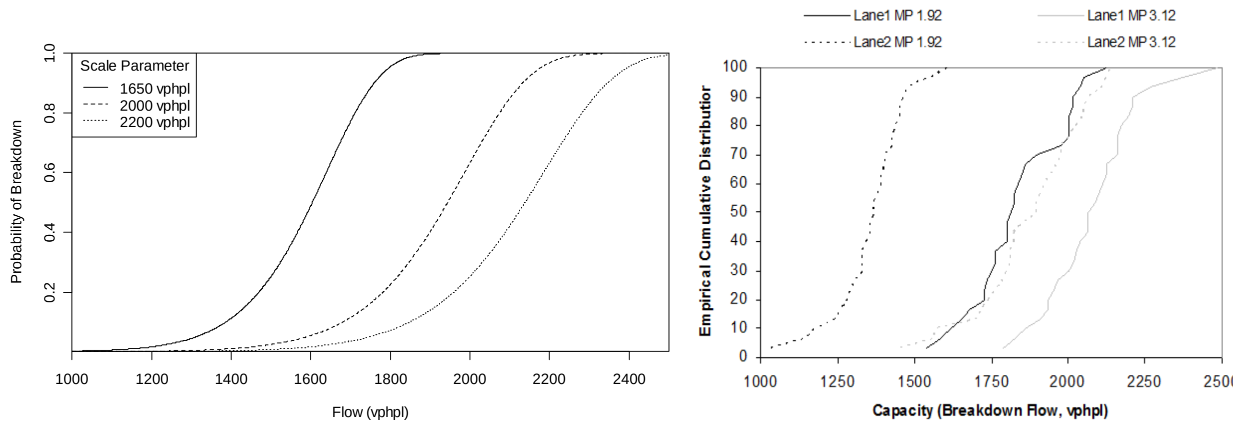
(b) Stochastic Freeway Capacity

The previous analysis assumes the travel speed is a function of the volume of vehicles but that traffic does not break down at any moment. Research has shown that after flow breakdown (a stochastic event) the traffic flow characteristics are altered (Zhang and Levinson 2004). Here we consider the case of a probabilistic breakdown in flow as traffic nears the roadway capacity.

Probability of Breakdown. We define $p(\lambda)$ as the traffic flow breakdown Bernoulli probability function, where the mean of the probability of failure is a function of λ. From Brilon, Geistefeldt, and Regler (2005), we can formulate the relationship as a Weibull-distributed cumulative distribution function

$$p(\lambda) = 1 - e^{-\left(\frac{\lambda}{\varphi}\right)^{\omega}} \tag{6}$$

where ω and φ are shape and scale parameters, respectively. They studied a 3-lane German motorway and found $\omega \approx 13$ consistently, while φ ranged from about 1,650 to 2,200 vphpl for a 1-hour steady flow interval. This agrees with recent research on flow breakdown on an urban freeway in Portland, Oregon (Saberi and Figliozzi 2010) – see Figure 2.



(a) (b)

Figure 2. Comparison of empirical breakdown probabilities from a) Brilon, et al. (2005) and b) Saberi & Figliozzi (2010)

Bottleneck Analysis. Assume that once flow breakdown occurs, a bottleneck is activated behind which a queue of slow-moving vehicles forms. This queue persists partly because of reduced throughput capacity after flow breakdown. If we assume a simple triangular shape on the space-time (*x*-*t*) plane for the extent of the bottleneck, we can estimate delay, fuel consumption, and emissions after flow breakdown using some additional parameters. We define the following parameters, illustrated in Figure 3:

T: duration of time where a bottleneck (*bn*) is present, from $t = 0$

v_f: free-flow traffic speed outside the queue (equal to $v(\lambda)$ for $\lambda < \lambda_c$)

v_b: traffic speed in the queue, where $v_f > v_b$

v_w: speed of queue propagation (a *negative* number)

$v_{w'}$: speed of queue dissipation

l: length of the freeway section under study

The dashed lines in Figure 3 are idealized vehicle trajectories, representing average traffic speed as constant-speed vehicles.

Assume that a vehicle starts at the upstream end of the freeway section under consideration at a time τ with respect to the start of a bottleneck at the downstream end of the section such that $0 \leq \tau + {}^{l}/_{v_f} \leq T$ in order to encounter the queue. The vehicle that reaches the queue at the initial activation of the bottleneck departs at time $\tau = -t_f$ where $t_f = {}^{l}/_{v_f}$. The vehicle that reaches the queue at the transition from formation to recovery wave of the bottleneck departs at time $\tau = t_r$ where $t_r = \frac{T v_{w'}(v_w - v_f)}{v_f(v_w - v_{w'})} - \frac{l}{v_f}$. The last vehicle to encounter the bottleneck departs at time $\tau = t_e$ where $t_e = T - {}^{l}/_{v_f}$.

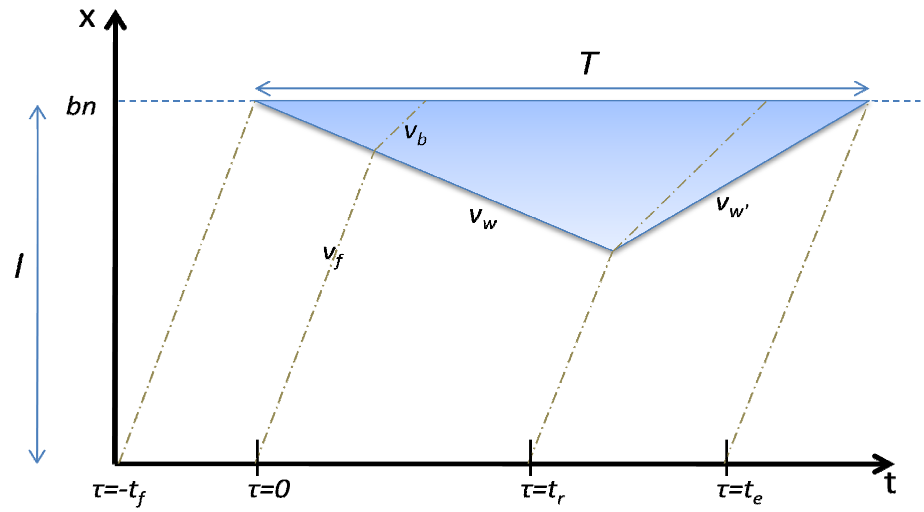


Figure 3. Illustrated breakdown flow parameters on the space-time plane

The travel time over the freeway segment for vehicles encountering the queue during formation (the propagation wave), where $-t_f \le \tau \le t_r$, is

$$t_1 = \frac{l}{v_b} \cdot \frac{v_b - v_w}{v_f - v_w} + \tau \cdot \frac{v_w}{v_b} \cdot \frac{v_b - v_f}{v_f - v_w} \tag{7}$$

and the travel time for vehicles encountering the queue during dissipation (the recovery wave), where $t_r \le \tau \le t_e$, is

$$t_2 = \frac{l}{v_b} \cdot \frac{v_b - v_{w'}}{v_f - v_{w'}} + (T - \tau) \cdot \frac{v_{w'}}{v_b} \cdot \frac{v_f - v_b}{v_f - v_{w'}} \,. \tag{8}$$

The travel time during the queue existence is then a piecewise linear function of the vehicle's departure time from the start of the section, with a maximum travel time for vehicles departing at t_r. These vehicles experience the maximum delay $D_{max} = l_{q\_max}\left(\frac{1}{v_b} - \frac{1}{v_f}\right)$, where max queue length $l_{q\_max} = \frac{T v_w v_{w'}}{v_w - v_{w'}}$, and a travel time of $t_{max} = t_f + D_{max}$. The delay in the queue, averaged over time for vehicles encountering the queue, is $\bar{D} = {}^{D_{max}}/_{2}$.

Assume that if there is a bottleneck, the duration of the bottleneck $T = \delta\Delta$ where Δ is the time period of study with constant demand λ maintained, and $0 \le \delta \le 1$ is a coefficient that indicates the relative duration of the bottleneck in the period of study Δ. The average travel rate (time/distance) during Δ accounting for bottleneck delay is then

$$t_b(\lambda) = t(\lambda) + \delta \frac{\bar{D}}{l} = \frac{\delta \cdot l_{q\_max}}{2 l v_b} + t(\lambda)\left(1 - \frac{\delta \cdot l_{q\_max}}{2l}\right) \tag{9}$$

since $v_f = {}^{1}/_{t(\lambda)}$. Substituting for l_{q_max}, T, and $t(\lambda)$ we get

$$t_b(\lambda) = t_o + \frac{\delta^2 \Delta v_w v_{w'} (1 - t_o v_b)}{2 l v_b (v_w - v_{w'})} + t_o a \left[1 - \frac{\delta^2 \Delta v_w v_{w'}}{2l(v_w - v_{w'})}\right] \left({}^{\lambda}/_{\lambda_c}\right)^b \tag{10}$$

which is an increasing convex function of λ, with parameters $l, \delta, \Delta, v_w, v_{w'}, v_b, t_o, a, b,$ and λ_c and physical constraints $l_{q\_max} \le l$ (the queue must be contained in the segment) and ${}^{1}/_{v_b} \ge t(\lambda)$ (the queue speed must be less than the un-queued speed). We can simplify the notation with a new dimensionless parameter

$$\theta = \frac{\delta l_{q\_max}}{2l} = \frac{\delta^2 \Delta v_w v_{w\prime}}{2l(v_w - v_{w\prime})} \quad (11)$$

which indicates the fractional effective bottleneck length in the context of the study period and road segment length. Using θ,

$$t_b(\lambda) = \frac{\theta}{v_b} + (1-\theta)\cdot t(\lambda) \quad (12)$$

and the average travel rate if flow breakdown occurs is a function of λ with parameters $\theta, v_b, t_o, a, b,$ and λ_c.

Using (1) and (2), the average emissions rates e_q (per vehicle, per mile) for vehicles inside a queue with average speed v_b can be estimated as:

$$e_q = e\left(\lambda_c \left(\frac{1-v_b t_o}{v_b t_o a}\right)^{1/b}\right) = \alpha_0 + \alpha_1 \left(\frac{1-v_b t_o}{v_b t_o a}\right)^{1/b} + \alpha_2 \left(\frac{1-v_b t_o}{v_b t_o a}\right)^{n/b} \quad (13)$$

with parameters $a, b, t_o, \alpha_0, \alpha_1, \alpha_2,$ and n. The emissions for vehicles outside the queue is the same as $e(\lambda)$. Neglecting the transitions in/out of the queue, by a parallel process as the development of (12) we can estimate the emissions when a bottleneck occurs as

$$e_b(\lambda) = \theta \cdot e_q + (1-\theta)\cdot e(\lambda) \quad (14)$$

If we estimate the excess queue transition emissions for a vehicle entering and exiting the queue as e_t in mass per vehicle encountering the queue, then the average emissions rate after breakdown (per vehicle, per mile) becomes

$$e_b(\lambda) = \theta \cdot e_q + (1-\theta)\cdot e(\lambda) + \frac{\delta}{l}\cdot e_t \quad (15)$$

The equations for $e_b(\lambda)$ and $t_b(\lambda)$ are linear functions of $e(\lambda)$ and $\mathrm{t}(\lambda)$, and so themselves increasing convex.

We can estimate the size of e_t using an assumption of constant deceleration/acceleration for vehicles encountering the queue. Let the emissions rates (per vehicle-mile) with constant acceleration a and constant deceleration d be e_a and e_d, respectively, and free-flow emissions be e_f. Then the *excess* emissions in mass per vehicle during the transitions are

$$e_t = \frac{v_b^2 - v_f^2}{2d}\left(e_d - e_f\right) + \frac{v_f^2 - v_b^2}{2a}\left(e_a - e_f\right). \quad (16)$$

Long (2000) discusses various acceleration characteristics of vehicles, and sites NCHRP report 270 for average accelerations around 2 mph/sec in the 30-60mph speed range (Olson et al. 1984). Assuming this value for both transition accelerations and decelerations, we modeled constant accelerations and decelerations in the 30-60mph speed range using the project-level methodology of the MOVES 2010 emissions model (U.S. Environmental Protection Agency 2009b), with the same fleet and other characteristics as above from Bigazzi and Figliozzi (2011). MOVES outputs for CO_2 generated, on average, $e_a = 0.957$ kg/veh-mi and $e_d = 0.156$ kg/veh-mi. Using $e_f = 0.404$ kg/veh-mi at 60mph free-flow speed (see Table 1), these lead to

$$e_t = 2.12 \times 10^{-5}\left(v_f^2 - v_b^2\right) \quad (17)$$

with speeds in mph and e_t in kg/veh. For free-flow speed of 60mph and queued speeds in the 10-40mph range, this results in an equivalent emissions distance penalty of $d_e = \frac{e_t}{e_f} = 0.10$ to 0.18 miles (the distance at free-flow speed that produces the same amount of excess emissions as those produced by the queue transition).

Modified Cost Functions. Utilizing the probabilistic function $p(\lambda)$ we generate the revised travel rate function

$$t'(\lambda) = p(\lambda) \cdot t_b(\lambda) + \left(1 - p(\lambda)\right) \cdot t(\lambda)$$
$$= t(\lambda) + p(\lambda)\theta\left[\frac{1}{v_b} - t(\lambda)\right] \tag{18}$$

Similarly, the revised average emissions rate function is

$$e'(\lambda) = p(\lambda) \cdot e_b(\lambda) + \left(1 - p(\lambda)\right) \cdot e(\lambda)$$
$$= e(\lambda) + p(\lambda)\left[\theta\left(e_q - e(\lambda)\right) + \frac{\delta}{l}e_t\right] \tag{19}$$

The fuel consumption estimate can also be revised as per (3) using $e'(\lambda)$ for $e(\lambda)$. For net social benefits considering probabilistic flow breakdown we then have

$$NB'(\lambda) = \beta l\lambda - c_t t'(\lambda) l\lambda - c_\varepsilon e'(\lambda) l\lambda \tag{20}$$

where $c_\varepsilon = c_e + c_f/F$.

Value of Reliability

The *value of reliability* (or cost of unreliability) with respect to traffic flow instability is defined as the decrease in social benefit due to stochastic freeway capacity. In units of $ per hour of analysis,

$$VR(\lambda) = c_t l\lambda[t'(\lambda) - t(\lambda)] + c_\varepsilon l\lambda[e'(\lambda) - e(\lambda)]$$
$$= \lambda l\theta p(\lambda)\left[c_t\left[\frac{1}{v_b} - t(\lambda)\right] + c_\varepsilon\left[e_q - e(\lambda) + \frac{\delta}{\theta l}e_t\right]\right] \tag{21}$$

This can be put into units of $/veh-mi by dividing by l and λ. So the value of reliability (related to stochastic capcity) can be estimated as a function of λ with parameters of section length, the cost coefficients (except β), the breakdown probability function, the bottleneck parameters, the BPR parameters, the emissions formulation parameters, and the queue transition emissions.

CASE STUDY

We now present a case study application using archived loop detector data from OR-217, a congested freeway corridor in the Portland, Oregon metropolitan region. The parameter values for use in the above equations and their sources are shown in Table 2. Much of the data come from PORTAL, a transportation data archive at Portland State University: http://portal.its.pdx.edu. The freeway stochastic capacity data come from Saberi and Figliozzi (2010), who recently analyzed traffic characteristics on this corridor utilizing PORTAL data.

From (6), we can estimate φ to make the likelihood of breakdown at nominal capacity a certain value p_c, $p(\lambda_c) = p_c$ using $\varphi = \lambda_c\left[ln\left({}^{1}/_{1-p_c}\right)\right]^{-1/\omega}$. For a median value of $p_c = 0.5$, $\varphi_{0.5} = 1.0286\lambda_c$, for $p_c = 0.90$, $\varphi_{0.9} = 0.9379\lambda_c$. Here we assume the nominal capacity (used in the BPR equation) equates to a 90% likelihood of flow breakdown, $p_c = 0.90$. We initially assume travel benefits $\beta = \$0.50$ per vehicle-mile. Theta, a function of the breakdown flow parameters (Δ, δ, v_w,and $v_{w'}$) and segment length l – see (11), is calculated from the parameters in Table 2 as 0.27; the corresponding maximum queue length is 4.8miles.

Table 2. Parameters used in the case study

Parameter	Value	Units	Source
l	7	mi	roadway
Δ	1	hours	approximated from PORTAL data
δ	0.8	-	approximated from PORTAL data
FFS	60	mph	approximated from PORTAL data
a	0.15	-	(Saberi and Figliozzi 2010)
b	7	-	(Saberi and Figliozzi 2010)
λ_c	2200	vphpl	(Saberi and Figliozzi 2010)
v_b	26	mph	(Saberi and Figliozzi 2010)
v_w	-12	mph	(Lu and Skabardonis 2007; Castillo and Benítez 1995)
$v_{w'}$	12	mph	assumed to be the same as v_w
ω	13	-	(Brilon, Geistefeldt, and Regler 2005)
φ	2063	vphpl	Makes λ_c the 90th percentile from (6), see above
e_t	Eq'n (17)	kg/veh	MOVES2010 modeling (see above)
c_t	15	\$/veh-hr	assumed from (Schrank and Lomax 2009)
c_e	0.02	\$/kg CO_2	assuming US\$20/tonne CO_2, from EU ETC
c_f	3	\$/gal	assumed from (Schrank and Lomax 2009)
F	10	kgCO_2/gal	(U.S. Environmental Protection Agency 2009a)
β	0.50	\$/veh-mi	assumed

Net Benefits, Optimal Flow, and the Value of Reliability

The results of applying the parameters in Table 2 to calculate net benefits as embodied in (20) are shown below. The cost components, total costs, benefits, and net benefits are illustrated in Figure 4. The total costs are dominated by travel time costs, and emissions costs are negligible. All cost components increase more rapidly as the flow approaches capacity and the likelihood of flow breakdown increases. The optimal flow here to maximize net benefits is 1,658vphpl (75% of λ_c).

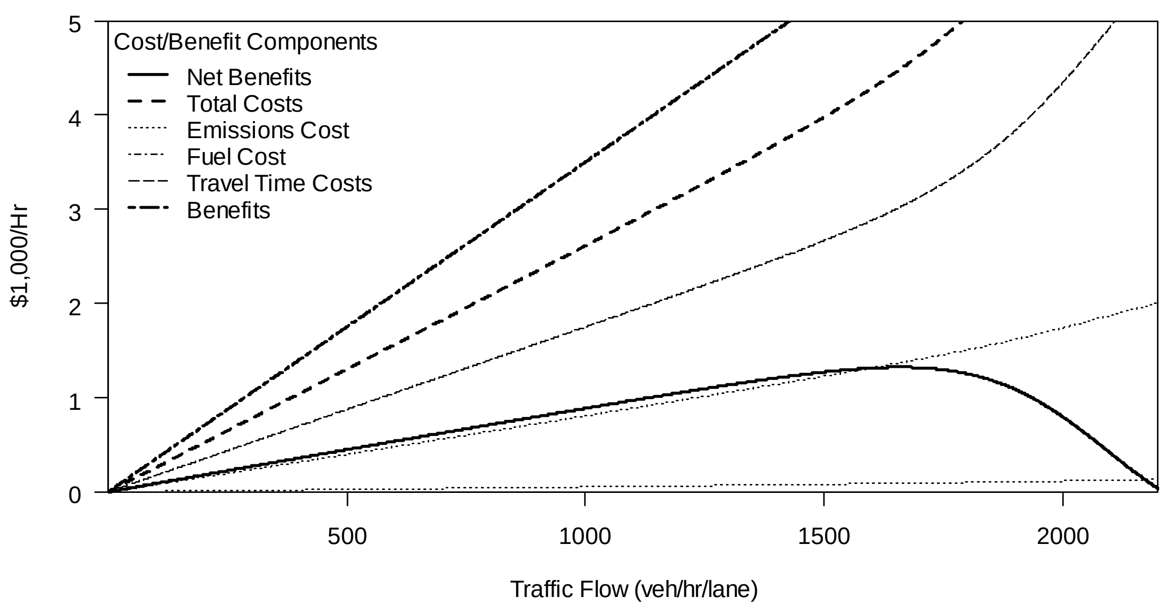


Figure 4. Case study cost and benefit curves

The parameter with the highest uncertainty is β, which would require some knowledge of the trips and travelers to estimate accurately. This parameter impacts both net benefits and the optimal flow rate. Given this uncertainty, it is interesting (and perhaps most useful) to look at how optimal flows vary with β. To this end Figure 5 illustrates the impacts of varying β on optimal flow. Figure 5 also shows the impacts of the probabilistic breakdown formulation with two different optimal flow curves – with and without considering $p(\lambda)$.

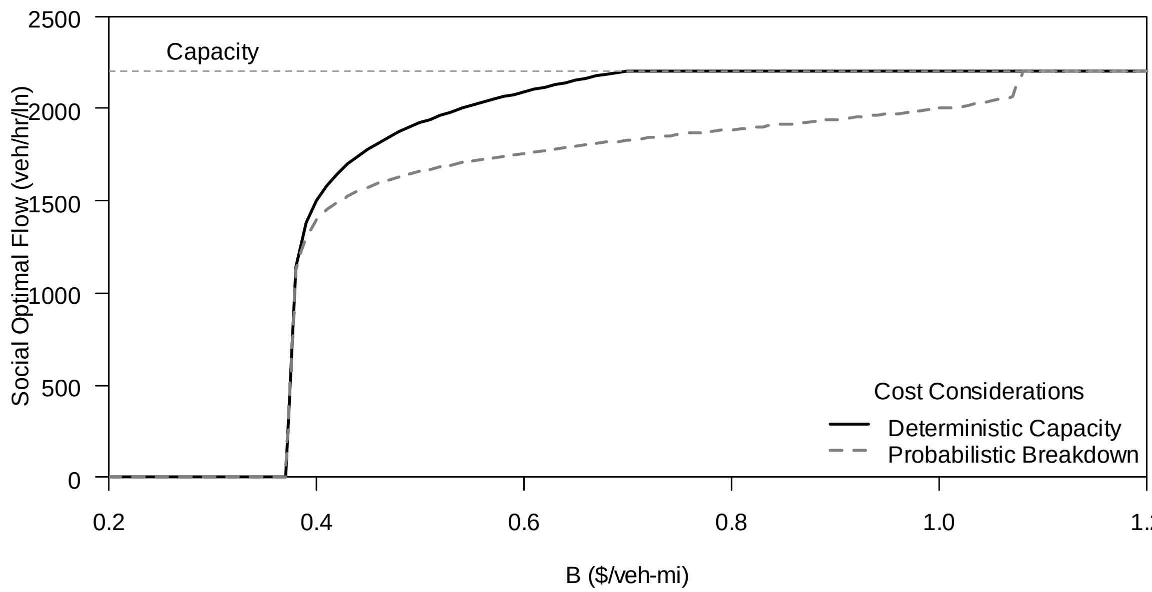


Figure 5. Impact of considering probabilistic breakdown on optimal flows

Here we see that there is an initial per-mile benefit threshold, below which auto travel is not worthwhile. For β just above this threshold, optimal flows quickly rise to the lower probabilities of breakdown flow, around 1,400 vphpl (which equates to $p(1400) = 0.01$). As β increases, higher flows are optimal because the value of additional trips outweighs the increased marginal costs for all vehicles. Optimal flows increase more slowly with β when using the probabilistic formulation, which considers the possibility of flow breakdown below capacity. Using stochastic costs a doubling of the benefits of travel from $0.40 to $0.80/veh-mi results in an optimal flow increase of only about 30%, while the deterministic curve increases to capacity (more than 45%). Lower optimal flows reduce the likelihood of flow breakdown at the sacrifice of additional throughput; up to $\beta = \$0.80$/veh-mi the optimal flow is still below $p(\lambda) = 0.27$. The optimal flow difference between the curves in Figure 5 shows that reducing $p(\lambda)$ is a key factor to increasing optimal traffic flow volumes.

As the optimal flow rates approach the roadway capacity where breakdown is nearly certain, there is a "Capacity Point" for β at which, despite the increased costs of queued conditions, the value of travel supersedes flow restrictions. This occurs at around $0.70/veh-mile considering deterministic costs and $1.06/veh-mile considering stochastic costs. The "Capacity Point" considering probabilistic breakdown is 50% greater than for deterministic conditions – indicating that trip values must be much higher in order to warrant high volumes if we consider traffic instability below the capacity threshold. For probabilistic breakdown there is a sudden change in the optimal flow curve as traffic flows near capacity that reflects the flattening of the Weibull distribution near capacity flows (see Figure 2).

The value of reliability (or cost of unreliability), here computed by (21), increases with $p(\lambda)$ as we approach the roadway capacity. The marginal social value of reliability increases from essentially zero at flows below 1500vphpl to about \$0.08/veh-mi at flows just below capacity. This value of reliability is about 16% of total estimated stochastic costs (per vehicle-mile) near capacity for the study corridor. The value of reliability is alternatively expressed as \$0.56 per vehicle throughput on the segment, since it is negligibly sensitive to segment length. As a reminder, this is only the unreliability due to stochastic capacity, not due to crashes or other incidents.

Finally, Figure 6 presents optimal flows versus β when considering different combinations of cost components. While emissions costs are negligible (at present valuations), the impacts of considering fuel costs are substantial. For example, the "Capacity Point" for total costs is about 20% higher than when neglecting fuel costs. In the other dimension, at $\beta =$ \$0.80/veh-mi the optimal flow is about 5% lower when considering fuel costs as compared to neglecting them.

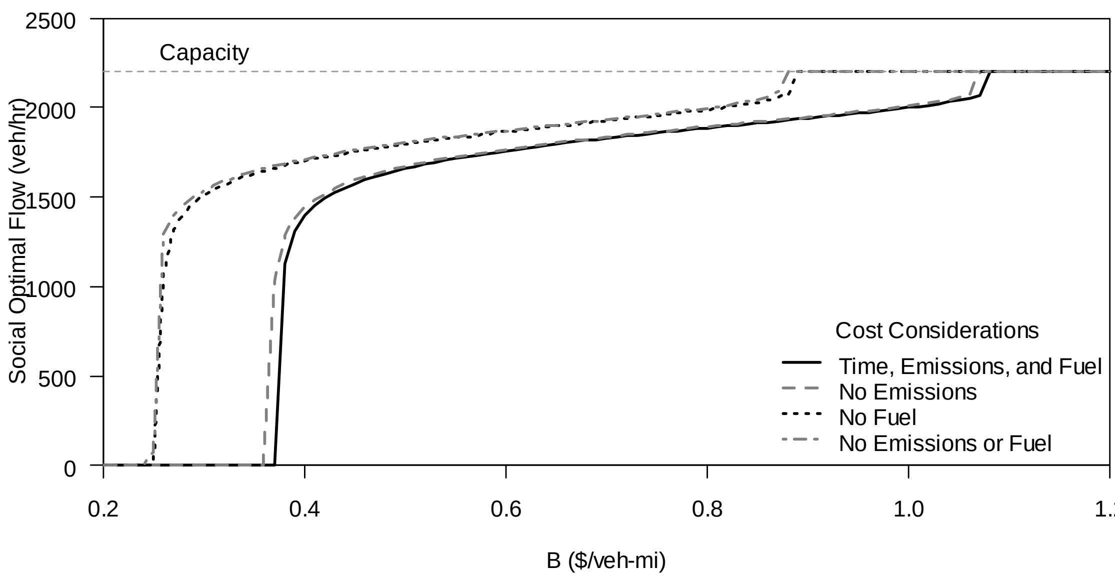


Figure 6. Optimal flow versus β, with different cost components

Case Study Sensitivity

Elasticities of net benefits, optimal flows, and the value of reliability to changes in various parameters were calculated, as presented in Table 3. The elasticity is the percent change in the dependent variable (benefit/flow/reliability) with each percent change in the parameter value, with respect to initial parameter values from Table 2. The initial optimal flow is 1,658vphpl with the initial net benefit of \$1,323/hour estimated at this optimal flow. The initial value of reliability is estimated at capacity flow (2,200vphpl) as \$1,273/hour.

The cost coefficient for time and the benefits per mile (c_t and β) are important factors for net benefits. The free-flow speed (FFS) is also important for net benefits as it greatly impacts the travel time on the segment. Optimal flow is generally less sensitive to parameters than net benefits are, though c_t and β are still among the more important factors. The scale parameter of the breakdown probability function φ is the most important factor for optimal flow, which is to be expected from the importance of flow breakdown likelihood illustrated in Figure 5. The value of reliability is most impacted by bottleneck characteristics such as δ and v_b, as well as the breakdown probability scale parameter φ.

Table 3. Elasticities of net benefits, optimal flow, and the value of reliability to parameters

Parameter	Net Benefit Elasticity	Optimal Flow Elasticity	Value of Reliability Elasticity
L	1.00	0.06	0.00
Δ	-0.05	-0.07	0.90
δ	-0.09	-0.13	**1.66**
FFS	**2.00**	0.17	0.83
a	-0.04	-0.03	-0.11
b	0.08	0.03	0.00
λ_c	0.32	0.20	0.76
v_b	0.08	0.11	**-1.67**
v_w	-0.03	-0.04	0.47
$v_{w'}$	-0.03	-0.04	0.47
ω	0.12	0.10	0.00
φ	0.55	0.71	**-2.50**
e_t	0.00	0.00	-0.02
θ	-0.05	-0.07	0.90
c_t	**-2.00**	-0.28	0.83
c_e	-0.07	0.00	0.01
c_f	-1.00	-0.10	0.13
β	**3.33**	0.38	0.00

Urban Area Comparisons

Our final analysis looks at how city size and density can impact optimal traffic flows. To do this we gather macroscopic characteristics of peak-period freeway volumes in different cities from the data tables of the Texas Transportation Institute's 2009 Urban Mobility Report (UMR) (Schrank and Lomax 2009). The urban areas selected are the most and least "traveler-dense" urban areas in the three top size categories: "Medium" (0.5-1 million people), "Large" (1-3 million), and "Very Large" (>3 million). Traveler density is assessed as the number of peak period travelers per square mile, easily extractable from the UMR data tables.

From the UMR data tables we can estimate average peak-period trip distance on major facilities by dividing total peak period freeway and arterial vehicle-miles traveled (VMT) by the number of peak period travelers. Since the number of travelers will exceed the number of vehicles, this is a low-end approach to estimating miles per person, per day. We also use the UMR data to calculate average congested peak period freeway volumes, in vehicles per hour per lane (vphpl), by assuming the portion congested (in VMT and lane-miles) is equivalent on freeways and arterials, and an even directional split. For each Urban Area the UMR provides estimates of freeway and arterial VMT and lane-miles, fractions of VMT and lane-miles congested, and number of "rush hours"- assumed to be congested. By assuming even distributions this is a conservative approach to volume estimates. These assumptions provide an admittedly rough approximation, but one which can be used to illustrate the differences among urban areas.

Table 4. Urban areas' average characteristics

Urban Area	Population	Peak Traveler Density	Lane-mi Congested	Peak Trip Distance	Peak Freeway Volume
	(1,000's)	(per mi^2)	(%)	(mi)	(vphpl)
Atlanta	4,440	771	58	19.5	1,570
Los Angeles	12,800	3,087	61	19.2	2,098
Raleigh-Durham	1,025	671	53	20.6	1,089
Las Vegas	1,405	2,539	53	16.4	1,700
Nashville	995	725	43	23.8	1,061
Honolulu	705	2,771	51	12.2	1,174

Table 4 shows the six urban areas analyzed along with their population, peak-period traveler density, percent of lane-miles congested, average peak-period trip distance, and average congested peak-period freeway volume – all extracted or calculated from the UMR data tables for 2007. We next use these data to calculate the per-traveler dollar values of daily peak period trips that would warrant capacity flows and optimize existing average peak period freeway flows – each for both deterministic and stochastic conditions. For θ in each urban area (the effective fraction of roadway in queued conditions after flow breakdown) we assume a value equal to the percent of lane-miles congested during the peak period (Table 4), and for v_b we assume a value of 35mph (from the UMR methodology – see Appendix A of the UMR (Schrank and Lomax 2009)). The other parameters for cost coefficients, $t(\lambda)$, $e(\lambda)$, and $p(\lambda)$ are assumed to be the same as above for the case study (bottleneck parameters do not apply since we are using θ).

Table 5. Comparison of travel values across urban areas

	β Values ($ per veh-mi)				Trip Values ($ per peak period traveler)			
Urban Area	To warrant capacity flow		To optimize existing avg peak period freeway flows		To warrant capacity flow		To optimize existing avg peak period freeway flows	
	Det.	Sto.	Det.	Sto.	Det.	Sto.	Det.	Sto.
Atlanta	$0.70	$1.10	$0.41	$0.46	$13.68	$21.49	$8.01	$8.99
Los Angeles	$0.70	$1.13	$0.60	$1.12	$13.43	$21.68	$11.51	$21.49
Raleigh-Durham	$0.70	$1.06	$0.38	$0.38	$14.44	$21.87	$7.84	$7.84
Las Vegas	$0.70	$1.06	$0.43	$0.54	$11.46	$17.35	$7.04	$8.84
Nashville	$0.70	$0.98	$0.38	$0.38	$16.67	$23.33	$9.05	$9.05
Honolulu	$0.70	$1.04	$0.38	$0.38	$8.52	$12.66	$4.62	$4.62

The necessary β to warrant capacity flow in each urban area is then computed as above for Figure 5, and the average trip value to warrant capacity flow calculated using β and the average peak-period trip distances. Results are shown in Table 5 and Figure 7. The values for θ and v_b here are higher than in the case study, which have opposite (and off-setting) effects on β at the "Capacity Point".

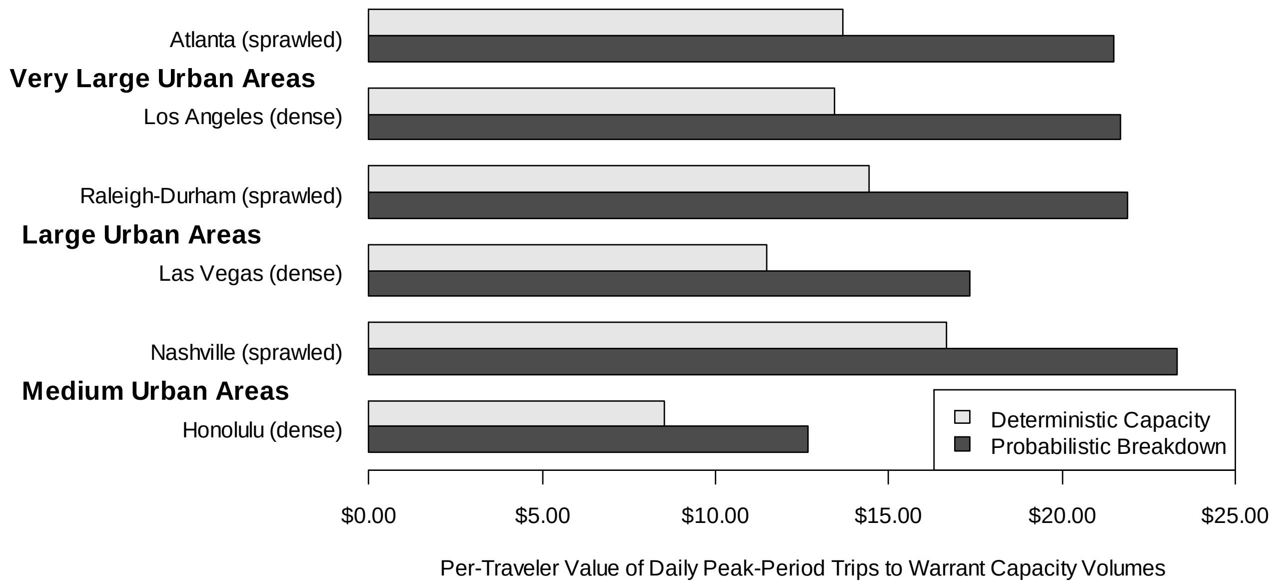


Figure 7. Comparison of peak period trip values that warrant capacity flows

Similarly, the β at which observed flows are optimal can be calculated (again as from Figure 5), and converted to travel values using average peak-period trip distances. Table 5 and Figure 8 show the results for each city for both deterministic and stochastic conditions. These trip values can be interpreted as the 'break-even' trip values, above which increasing freeway flows are still warranted, but below which the observed flows are inefficiently high.

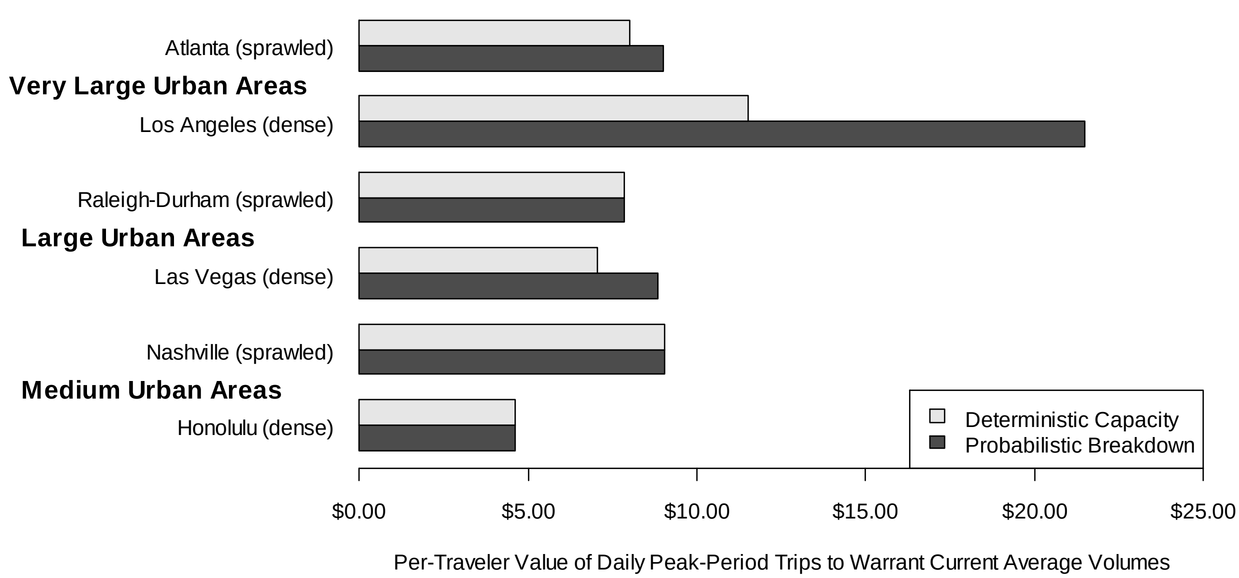


Figure 8. Comparison of peak period trip values that optimize existing freeway volumes

Less dense urban areas require higher trip values to warrant capacity flow since the trip lengths tend to be longer (which lowers value per mile). Conversely, for a given peak-period trip value denser areas will have higher optimal flow rates because the β value is larger. This is particularly true for mid-sized urban areas, since the larger the population the less of a difference in trip distance is observed for areas with different traveler densities.

Dense but smaller-sized urban areas have the lowest required trip value to optimize existing flows (Honolulu) since trips are short and congestion is light. Higher existing volumes increase marginal costs, and so increase the required trip values – particularly as flows approach capacity and the probability of breakdown increases. The denser Large and Very Large urban areas have markedly higher volumes than lower density areas of the same size (since travel is less spatially distributed), which increases the marginal costs of travel and can offset shorter trip lengths. Despite higher volumes, Las Vegas has comparable break-even trip values to the less dense urban areas because the average trip length is shorter. Finally, urban areas with low peak volumes (below 1,200 vphpl) have no observable difference between probabilistic and deterministic conditions, while those very near capacity (e.g. Los Angeles) are greatly affected by the uncertainty of breakdown conditions. The high likelihood of traffic flow breakdown combined with long trip lengths makes the break-even trip value for existing conditions in Los Angeles double that of any other urban area when considering stochastic capacity.

As stated above, these comparisons by urban area based on a set of loose assumptions. To see the sensitivity of these results, we varied these assumptions and the key parameters as indicated by Table 3. Using the Portland case study values for θ and v_b (both of which are lower) has no large impact on the results. Decreasing θ reduces stochastic costs at capacity and for high-volume areas, but decreasing v_b has the opposite (and here offsetting) effect. This suggests that these results are consistent with varying thresholds of congestion.

Increasing the roadway capacity (or scale parameter of the probability of breakdown function) reduces the stochastic costs for high-volume areas since they are less exposed to congestion or flow breakdown. On the other hand, assuming less homogenous flow distribution increases the costs for higher-volume urban areas. For example, assuming a 60/40 directional split increases the stochastic costs for Las Vegas and Atlanta by around 80%; the cost increase for Los Angeles is muffled since a 60/40 directional split puts the congested volumes over the assumed capacity, violating the model assumptions. Varying other parameters such as free-flow speed and cost coefficients has little to no effect on the comparison among urban areas.

The ratio of stochastic to deterministic costs for each urban area similarly increases with parameters that increase congestion penalties. For observed volumes the stochastic/deterministic cost ratio ranges from 1.0 for low-volume areas to 1.9 for Los Angeles. This ratio tends to increase with higher volumes (which themselves increase with traveler density).

Given the uncertainty in parameter estimation, the above results comparing urban areas are presented as conservative estimates, with the caveat that they are highly sensitive to assumptions about θ, v_b, and the directional split. The estimates are conservative in that they will tend to underestimate trip distances, peak volumes, and stochastic costs, as explained above. While the absolute cost estimates are highly sensitive to the assumptions, the same general trends in the results hold (though possibly magnified) for varying parameter values.

CONCLUSIONS

In this paper we model the costs and benefits of freeway traffic flows with stochastic freeway capacity. We apply this model to a congested freeway corridor in Portland, Oregon using real-world archived traffic data. Case study results show that unreliability decreases optimal traffic flow volume – and increases travel value that is required to warrant flow at capacity. Travel time is the dominant cost, followed by fuel costs; emissions costs are negligibly small at present valuations. Over a wide range of trip values, optimal flow remains at levels with a low probability of breakdown – indicating the importance of the breakdown probability function. A sensitivity analysis indicates that net social benefits and optimal flow are most sensitive to the travel time cost coefficient, the travel benefit coefficient, the free-flow speed, and the probability of breakdown. The value of reliability is most sensitive to the breakdown flow characteristics and the probability of breakdown function.

Comparing macroscopic peak-period traffic characteristics among urban areas of varying size and density, results indicate that lower density areas require higher trip values to warrant flow near capacity since average trip distances are longer. The large and dense urban areas have markedly higher flows which increase the marginal costs of travel and can offset shorter trip lengths when estimating net benefits. These results indicate that there is a trade-off between trip length and traffic intensity in urban areas with different density. The urban comparisons are conservative with respect to cost estimates and highly sensitive to several key assumptions. Still, the trends of the results hold across varying parameter values.

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