Bus Fleet Type and Age Replacement Optimization: A case study utilizing King County Metro fleet data

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Abstract:
Bus fleet data have consistently shown that vehicle operating and maintenance costs increase as vehicles age. A fleet manager has to deal with the tradeoff between the lower operating and maintenance costs of newer fleets and their higher initial capital costs as well as the tradeoff between conventional and fuel efficient bus technologies. This study formulates and implements a fleet replacement optimization framework that is applied to a case study that compares two bus types: a conventional diesel and a hybrid bus. Employing real-world bus fleet data from King County Metro (Washington State, USA) multiple scenarios are examined to account for uncertainty and variability in the model parameters. In addition sensitivity analyses are performed to study the impacts of parameter values on optimal replacement policies and the per-mile costs. Key findings include: the Federal Transit Administration (FTA) purchase cost subsidy has the highest impact on the optimal replacement policies; without FTA subsidy it is always cost effective to adopt diesel buses and replace them every 20 years. With an 80% purchase cost FTA subsidy, hybrid buses are the best choice; the optimal hybrid bus replacement cycle decreases from 18 to 14 years with increasing annual utilizations and operating and maintenance costs or decreasing hybrid purchase price and fuel economy. Fuel price, emissions costs, and initial bus age have little impact on optimal replacement policies. However, discount rate and diesel bus price, annual utilization (in 0% FTA subsidy scenario) and fuel price (in 80% FTA subsidy scenario) have the highest impacts on per-mile costs.

1 Introduction

Transit agencies typically own hundreds or thousands of buses, large transit agencies may have multiple fleets of buses with different types of buses serving different routes. For example, King County Metro (Washington State, USA) operates about 1,300 vehicles with multiple bus technologies (electric trolley, diesel, hybrid, etc.), designs (60ft articulate, 30ft or 40 ft standard, etc.) and models (New Flyer, Gillig, etc.). Large fleets' capital and operational costs are a significant expense for transit agencies. Transit agencies have to consider two important tradeoffs when making bus fleet replacement decisions. First, as buses age, the per-mile operating and maintenance (O&M) costs tend to increase. Replacing old vehicles with new ones reduces these costs but significantly increases capital costs. Therefore, there is an optimal replacement age that minimizes the total net cost over a planning time horizon. Second, vehicle purchase price, per-mile operating, maintenance and fuel costs vary across bus types (conventional diesel, hybrid), bus designs, and operating environments (congested or not congested routes, hilly or flat routes). There is an optimal bus type of all the candidates that minimizes total future net fleet costs. The problem can become significantly more complicated when replacement decisions have to be made on multiple fleets simultaneously, and when budget and demand constraints have to be considered.

In practice, many transit agencies replace their vehicles based on polices derived from rules of thumb (e.g. every 12 or 20 years), which may not be an optimal policy. From a pure economic perspective, to best address the two tradeoffs mentioned above, this paper proposed an optimization model that can provide optimal replacement decisions for fleets with any mixed bus types and ages under any budget or demand constraints. In this optimization model, five major cost components are considered: capital (purchase) cost, salvage revenue (negative cost), energy (fuel) cost, O&M costs, and emissions costs. The objective function of this model is to minimize the discounted sum of all five cost components for all buses that are considered over a planning time horizon. The decision variables are when and which buses should be replaced with what type of new buses. In other words, the optimal solution includes the optimal bus type of all candidates and optimal replacement cycle. In addition, once the optimal solutions are calculated, costs breakdown and bus usage statistics can be easily calculated accordingly.

The optimization model requires three types of inputs: economic factors, vehicular characteristics, and initial fleet composition. Economic factors include planning time horizon, annual number of vehicles (demand) or annual miles that have to be traveled, discount rate, energy price and forecast in the future (fuel, electricity). Vehicular factors include types of new bus candidates, and for each bus type, the maximum physical life, capital cost and its salvage value function over age, energy efficiency (fuel economy or electricity efficiency) as a function of age, O&M costs as a function of age, annual utilization (miles traveled) as a function of age. Initial fleet composition includes the numbers and ages of all bus types. Once all of the inputs are specified, the model can provide an optimal solution, together with cost breakdown and usage statistics.

In order to minimize the total fleet cost, transit agencies need to have an good understanding of bus performance and cost structure to provide accurate inputs for the optimization model. Although the model is able to provide an optimal solution for any given inputs, the stochasticity associated to forecasting some variables require the use of scenarios. A sensitivity analysis on input variables is also necessary to understand how optimal solutions vary with changes in the input variables. The optimization model and sensitivity analysis are illustrated in a case study comparing the 60ft New Flyer diesel and hybrid buses. Real world cost data were provided by King County Metro (Washington State, USA).

To the best of the authors' knowledge there is no published research that simultaneously analyzes real world bus fleet operating, maintenance, administrative and fuel costs, and models the impacts of various deterministic and stochastic cost components, vehicle utilization, emissions, market fluctuations, and USA government subsidies into a bus fleet replacement model. This paper will address these issues. The objectives of this research are to: 1) develop an optimization framework to minimize fleet total net cost with input, output and sensitivity analysis functions; 2) study the optimal replacement strategy for King County Metro (Washington State, USA) utilizing the model and real world data; 3) study the impacts of government subsidy policy, fuel price, fuel efficiency, vehicle utilization, O&M costs, emissions cost and initial fleet composition on the optimal replacement decisions, including the optimal bus candidate choice, optimal replacement cycle, and per-mile net cost.

The reminder of this paper is organized as follows: section two briefly introduces the background of bus replacement practices and replacement optimization models. Section three explains the model formulation. In section four, bus fleet data from King County Metro (Washington State, USA) and assumptions of the case study are stated. Section five presents the sensitivity analysis of different input variables on optimal replacement solutions and per-mile net cost. Finally, section six wraps up with conclusions.

2 Literature Review

Studies have shown how fuel efficiency, operating and maintenance costs change when vehicles age (Lammert, 2008; Chandler and Walkowicz, 2006; Schiavone, 1997). However, significant differences have been found across bus models, transit agencies and service environments. Bus life cycle costs and breakdown cost components have been previously compared across bus engine types and design models (Clark et al., 2007; Laver et al., 2007; Clark et al., 2009; Kim et al., 2009). However, these studies focus on vehicular characteristics and cost estimation for some fixed replacement cycles. Optimal replacement schedules and choice of bus type that aim at minimizing bus fleet's total net cost were not studied.

On the other hand, there is a good amount of literature on vehicle replacement optimization models in the operations research field. These models can be broken into two categories based on different fleet characteristics: homogeneous and heterogeneous models. In the former, the objective is to find the best policy in terms of replacement timing for a set of homogeneous vehicles, in other words, buses with the same type and age have to be replaced together (also known as "no cluster splitting rule"). These models are usually solved by the dynamic programming (DP) approach (Bellman, 1955; Oakford et al., 1984; Bean et al, 1985; Bean et al., 1994; Hartman, 2001; Hartman and Murphy, 2006), which has the advantage of integrating probabilistic distributions of input variables into the optimization model. Whereas heterogeneous models are more appropriate when multiple bus fleets have to be optimized simultaneously, or when budget constraints have to be considered. For example, buses with the same type and age may be replaced in different years due to restricted budget for purchasing new vehicles. These models are able to solve more practical problems, but input variables are deterministic and the model may be too complicated to solve when too many types of buses are considered. Integer programming (IP) models are usually used to solve such problems (Simms et al., 1984; Karabakal et al., 1994; Hartman, 1999; Hartman, 2000; Hartman, 2004) though a simplified DP approach was used with special assumptions in Jones et al. (1991). However, none of these theoretical models deal with real world fleet data. Keles and Hartman (2004) adopted an IP model in a transit fleet replacement problem with multiple types of buses. However, many cost functions were highly simplified or not based on real data, and variability in vehicular characteristics, utilizations, and market fluctuations were not studied. Fan et al. (2012) formulated and implemented a fleet optimization framework using DP approach; however sensitivity analysis of input variables was not addressed. Figliozzi et al. (2011) and Feng and Figliozzi (2012) adopted IP models to study a fleet of heterogeneous passenger cars and delivery trucks with real world operational data. Impacts of policy, market, utilization, emissions, and technological factors were analyzed using scenario analysis and elasticity analysis. Boudart and Figliozzi (2012) studied how various economic and vehicular factors affect a single bus's optimal replacement cycle.

3 Model Formulation

The optimization model is formulated as a deterministic model, which means all input variables are a known priori.

Indexes
Type of bus: $k \in \mathbf{K} = \{1, 2, \dots, K\}$.
Age of a bus type *k* in years: $i \in \mathbf{A}_k = \{0, 1, 2, \dots, A_k\}$,
Time periods: $j \in \mathbf{T} = \{0, 1, 2, \dots, T\}$, and

Decision Variables
$X_{ijk} =$ the number of i-year old, k-type buses used in year j,
$Y_{ijk} =$ the number of i-year old, k-type buses salvaged at the end of year j, and
$P_{jk} =$ the number of k-type buses purchased at the beginning of year j.

Parameters
(a) Constraints
$A_k =$ maximum age of bus type k (it must be salvaged when a bus reaches this age),
$u_{ik} =$ utilization (annual miles traveled by an i-year old, k-type bus),
$d_j =$ demand (miles traveled by all buses) in year j,
$b_j =$ budget (available for purchasing new buses) constraint in year j,

(b) Cost or revenue
$v_k =$ purchase cost of a k-type bus,
$f_{ik} =$ fuel economy (mpg) for an i-year old, k-type bus,
$fc_j =$ fuel price($/gallon) in year j,
$om_{ik} =$ per-mile operation and maintenance costs for an i-year old, k-type bus,
$s_{ik} =$ salvage revenue (negative cost) from selling an i-old, k-type bus,
$ec =$ emissions cost per ton of GHG,
$\delta =$ discount rate.

(c) Emissions
$em_{ik} =$ utilization emissions in GHG equivalent tons per mile for an i-year old, k-type bus, and

(d) Initial conditions
h_{ik} = the number of i-year old, k-type buses available at the beginning.

Objective Function, minimize:

$$\sum_{j=0}^{T-1}\sum_{k=1}^{K} v_k P_{jk}\,(1+\delta)^{-j} + \sum_{i=0}^{A_k-1}\sum_{j=0}^{T-1}\sum_{k=1}^{K}\left(\frac{fc_j u_{ik}}{f_{ik}}\right)X_{ijk}\,(1+\delta)^{-j} + \sum_{i=0}^{A_k-1}\sum_{j=0}^{T-1}\sum_{k=1}^{K} om_{ik}u_{ik}X_{ijk}\,(1+\delta)^{-j}$$
$$-\sum_{i=1}^{A_k}\sum_{j=0}^{T}\sum_{k=1}^{K} s_{ik}Y_{ijk}\,(1+\delta)^{-j} + \sum_{i=0}^{A_k-1}\sum_{j=0}^{T-1}\sum_{k=1}^{K} em_{ik}u_{ik}ecX_{ijk}\,(1+\delta)^{-j} \tag{1}$$

Subject to:

$$\sum_{k=1}^{K} v_{jk}\cdot P_{jk} \le b_j \forall j \in \{0,1,2,\dots,T-1\} \tag{2}$$

$$\sum_{i=0}^{A_k-1}\sum_{k=1}^{K} X_{ijk}\cdot u_{ik} \ge d_j \forall j \in \{0,1,2,\dots,T-1\} \tag{3}$$

$$P_{jk} = X_{0jk} \forall j \in \{1,2,\dots,T-1\} \forall k \in \mathbf{K} \tag{4}$$

$$P_{0k} + h_{0k} = X_{00k} \forall k \in \mathbf{K} \tag{5}$$

$$X_{i0k} + Y_{i0k} = h_{ik} \forall i \in \{1,2,\dots,A_k\}, \forall k \in \mathbf{K} \tag{6}$$

$$X_{(i-1)(j-1)k} = X_{ijk} + Y_{ijk} \forall i \in \{1,2,\dots,A_k\}, \forall j \in \{1,2,\dots,T\}, \forall k \in \mathbf{K} \tag{7}$$

$$X_{iTk} = 0 \quad \forall i \in \{0,1,2,\dots,A_k-1\} \forall k \in \mathbf{K} \tag{8}$$

$$X_{A_k jk} = 0 \quad \forall j \in \{0,1,2,\dots,T\} \forall k \in \mathbf{K} \tag{9}$$

$$Y_{0jk} = 0 \quad \forall j \in \{0,1,2,\dots,T\} \forall k \in \mathbf{K} \tag{10}$$

$$P_{jk}, X_{ijk}, Y_{ijk} \in \mathbf{I} = \{0,1,2,\dots\} \tag{11}$$

The objective function, expression (1), minimizes the sum of purchasing, energy (fuel) cost, O&M costs, salvage, and emissions costs over the period of analysis, i.e. from time zero (present) to the end of year T. Purchase costs cannot exceed the yearly budget, expression (2). The number of vehicles in the fleet at any time must equal or exceed the minimum needed to cover the demand in terms of annual number of buses or annual miles traveled, expression (3). The number of vehicles purchased must equal the number of new vehicles for each vehicle type and year, except for the current time, expression (4). The number of new vehicles utilized during year zero must equal the sum of existing new vehicles plus purchased vehicles, expression (5). Similarly, expression (6) ensures the conservation of vehicles (i.e., the initial vehicles—not 0-age ones—must be either used or sold). The age of any vehicle in use will increase by 1 year after each time period either be used or sold (7). At the end of the last time period, there will be no vehicle in use for any age or type of vehicles (i.e., all vehicles will be sold at the corresponding salvage value, which is a function of vehicle type and age) (8). When a vehicle reaches its allowable maximum age, a function of vehicle type, the vehicle must be sold at the corresponding salvage value (9). A newly purchased vehicle should not be sold before use (10). Finally, the decision variables associated with purchasing, utilization, and salvaging decisions must be integer non-negative numbers, expression (11).

4 Baseline Scenarios

To illustrate how this model can help transit agencies make optimal replacement decisions, real world bus data from King County Metro (Washington State, USA) were analyzed and incorporated into the model. King County Metro plans to replace some of their existing bus fleet in 2014, including 40 ft and 60 ft electric trolley buses, conventional diesel buses and hybrid diesel buses. King County Metro requested that the model provides insight into which bus should be chosen and what would be the expected replacement cycle. Data from King County Metro were analyzed and input into the optimization model described in the previous section to assist King County Metro to make optimal replacement decisions. The specification of model input variables are explained in the following three parts: economic factors, vehicular characteristics, and fleet initial compositions.

4.1 Economic factors

The baseline scenario economic factors are summarized in Table 1. A long planning time horizon of 100 years ($T = 100$) was used to unify the effect of the last incomplete vehicle life cycle. A 7.0% annual discount rate (APR) and 2.55% consumer price index (CPI) are assumed to be constant throughout the planning time horizon according to King County Metro's request, which yields a 9.55% nominal annual discount rate ($\delta = 9.55\%$). Three fuel price (diesel) forecast functions are utilized according to a recent report by King County (2011), where these three fuel cost forecast functions are obtained by combining the 2011 through 2015 fuel price forecast provided by Linwood Capital (2011) and long term petroleum projection provided by US EIA (2011). Fig. 1 shows the three fuel price forecast functions. The initial prices are \$2.64/gal (low), \$3.48/gal (mid) and \$4.46/gal (high) with a constant annual inflation rate of 2.6%, therefore, $fc_j = fc_0 \cdot (1 + 2.6\%)^j, \forall j \in \{1, 2, \dots, T\}$. Transit agencies usually purchase a group of buses in certain years instead of purchasing only a few buses annually, this is also true for King County Metro. In this case, budget constraint is not considered ($b_j = +\infty, \forall j \in \{0, 1, 2, \dots, T\}$). Emissions costs are not considered in the baseline scenarios but will be analyzed in the sensitivity analysis section later ($ec = 0$).

Table 1 Baseline scenario economic factors

Planning horizon	Nominal annual discount rate	Base Fuel price (\$/gal)			Fuel inflation rate	Emission cost (\$/ton)	Budget constraint
		Low	Mid	High			
100 years	9.55%	2.64	3.48	4.46	2.6%	0	No constraint

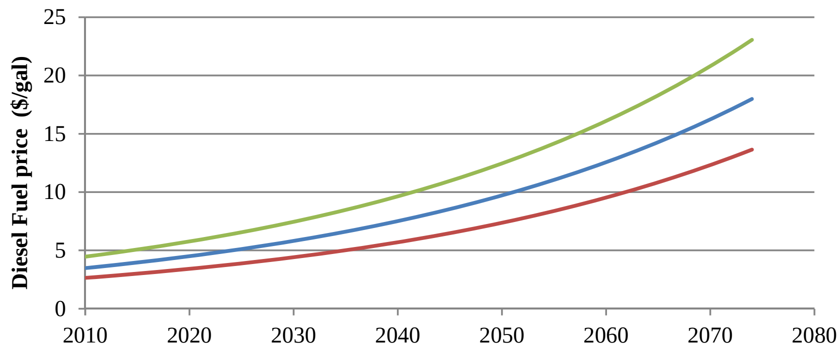


Fig. 1 Diesel fuel price forecast

4.2 Vehicular characteristics

For simplicity in reporting and comparing results, in this paper, only two bus technologies (types) are selected to replace existing buses: New Flyer 60ft hybrid diesel bus ($k = 1$) and New Flyer 60ft conventional diesel bus ($k = 2$). Detailed vehicular characteristics of the two bus types are summarized in Table 2.

Table 2 Baseline scenario vehicular characteristics

Bus type index	Bus type	Max age (years)	Purchase cost (\$)	Salvage value (\$) $i =$ age	Annual utilization (miles)	Fuel economy (mpg)	Per-mile O&M costs (\$/mile)	Tailpipe emissions (kg/mile)
$k = 1$	Hybrid	$A_1 = 20$	$v_1 = 958{,}000$	$s_{i1} = 1000$	$u_{i1} = 33{,}045$	$f_{i1} = 3.65$	$om_{i1} = 1.458 + 0.0661 \cdot i$	$em_{i1} = 2.504$
$k = 2$	Diesel	$A_2 = 20$	$v_2 = 737{,}000$	$s_{i2} = 1000$	$u_{i2} = 33{,}045$	$f_{i2} = 3.32$	$om_{i2} = 1.706 + 0.0463 \cdot i$	$em_{i2} = 3.407$

The maximum age is assumed to be 20 years for both buses ($A_1 = A_2 = 20$), because most transit agencies in the U.S. replace their buses in less than the 20-year cycle (Laver et al., 2007). The purchase costs for the two buses are $v_1 =$ \$958,000 for hybrid bus and $v_2 =$ \$737,000 for diesel bus, ordering costs and other related costs already included. Also, transit agencies can receive purchase subsidies from Federal Transit Administration (FTA) with additional stipulations that must be met, for example, if an 80% purchase cost subsidy is received, the bus must be kept for a minimum of 12 years. The salvage values for the two buses are assumed to be \$1,000 regardless of bus type or bus age according to King County Metro's request ($s_{ik} = \$1{,}000, \forall i \in \{1, 2, \dots, A_k\}, \ \forall k \in \mathbf{K}$). Because the two competing buses are going to serve the same bus routes, their annual utilizations (miles traveled) have to be equal, and this annual utilization does not vary with bus age ($u_{ik} =$ 33,045 miles/year according to current fleet data, $\forall i \in \{0, 1, 2, \dots, A_k - 1\}, \ \forall k \in \mathbf{K}$). The annual statistical data from King County Metro indicates that the hybrid bus fuel economy is 3.65 mpg and the diesel bus is 3.32 mpg on average if they were operated in the same existing routes, and fuel economy does not significantly vary with age, therefore, $f_{i1} = 3.65mpg$, $f_{i2} = 3.32mpg$,

$\forall i \in \{0, 1, 2, \dots, A_k - 1\}$. The per-mile O&M costs for the two bus type candidates vary significantly, the baseline scenario uses the per-mile O&M cost functions estimated by King County Metro (2011), $om_{i1} = 1.4580 + 0.0661 \times i$; $om_{i2} = 1.7060 + 0.0463 \times i$, $\forall i \in \{0, 1, 2, \dots, A_k - 1\}$, other O&M cost functions will be tested in the sensitivity analysis section. Only the tailpipe CO_2 emissions are considered into the model and the generation rates are 2.504 kg/mile for hybrid buses and 3.407 kg/mile for diesel buses according to Clark et al. (2007), therefore, $em_{i1} = 2.504\ kg/mi$, $em_{i2} = 3.407\ kg/mi, \forall i \in \mathbf{A}_k$.

4.3 Initial fleet Composition

According to King County Metro, in 2014, the existing buses will be replaced with new ones, therefore, these buses will be salvaged for certain by 2014, and their replacement cycles are not decision variables anymore. The initial fleet composition in year 2014 is equivalent to no initial buses ($h_{ik} = 0, \forall i \in \{0, 1, 2, \dots, A_k - 1\}$, $\forall k \in \mathbf{K}$). The problem thus becomes simple: which new bus type should King County Metro buy in 2014? The New Flyer 60ft hybrid bus or the New Flyer 60ft diesel bus? What will be the optimal replacement cycle? Also, because King County Metro assumes homogeneous bus fleet and no budget constraints, a group of buses that are purchased together have to be used and salvaged together. Therefore, instead of optimizing for the actual number of buses in a fleet, a constant number of buses is set to one ($d_j = 1, \forall j \in \{0, 1, 2, \dots, T - 1\}$), and results are presented on a per bus basis.

4.4 Baseline scenario results

The baseline scenarios include six scenarios: three fuel price functions and two levels of subsidies (0% and 80%), all other parameters are kept constant. The optimal replacement solutions for each of the six baseline scenarios are summarized and shown in Table 3.

Table 3 Baseline scenarios optimal replacement results

Purchase subsidy	0%			80%		
Fuel price	Low	Mid	High	Low	Mid	High
Discounted annualized costs						
Total cost ($)	20,574	21,874	23,238	13,173	14,369	15,774
Purchase cost ($)	8,788	8,788	8,797	2,495	2,495	2,495
Fuel cost ($)	4,134	5,450	6,975	3,762	4,959	6,355
O&M cost ($)	7,628	7,628	7,626	6,935	6,935	6,935
CO2 cost ($)	0	0	0	0	0	0
Salvage revenue ($)	-1	-3	-1	-3	-1	-3
Per-mile net cost ($/mile)	0.623	0.662	0.708	0.399	0.435	0.477
Not-discounted annualized costs						
Total cost ($)	226,464	263,339	303,368	186,837	221,985	263,245
Purchase cost ($)	39,060	39,060	41,270	11,495	11,495	11,495
Fuel cost ($)	116,739	153,883	192,387	110,530	145,699	186,730
O&M cost ($)	70,518	70,518	70,121	65,372	65,372	65,372
CO2 cost ($)	0	0	0	0	0	0
Salvage revenue ($)	-50	-60	-50	-60	-50	-60
Not-discounted per-mile cost ($/mile)	6.853	7.969	9.180	5.654	6.718	7.966
Annual fuel (gallons)	9,773	9,773	9,593	9,053	9,053	9,053
Annual CO2 (tons)	107	107	101	83	83	83
Annual miles	33,045	33,045	33,045	33,045	33,045	33,045
Hybrid replacement age	-	-	-	16	16	16
Diesel replacement age	20	20	20	-	-	-

The five cost components and their sum (total cost) are shown explicitly with both discounted annualized costs and not discounted annualized costs, the discounted and not discounted per-mile costs are also shown. Note that the discounted annualized costs are much smaller than the not discounted costs due to the long planning time horizon and high nominal discount rate. Also the cost breakdown of the five cost components are different between discounted and not discounted annualized costs because of the combined effects of discount rate, fuel inflation rate and planning time horizon. Recall the optimal solutions are solved to minimize the total discounted sum of all the cost components. The optimal replacement decisions are shown in the last two rows in Table 3. If no purchase cost subsidy is received, the optimal solution is to purchase diesel buses and replace every 20 years (maximum age), if 80% purchase cost subsidy can be received, the optimal solution switched to purchase hybrid buses and replace every 16 years. Note that these replacement ages shown in Table 3 are the first replacement cycle in the planning time horizon, further replacement bus type and age are not shown for simplicity, though they almost repeat the first cycle solutions. Results indicate that government subsidy affects the optimal replacement solution significantly. This is because when no subsidy is received, purchase cost dominates other cost components. The savings from lower fuel costs and O&M costs cannot compensate for the high purchase cost of a hybrid bus. On the other hand, if 80% purchase subsidy is received, the purchase cost drops significantly and savings in fuel cost and O&M costs from choosing hybrid buses outweigh their higher purchase cost. The subsidy affects the optimal replacement age in a similar way, no subsidy results in high capital cost in purchasing new buses and thus tend to extend replacement age, while low capital cost tends to reduce replacement cycle because the savings from operating newer buses may outweigh the low capital cost. Results also show that fuel price has no effect on the optimal replacement solution either with or without government subsidy.

5 Sensitivity Analysis

Although the model is able to provide the optimal solution given a set of input variables, the variability and uncertainty of the input variables requires additional sensitivity analysis to understand how optimal solutions are affected by changes in each of the input variables. Holding input variables in the baseline scenarios constant, we evaluate the effects of each input variable on the optimal replacement solution: optimal choice of bus type and replacement age, as well as per-mile net cost respectively. Section 5.1 analyzes the impacts of some key parameters on optimal bus type choice and replacement age by evaluating the relationship between each parameter and the optimal solution, and section 5.2 studies the impacts of each variable on the per-mile net cost by computing the elasticity, then section 5.3 calculates the breakeven values of several selected input variables that separate the preference of diesel and hybrid buses in the optimal solution.

5.1 Impacts of key parameters on optimal replacement policy

5.1.1. Fuel economy

According to the data provided by King County Metro, the 60ft New Flyer hybrid bus fuel economy varies slightly between 3.59 mpg and 3.65 mpg; however, the 60 ft New Flyer diesel bus fuel economy varies significantly between 2.49 mpg and 3.32 mpg. Therefore, to investigate the impact of relative fuel economies between diesel and hybrid buses, different fuel economies for both diesel and hybrid buses were tested within ranges that cover the observed fuel economy records. Sensitivity results are summarized in Table 4. Diesel bus fuel economy ranges from 2.5 mpg to 3.5 mpg with 0.1 mpg interval, hybrid buses fuel economy ranges from 3.15 mpg to 4.15 mpg with 0.1 mpg interval. Results are shown in Table 4.

Table 4 Impacts of diesel bus fuel economy on optimal replacement plan

Diesel bus fuel economy (mpg)	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5
Hybrid fuel economy: 3.65 mpg											
0% subsidy	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D
80% subsidy	16H	16H	16H	16H	16H	16H	16H	16H	16H	16H	16H
Hybrid bus fuel economy (mpg)	3.15	3.25	3.35	3.45	3.55	3.65	3.75	3.85	3.95	4.05	4.15
Diesel fuel economy: 3.32 mpg											
0% subsidy	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D
80% subsidy	17D	17D	17D	16H	16H	16H	16H	16H	16H	16H	16H

Table 4 shows how optimal replacement solutions change with varying diesel and hybrid bus fuel economies in both 0% and 80% subsidy scenarios. The number in the table, “16H” for example, indicates that the optimal solution is to choose a hybrid bus and replace it every 16 years. When hybrid bus fuel economy is held constant as 3.65 mpg, even if diesel bus fuel economy reduces to 2.5 mpg, the optimal solution is to choose a diesel bus and replace it every 20 years in the 0% subsidy scenario; this means that the savings of reduced fuel cost by using a hybrid cannot compensate for the additional capital cost of purchasing hybrid buses. In the 80% subsidy scenario, even if the diesel bus fuel economy increases to 3.5 mpg, the optimal solution is still to purchase a hybrid bus. When diesel bus fuel economy is held constant as 3.32 mpg, and hybrid bus fuel economy varies from 3.15 mpg to 4.15 mpg, the optimal solution is always to choose diesel bus and replace every 20 years in the 0% subsidy scenario. In 80% subsidy scenario, the optimal solution chooses hybrid bus and replacement every 16 years, if the hybrid bus fuel economy is higher than 3.45 mpg, however when hybrid bus fuel economy reduces to 3.35 mpg or below (or a 10% decrease), the optimal solution switched to diesel buses and replace every 17 years.

5.1.2. Annual utilization

Historical data provided by King County Metro indicated that the average annual utilization ranges between 28,379 miles and 39,679 miles per bus. Therefore, to investigate whether and how annual utilization affects the optimal replacement solutions, eleven different annual utilizations are tested from 28,379 miles/year/bus to 39,679 miles/year/bus with equal increment interval 1,130 miles/year/bus. Results are shown in Table 5.

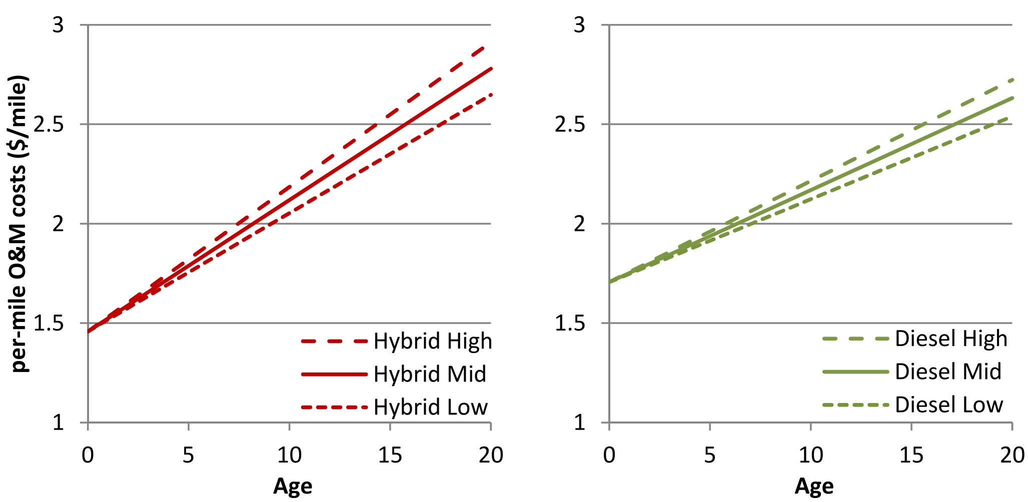
Table 5 Impacts of annual utilization on optimal replacement plan

Annual utilization (miles/year/bus)	28,379	29,509	30,639	31,769	32,899	34,029	35,159	36,289	37,419	38,549	39,679
0% subsidy	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D	20D
80% subsidy	18H	17H	17H	16H	16H	16H	15H	15H	15H	15H	14H

Results from Table 5 indicate a general trend that as annual utilization increases, hybrid buses are more favorable because savings from fuel cost and O&M costs increase. If there is no government subsidy, the optimal solution is always to buy diesel buses and replace them every 20 years regardless of annual utilization. If 80% purchase cost subsidy is received, the optimal bus candidate is always the hybrid bus. The optimal hybrid bus life cycle decreases with increasing annual utilization, because the additional O&M costs to operate older buses also increases along with annual utilization.

5.1.3. O&M Costs

Per-mile O&M costs as a function of age are the most difficult cost functions to estimate because of the high variance among buses and the lack of data for old buses (more than 12 years old). Therefore, average values for hybrid and diesel buses are used and linear increasing function extrapolations are assumed to predict the per-mile O&M costs as a function of age. The variance between buses is represented by creating two additional per-mile O&M costs functions that are lower and higher than their average functions. As shown in Fig. 2 (a) and (b), the solid lines represent the "Mid" functions, which are the baseline per-mile O&M costs functions. The two dashed lines represent "High" and "Low" per-mile O&M cost functions. The intercepts for the three functions are the same for each bus type, but the slopes of "Low" and "High" functions are 10% lower and higher than their "Mid" per-mile O&M costs function slopes. This generates nine scenarios. Each of the nine scenarios is tested with the model to investigate the impact of relative per-mile O&M cost functions on optimal replacement solution. Results are shown in Table 6.



(a) Hybrid bus (b) Diesel bus

Fig. 2 Per-mile O&M cost functions (a) Hybrid bus and (b) Diesel bus

Table 6 Impacts of per-mile O&M costs function slopes on optimal replacement plan

Hybrid slope	High	High	High	Mid	Mid	Mid	Low	Low	Low
Diesel slope	Low	Mid	High	Low	Mid	High	Low	Mid	High
0% subsidy	20D	20D	20D	20D	20D	20D	20D	20D	20H
80% subsidy	15H	15H	15H	16H	16H	16H	17H	17H	17H

Table 6 shows that without any purchase cost subsidy, the optimal replacement solution is to choose diesel buses and replace them every 20 years for all combinations except for one that the hybrid per-mile O&M cost function slope is low and the diesel per-mile O&M cost function slope is high, in which the optimal solution is to choose hybrid buses and replace them every 20 years. On the other hand, while 80% purchase cost subsidy is received, the optimal candidate is always the hybrid bus, and the optimal hybrid bus replacement cycle increases from 15 years to 17 years as the per-mile O&M cost function slope decreases. The results indicate that within these relative ranges of per-mile O&M cost function slopes, the relative slopes affect the optimal bus type choice but not the optimal replacement cycle in the 0% subsidy scenario. On the other hand, the relative slopes affect the optimal replacement cycle but not the optimal bus type in the 80% subsidy scenario.

5.1.4. Capital purchase cost

The capital costs of purchasing new buses may vary due to market fluctuations, technology improvements, and purchase quantity. It has also been shown in the baseline scenario results (Table 3) that purchase costs share a large percent of the total life cycle costs. Therefore, it is necessary to evaluate how sensitive the optimal replacement plan is in response to varying capital purchase costs. Up to 20% under and over the current purchase cost for diesel and hybrid buses in King County Metro are tested, and results are shown in Table 7.

Table 7 Impacts of capital purchase cost on optimal replacement plan

Capital cost percent change	-20%	-15%	-10%	-5%	0%	5%	10%	15%	20%
0% subsidy	20H	20H	20H	20D	20D	20D	20D	20D	20D
80% subsidy	14H	15H	15H	16H	16H	16H	17H	18H	18H

Results from Table 7 indicates that if no purchase cost subsidy can be received, when the purchase cost of both hybrid and diesel buses reduces 10% or more of their current prices, hybrid buses will be the best choice; otherwise diesel buses will be better. The optimal replacement cycles are equal to 20 years. If 80% subsidy can be received, the optimal bus candidate will always be hybrid buses, and the optimal replacement cycle increases with increasing capital costs.

5.1.5. CO_2 emissions

The CO_2 emissions costs are not considered in the baseline scenarios. In order to test whether CO2 emissions have a significant impact on optimal solutions, CO_2 emissions costs were added to the model objective functions. Two penalty costs for CO_2 emissions are tested: $30/ton as suggested by King County Metro, and $100/ton as a higher value to test if it has a significant impact on the optimal replacement solution. Results are shown in Table 8.

Table 8 shows that CO_2 emissions costs contribute a small part of total costs in all scenarios, optimal bus candidate and replacement cycle are the same as the baseline scenario where CO_2 emissions penalty costs are not considered. Results indicate that even with a high penalty cost ($100/ton) for CO_2 emissions, CO_2 has little impact on optimal replacement solution.

Table 8 Results after including CO_2 emissions costs

Subsidy	0%		80%	
CO_2 penalty cost ($/ton)	100	30	100	30
Discounted annualized				
Total cost ($)	23,161	22,253	15,317	14,672
Purchase cost ($)	8,797	8,797	2,495	2,495
Fuel cost ($)	5,443	5,443	4,959	4,959
O&M cost ($)	7,626	7,626	6,935	6,935
CO2 cost ($)	1,287	378	947	303
Salvage revenue ($)	-1	-1	-3	-3
Per-mile net cost ($/mile)	0.701	0.673	0.464	0.444
Not discounted annualized				
Total cost ($)	271,221	264,215	230,246	224,628
Purchase cost ($)	41,270	41,270	11,495	11,495
Fuel cost ($)	150,113	150,113	145,699	145,699
O&M cost ($)	70,121	70,121	65,372	65,372
CO_2 cost ($)	10,145	3,040	8,261	2,643
Salvage revenue ($)	-50	-50	-60	-60
Not discounted per-mile cost ($/mile)	8.208	7.996	6.968	6.798
Fuel (gallons)	9,953	9,953	9,053	9,053
CO_2 (tons)	112	112	82	82
Miles	33,045	33,045	33,045	33,045
Hybrid replacement age	-	-	16	16
Diesel replacement age	20	20	-	-

5.1.6. Initial age and bus type

The baseline scenarios assume that there are no existing buses; therefore, the initial fleet composition is always empty. However, it is interesting to evaluate some scenarios with an existing fleet of buses. Scenarios with different initial fleet compositions (types and ages) are also tested. The initial fleet composition is assumed to be one bus, hybrid or diesel bus, with any of the six ages (3,6,9,12,15,18), results for the 24 scenarios are shown in Table 9.

Results from Table 9 indicates that initial age has little impact on replacement age or optimal bus type. In the 0% subsidy scenario, if the initial bus is a hybrid, the optimal solution will continue to use the existing hybrid bus until it reaches age 20, and then replace it with a diesel bus. If the initial bus is diesel, the optimal solution will be to keep using that diesel bus and replace it every 20 years. In the 80% subsidy scenario, if the initial bus is hybrid, the optimal solution will be to keep using the hybrid bus until age 16. If the initial bus is diesel, the optimal solution will be to keep using the diesel bus until it reaches age 15 (or age 18 if the initial diesel bus age is 18), and then replace it with a hybrid bus every 16 years.

Table 9 Impacts of initial fleet composition on optimal replacement plan

Subsidy	0%						80%					
Initial age (Hybrid)	3	6	9	12	15	18	3	6	9	12	15	18
Hybrid replacement age	20	20	20	20	20	20	16	16	16	16	16	18
Diesel replacement age	20	20	20	20	20	20	-	-	-	-	-	-
Initial age (Diesel)	3	6	9	12	15	18	3	6	9	12	15	18
Hybrid replacement age	-	-	-	-	-	-	16	16	16	16	16	16
Diesel replacement age	20	20	20	20	20	20	15	15	15	15	15	18

5.2 Impacts of key parameters on per-mile costs

Section 5.1 focuses on the impacts of fuel economy, annual utilization, O&M costs, capital purchase costs, CO_2 emissions costs and initial age and bus type on the optimal replacement plan. It is also necessary to analyze which input variable has the highest impact on the optimal per-mile net cost. Elasticity of per-mile net cost to each of the above input factors was calculated using the following arc elasticity formula (12), where η_x^c is the elasticity of per-mile net cost *c* to parameter *x*:

$$\eta_x^c = \frac{(x_1 + x_2)/2}{(c_1 + c_2)/2} \cdot \frac{\Delta_c}{\Delta_x} = \frac{(x_1 + x_2)}{(c_1 + c_2)} \cdot \frac{(c_2 - c_1)}{(x_2 - x_1)} \tag{12}$$

Elasticity values and the evaluation range of each factor are summarized in Table 10. For example, with an annual utilization range between 28,379 miles/year/bus and 39,679 miles/year/bus, each additional 1% increase in annual utilization, decreases 0.41% per-mile net cost (0% subsidy scenario) or decreases 0.17% (80% subsidy scenario). Results show that nominal annual discount rate and planning time horizon have the highest absolute elasticity values in both 0% and 80% subsidy scenarios, followed by annual utilization in the 0% subsidy scenario, diesel bus price in the 0% subsidy scenario, and fuel price in the 80% scenario.

Table 10 Elasticity between various input variables and per-mile net cost

Factors	0% subsidy	80% subsidy
Vehicular Factors		
Diesel bus mpg (2.5 – 3.3)	-0.24	0.00
Hybrid bus mpg (3.15 – 4.15)	0.00	-0.26
Diesel bus O&M cost function slope ($0.0417/mi/year – $0.0509/mi/year)	0.06	0.00
Hybrid bus O&M cost function slope ($0.0595/mi/year – $0.0727/mi/year)	0.00	0.09
Diesel bus price ($589,600 – $737,000)	0.38	0.00
Hybrid bus price ($766,400 – $958,000)	0.13	0.17
General Factors		
Annual utilization (28,379 miles/year – 39,679 miles/year)	-0.41	-0.17
CO2 emissions penalty cost ($0/ton – $100/ton)	0.03	0.03
Fuel price ($2.64/gallon – $4.46/gallon)	0.25	0.35
Fuel inflation rate (0% – 5%)	0.09	0.13
Nominal annual discount rate (5% – 15%)	-0.85	-1.01
Planning time horizon (30 years – 100 years)	-0.95	-0.94

5.3 Breakeven analysis

All scenarios have consistently shown that it is more economical to buy diesel buses without government subsidy. However, with 80% purchase cost subsidy, the best option is to buy the hybrid bus. Thus, there is a breakeven value of the government subsidy larger than which the hybrid bus will be chosen and lower than which the diesel bus will be chosen. The breakeven subsidy values are calculated for the three fuel price scenarios. Results are shown in Table 11.

Table 11 Breakeven values of government subsidies

fuel price ($/gallon)	2.64	3.48	4.46
subsidy breakeven value	72%	69%	66%

For example, with the mid fuel price forecast functions (initial value $3.48/gal), it is more economical to buy a hybrid bus if the purchase cost subsidy is more than 69%. It is more economical to buy a diesel bus if the subsidy is less than 69%, with all other variables held constant as in the baseline scenario. Results show that higher fuel prices favor the hybrid bus though significant government subsidy (at least 66%) is required in all cases. Similarly, breakeven values for other input variables have been calculated for baseline scenarios with mid ($3.48/gal) fuel price, 0% and 80% subsidies. Results are summarized in Table 12 and Table 13.

Table 12 Breakeven values for 0% subsidy scenario

Factors	Baseline values	Breakeven values
Vehicular factors		
Diesel bus mpg	3.32	≤ 2.43
Hybrid bus mpg	3.65	≥ 6.16
Diesel bus per-mile O&M cost function slope	0.0436	≥ 0.1155
Hybrid bus per-mile O&M cost function slope	0.0661	≤ inf.
Diesel bus purchase cost ($)	737,000	≥ 882,784
Hybrid bus purchase cost ($)	958,000	≤ 812,215
General factors		
Annual utilization (miles/bus)	33,045	≥ 97,093
Fuel price ($/gal)	3.48	≥ 17.88
Fuel inflation rate	2.6%	≥ 20.9%
CO2 emissions penalty cost ($/ton)	0	≥ 506
Nominal annual discount rate	9.55%	≤ inf.
Planning time horizon (years)	100	≥ inf.

inf. means infeasible, there is no realistic value of the parameter that can change the optimal solution

Since in the baseline scenarios, diesel buses win without government subsidy, the breakeven values in Table 12 indicate when hybrid buses would win if any of the factors meet the condition. For example, with 0% subsidy, if the diesel bus fuel economy is less than or equal to 2.43 mpg compared to the hybrid bus baseline fuel economy of 3.65 mpg, or if the hybrid bus fuel economy is greater than or equal to 6.16 mpg compared to the diesel bus baseline fuel economy of 3.32 mpg, the optimal solution will be to choose the hybrid bus. If the annual utilization per bus is higher than 97,093 miles/year/bus (unrealistically high), it will be cost effective to adopt hybrid buses; however, even if the nominal annual discount rate is 0%, the planning time horizon is infinitely long, or hybrid bus O&M costs do not increase with age, the hybrid bus will not be chosen in the optimal solution.

Table 13 Breakeven values for 80% subsidy support scenario

Factors	Baseline values	Breakeven values
Vehicular factors		
Diesel bus mpg	3.32	≥ 3.60
Hybrid bus mpg	3.65	≤ 3.36
Diesel bus per-mile O&M cost function slope	0.0436	≤ 0.0299
Hybrid bus per-mile O&M cost function slope	0.0661	≥ 0.0852
Diesel bus purchase cost ($)	737,000	≤ 593,075
Hybrid bus purchase cost ($)	958,000	≥ 1107,625
General factors		
Annual utilization (miles/bus)	33,045	≤ 19,418
Fuel price ($/gal)	3.48	≤ inf.
Fuel inflation rate	2.6%	≤ inf.
CO2 emissions penalty cost ($/ton)	0	≤ inf.
Nominal annual discount rate	9.55%	≥ 27.25%
Planning time horizon (years)	100	≤ 2

Since hybrid buses win 80% purchase cost subsidy, the breakeven values indicate when diesel buses would win if any of the factors meet the condition. For example, if the diesel bus fuel economy is greater than or equal to 3.60 mpg compared to the hybrid bus baseline fuel economy of 3.65 mpg, or if the hybrid bus fuel economy is less than or equal to 3.36 mpg compared to the diesel bus baseline fuel economy of 3.32 mpg, the optimal solution will be to choose the diesel bus. If the annual utilization is smaller than 19,418 miles/year/bus, it will be cost effective to adopt diesel buses; however, even if fuel price, fuel inflation rate or CO_2 emissions penalty cost are as low as 0, the diesel bus will not be chosen.

In general, the breakeven values for those general factors shown in Table 12 and Table 13 are way too far from realistic values in either the 0% or 80% subsidy scenarios. Also, the breakeven values of the vehicular factors in the 0% subsidy scenario are far from the real world vehicle performance, but the breakeven values of vehicular factors in the 80% subsidy support scenario are likely to happen. The breakeven values above indicate to what extent, each factor itself can change optimal vehicle type. Many breakeven values are unrealistic or infeasible, which means the optimal solution for the baseline scenarios are stable. However, if multiple parameters change toward their breakeven values together, all of their breakeven values will change simultaneously towards more realistic values.

6 Conclusions

This research presented and applied a fleet replacement optimization model that minimizes, over a planning horizon, total fleet costs, including capital cost, salvage revenue, energy cost, O&M costs and emissions costs. This model can provide fleet managers an optimal replacement strategy, for both homogeneous and heterogeneous fleets, regarding when and which vehicles should be replaced and what type of vehicles should be purchased.

Detailed real world operational data from King County Metro were analyzed to provide more realistic model parameters. Multiple scenarios and sensitivity analysis of input variables were analyzed to deal with parameter uncertainty and variability. Case study results indicate that FTA bus purchase cost subsidy levels have the most significant impact on optimal bus type and replacement age policies. Without FTA subsidy, the bus capital cost dominates other cost elements and in almost all scenarios optimal policies select diesel buses that should be replaced every 20 years.

With a FTA 80% subsidy, almost all optimal solutions select hybrid buses except when a hybrid bus fuel economy is less than a diesel bus (which is rather unrealistic). In addition, with a FTA 80% subsidy and baseline conditions it was found that: i) fuel price (low, medium, high) and CO_2 emissions costs (up to $100/ton) have no impact on the optimal replacement policies, ii) hybrid buses are preferred if the fuel economy is 4% (or more) higher than diesel bus fuel economy, iii) higher utilizations (from 28,379 mi/year to 39,679 mi/year) decrease optimal replacement ages for hybrid buses (from 18 years to 14 years), iv) higher per-mile O&M costs function slopes decrease hybrid bus optimal replacement age (from 17 years to 15 years), and v) initial bus age has no impact on optimal replacement policies.

The breakeven analysis of government subsidies suggests that i) hybrid buses will not be selected by optimal policies unless the FTA subsidy is equal or greater than 66% of the bus purchase price, ii) the breakeven values that separate the optimal bus choice are not likely to occur in reality with a 0% FTA subsidy level, but iii) an 80% FTA subsidy greatly facilitates the competitiveness of hybrid buses even with unfavorable fuel prices, discount rates, O&M costs and purchase prices.

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