Proceedings of the 2011 Industrial Engineering Research Conference
T. Doolen and E. Van Aken, eds.

The Recharging Vehicle Routing Problem

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Abstract

This research introduces the recharging vehicle routing problem (RVRP), a new variant of the well-known vehicle routing problem (VRP) where vehicles with limited range are allowed to recharge at customer locations mid-tour. The problem has potential practical applications in real-world routing problems where electric vehicles with fast recharging capabilities may be used for less-than-truckload (LTL) deliveries in urban areas. The general problem is introduced as a capacitated problem (CRVRP) and a capacitated problem with customer time window constraints (CRVRP-TW). A problem statement is formulated and experimental results along with derived solution bounds are presented. Intuitive results are observed when the vehicle range is constrained and when recharging time is lengthy. It is also shown that the average tour length highly correlates with derived solution bounds. Estimations of the average tour length can be used in planning application to estimate energy costs and consumption as a function of vehicle and customer characteristics.

Keywords
Vehicle routing problem, freight logistics, electric vehicles, battery recharging

1. Introduction

This research introduces the recharging vehicle routing problem (RVRP) wherein vehicles with limited range must service a predetermined set of customers but may recharge at certain customer locations in order to continue a tour. The problem is introduced as an extension of the distance-constrained VRP (DCVRP) and theoretical bounds are derived to predict and test solution behavior.

This research has potential applications in the routing of electric vehicle (EV) fleets utilized for less-than-truckload (LTL) deliveries in urban areas. A routing scheme as proposed above could potentially ameliorate the inherent range limitations that put EVs at a disadvantage to standard internal combustion engine (ICE) delivery vehicles. The emergence of newer battery technologies shows the potential for mid-tour recharging in as little as 15 minutes [1]. Though current usage of EVs in LTL deliveries is limited, research conducted in parts of Europe has shown practical applications of EVs for short distance deliveries in dense urban areas [2, 3].

A generalized problem statement is presented that models vehicle range and energy consumption. Individual customer locations can also be designated as “rechargeable”, i.e. whether or not vehicle recharging is allowed at a particular customer location. The impacts of recharging time and vehicle range are ascertained to determine trends in specific fleet characteristics including fleet size, average tour distance, and total fleet distance traveled.

2. Literature Review

A brief summary of the relevant vehicle routing problem (VRP) research is provided. To the best of the authors’ knowledge, the RVRP is a new variant of the VRP that has not been proposed in the existing literature. The basic distance-constrained DCVRP has been extensively studied; Li et al. [4] established bounds on the average tour distance for the distance-constrained traveling salesman problem (TSP) along with computational complexity. Laporte et al. [5] present problems with distance and vehicle capacity constraints for Euclidean and non-Euclidean problems. More recently, Kek et al. [6] formulated a multi-depot capacitated DCVRP. To the best of the authors’ knowledge, no existing VRP formulations allow for vehicle recharging at customer locations. Ichimori et al. solved a truckload (TL) refueling problem with separate refueling stations [7], but do not extend this to LTL delivery.

3. Problem Statement

3.1 General Formulation

The RVRP follows a traditional flow-arc formulation [8]: let $G(V,A)$ be a graph with vertices $V=(v_0,v_1,\dots,v_{n+1})$ and arcs $A=\{(v_i,v_j): i\neq j \wedge i,j\in V\}$. The vertices v_0 and v_{n+1} denote the depot where a set of vehicles K with capacity $q_{\max}$ and charge capacity $f_{\max}$ are based. The vertices $C=(v_1,v_2,\dots,v_n)$ denote customer locations. Each vertex $v_i\in V$ is assigned a quintuplet $\{q_i,\tilde{r}_i,g_i,e_i,l_i\}$; $q_i\le q_{\max}$ represents the demand; $\tilde{r}_i\in\{1,0\}$ is a binary value that indicates whether vehicle recharging is allowed; g_i is the service time; and lastly e_i and l_i represent the earliest and latest services times. Specifically the depot is assigned the values $\{q_0,\tilde{r}_0,g_0,e_0,l_0\}=\{0,0,0,e_0\le e_i,l_0\ge l_i\}\ \forall i\in C$.

This problem contains three decision variables: x_{ij}^k is a binary decision variable that indicates whether a vehicle k travels between locations v_i and v_j; the binary decision variable r_i^k indicates whether a vehicle k recharges at customer i incurring a recharging service time penalty g_r before continuing its tour; lastly, the positive real variable y_i^k represents the service start time at customer i. The full problem statement is as follows:

$$minimize \sum_{k\in K}\sum_{j\in C} x_{0j}^k, \tag{1}$$

$$minimize\ c_d\sum_{k\in K}\sum_{(i,j)\in A} d_{ij}^k x_{ij}^k + c_t\sum_{k\in K}\sum_{j\in C}(y_{n+1}^k-y_0^k)x_{0j}^k + c_r\sum_{k\in K}\sum_{i,\in C} r_i^k \tag{2}$$

subject to:

$$\sum_{i\in C} q_i\sum_{j\in V} x_{ij}^k \le q_{\max}, \forall k\in K \tag{3}$$

$$\sum_{k\in K}\sum_{j\in V} x_{ij}^k = 1, \forall i\in C \tag{4}$$

$$\sum_{i\in V} x_{il}^k - \sum_{j\in V} x_{lj}^k = 0, \forall l\in C, \forall k\in K \tag{5}$$

$$x_{i0}^k = x_{n+1,i}^k = 0, \forall i\in C, \forall k\in K \tag{6}$$

$$\sum_{j\in C} x_{0j}^k = \sum_{j\in C} x_{j,n+1}^k = 1, \forall k\in K \tag{7}$$

$$r_i^k \le \tilde{r}_i, \forall i\in C, \forall k\in K \tag{8}$$

$$f_j^k = x_{ij}^k\left[(1-r_i^k)f_i^k + r_i^k\max[f_i^k+\gamma f_{\max}, f_{\max}] - F(v_i,v_j)\right], \forall i,j\in C, \forall k\in K \tag{9}$$

$$f_0^k = f_{\max}, \forall k\in K \tag{10}$$

$$f_i^k \ge 0, \forall i\in V, \forall k\in K \tag{11}$$

$$e_i \le y_i^k \le l_i, \forall i\in V, \forall k\in K \tag{12}$$

$$x_{ij}^k\left(y_i^k + \max[g_r,g_i] + t_{ij}\right) \le y_j^k\ \forall i,j\in V, \forall k\in K \tag{13}$$

Decision variables:

$$x_{ij}^k \in \{1,0\}, \forall (i,j)\in A, \forall k\in K \tag{14}$$

$$r_i^k \in \{1,0\}, \forall i\in C, \forall k\in K \tag{15}$$

$$y_i^k \in \Re, \forall i\in V, \forall k\in K \tag{16}$$

The primary objective is the minimization of the number of routes or vehicles (1); the secondary objective is the minimization of total costs related to the travel distance, service time and vehicle recharging represented by the cost functions c_d, c_t and c_r, respectively (2). The recharging time g_r is incorporated into the cost function c_r.

Constraints (3) through (7) are standard constraints on the capacitated VRP: vehicle capacity constraints cannot be exceeded (3); all customers must be served (4); if a vehicle arrives at a customer it must also depart from that customer (5); Routes must start and end at the depot (6); Each vehicle leaves from and returns to the depot exactly once (7).

Additional constraints specific to the RVRP are described as follows: vehicles can recharge only where recharging is allowed (8). The variable f_j^k keeps track of remaining energy in a given vehicle k upon arriving at customer j and is defined recursively by (9). The function $F(v_i,v_j)$ is an arbitrary function that represents the amount of energy required to travel a distance d_{ij}; without loss of generality $f_{\max} \geq 2F(v_i, v_j), \forall(i,j) \in A$ so that every customer can be feasibly served by the depot regardless of the feasibility of recharging at any given customer. The value $0 < \gamma \leq 1$ indicates the percentage of the charge capacity that can be obtained via a "quick charge" at customer locations, i.e. $\gamma = .8$ indicates that a vehicle can recharge to 80% of its charge capacity at customer locations. Vehicles always leave the depot with a full charge (10). The charge level upon arriving at a customer location or returning to the depot must be a non-negative value (11). Constraint (12) requires that service start times be within customer time windows inclusive of the depot working time $[e_0, l_0]$. Service start times must also allow for travel time between customers (13) as well as servicing and recharging. Recharging is allowed while servicing customers, but vehicles must remain at customer locations until servicing is complete.

Decision variables for the problem are given by (14), (15) and (16).

3.2 Proposed Problems

For the sensitivity analysis provided in section 4, two distance-constrained problems are proposed. The first is a capacitated problem with relaxed customer time windows and is denoted the CRVRP; the second problem introduces hard customer time windows and is denoted the CRVRP-TW. Both problems assign a maximum range λ that each vehicle can travel before either recharging or a return trip to the depot is required. Therefore $f_{\max} = \lambda$ and $F(v_i, v_j) = d_{ij}$. Customer have identical assigned service times $g_i = g, \forall i \in C$ and unique demands $q_i < q_{\max}$. Vehicles are allowed to recharge at all customer locations such that $\tilde{r}_i = 1, \forall i \in C$ and obtain a full charge upon recharging ($\gamma = 1$). Lastly, the travel arcs $(v_i, v_j) \in A$ are assigned identical travel speeds $t_{ij} = \alpha$.

4. Experimental Results

The Solomon problems [9] for the standard capacitated VRP with time windows (CVRP-TW) are utilized to derive experimental instances. The Solomon distribution of time windows is assigned to each customer location in the CRVRP-TW and is relaxed in the CRVRP with $e_i = e_0, l_i = l_0, \forall i \in C$.

The proposed problems are tested on 30 sets of 40 customers. The problem sets are generated by taking random selections from the R101 Solomon benchmark problem set. The problem sets are then tested on pairs of values $\left[\lambda, \frac{g_r}{l_0 - e_0}\right], \lambda \in \{70,55, \ldots, 120\}, g_r \in \{15,20, \ldots, 35\}$ for, respectively, the vehicle range and recharging time expressed as a percentage of the depot working time $l_0 - e_0 = 230$. Customers are also assigned service times $g = 10$ and original Solomon customer demands with $q_{\max}$ =100.

A heuristic based on an iterative construction and improvement algorithm for the capacitated VRP with time windows (CVRP-TW) [10] is employed to obtain the solutions to the RVRP. The details of the new algorithm are not included in this paper due to length limitations.

Results for the CRVRP are provided in Table 1 arranged, respectively, as average fleet size, average fleet distance traveled and average number of vehicle recharges per tour. The upper right corner represents the most constraining conditions with short vehicle range and a large time penalty for each recharging. As such, the average total fleet distance and average fleet size are greatest under these conditions. Conversely, the least restrictive conditions (lower left-hand corner) show the smallest average fleet size and distance traveled. The average number of vehicle recharges also has a near-consistent increasing trend as vehicle range is constrained; trends with increasing recharging time are less clear as the need to recharge more frequently competes with maintaining tour duration feasibility.

Table 1: CRVRP results [Avg. fleet size, Avg. Avg. fleet distance traveled, Avg. No. of vehicle recharges]

		Time Penalty per Vehicle Recharge (% depot working time)				
		7%	9%	11%	13%	15%
Vehicle Range	70	4.83, 517, 1.11	4.87, 521, 1.32	4.97, 524, 1.28	5.10, 529, 1.16	5.23, 534, 1.17
	75	4.80, 518, 1.08	4.87, 517, 1.21	4.97, 528, 1.24	5.03, 528, 1.16	5.13, 537, 1.10
	80	4.77, 517, 1.04	4.90, 518, 1.16	4.97, 527, 1.20	5.00, 527, 1.14	5.03, 528, 1.09
	85	4.77, 517, 1.01	4.90, 518, 1.08	4.97, 523, 1.01	5.00, 527, 1.06	5.00, 528, 1.03
	90	4.73, 517, 0.99	4.90, 521, 0.99	4.97, 525, 0.96	5.00, 530, 0.96	5.00, 527, 0.97
	95	4.73, 517, 0.94	4.87, 519, 0.95	4.97, 526, 0.94	5.00, 529, 0.91	5.00, 531, 0.91
	100	4.73, 517, 0.84	4.83, 519, 0.86	4.93, 524, 0.81	4.97, 531, 0.81	5.00, 531, 0.83
	105	4.73, 517, 0.69	4.80, 516, 0.78	4.97, 520, 0.72	4.97, 522, 0.74	4.97, 529, 0.64
	110	4.67, 516, 0.64	4.83, 518, 0.63	4.90, 519, 0.68	4.93, 523, 0.58	4.93, 530, 0.57
	115	4.67, 516, 0.50	4.87, 517, 0.55	4.90, 517, 0.63	4.90, 519, 0.64	4.97, 524, 0.55
	120	4.67, 516, 0.49	4.80, 515, 0.59	4.77, 516, 0.48	4.87, 518, 0.51	4.87, 518, 0.51

Table 2 provides similar results for the CRVRP-TW. The most noticeable difference is a drastic increase in fleet size and fleet distance traveled compared to the CRVRP. The addition of time windows also impacts recharging feasibility as additional service time at a given customer location can greatly reduce the number of feasible candidate customers that can be added to a route while forcing late departures can limit tour length.

Table 2: CRVRP-TW results [Avg. fleet size, Avg. fleet distance traveled, Avg. No. of vehicle recharges]

		Time Required to Recharge (% depot working time)				
		7%	9%	11%	13%	15%
Vehicle Range	70	12.70, 974, 0.72	12.50, 966, 0.86	12.67, 980, 0.81	12.87, 974, 0.76	13.20, 982, 0.72
	75	12.63, 974, 0.67	12.50, 965, 0.79	12.57, 961, 0.76	12.80, 964, 0.71	13.07, 979, 0.69
	80	12.57, 974, 0.60	12.53, 973, 0.74	12.53, 969, 0.70	12.70, 974, 0.65	13.00, 983, 0.58
	85	12.53, 975, 0.54	12.50, 976, 0.69	12.63, 976, 0.66	12.63, 976, 0.61	12.70, 975, 0.53
	90	12.50, 970, 0.46	12.50, 973, 0.61	12.57, 971, 0.58	12.60, 974, 0.55	12.63, 973, 0.44
	95	12.47, 970, 0.40	12.47, 973, 0.56	12.53, 969, 0.56	12.53, 969, 0.50	12.60, 973, 0.39
	100	12.47, 971, 0.34	12.50, 979, 0.51	12.53, 971, 0.52	12.53, 968, 0.44	12.53, 972, 0.36
	105	12.50, 971, 0.31	12.57, 973, 0.47	12.40, 966, 0.51	12.53, 968, 0.39	12.60, 974, 0.32
	110	12.53, 972, 0.30	12.57, 970, 0.46	12.47, 968, 0.46	12.53, 967, 0.36	12.63, 973, 0.31
	115	12.53, 974, 0.27	12.57, 971, 0.45	12.50, 967, 0.45	12.53, 967, 0.34	12.60, 973, 0.30
	120	12.50, 973, 0.26	12.53, 971, 0.44	12.47, 967, 0.43	12.50, 967, 0.33	12.57, 972, 0.30

5. Solution Bounds

In this section solution bounds on the average tour distance are derived. These bounds are later used in Section 6 to predict average tour distance as a function of vehicle and customer characteristics using regression analysis.

5.1 CRVRP Solution Bounds

The means of establishing the lower solution bound for the CRVRP that relates the minimum fleet size k^* and the minimum fleet distance traveled L^* follows an approach extending the work by Li et al. [4] for the DCVRP. With $g_r \geq g_i = g$, the quantity $\max[g_r, g_i]$ can be written as $r_i^k g_r + (1 - r_i^k)g$. When depot working time is the binding constraint on tour duration and route minimization is the primary objective, the following constraint holds for any two constructed routes:

$$\left(\sum_{(i,j)\in A} x_{ij}^{k} t_{ij} + \sum_{i\in C}\left(r_i^k g_r + \left(1-r_i^k\right)g\right)\right) + \left(\sum_{(i,j)\in A} x_{ij}^{k+1} t_{ij} + \sum_{i\in C}\left(r_i^{k+1} g_r + \left(1-r_i^{k+1}\right)g\right)\right) > l_0 - e_0 \quad \forall\, k \in K = \{1,2,\dots,k,\dots,k^*\} \tag{17}$$

where k^* is the minimum number of vehicles with $k^*+1 \equiv 1$. If expression (17) did not hold for a given pair of routes $\{k, k+1\}$, the routes could be combined and the number of vehicles k^* could be reduced by one. Summing over all $k \in K$ yields

$$\begin{aligned}
&\sum_{k\in K}\left[\left(\sum_{(i,j)\in A} x_{ij}^{k} t_{ij} + \sum_{i\in C}\left(r_i^k g_r + \left(1-r_i^k\right)g\right)\right) + \left(\sum_{(i,j)\in A} x_{ij}^{k+1} t_{ij} + \sum_{i\in C}\left(r_i^{k+1} g_r + \left(1-r_i^{k+1}\right)g\right)\right)\right] \\
&= \left[\left(\sum_{(i,j)\in A} x_{ij}^{1} t_{ij} + \sum_{i\in C}(r_i^1 g_r + (1-r_i^1)g)\right) + \left(\sum_{(i,j)\in A} x_{ij}^{2} t_{ij} + \sum_{i\in C}(r_i^2 g_r + (1-r_i^2)g)\right)\right] + \\
&\left[\left(\sum_{(i,j)\in A} x_{ij}^{2} t_{ij} + \sum_{i\in C}(r_i^2 g_r + (1-r_i^2)g)\right) + \left(\sum_{(i,j)\in A} x_{ij}^{3} t_{ij} + \sum_{i\in C}(r_i^3 g_r + (1-r_i^3)g)\right)\right] + \dots + \\
&\left[\left(\sum_{(i,j)\in A} x_{ij}^{k^*-1} t_{ij} + \sum_{i\in C}\left(r_i^{k^*-1} g_r + \left(1-r_i^{k^*-1}\right)g\right)\right) + \left(\sum_{(i,j)\in A} x_{ij}^{k^*} t_{ij} + \sum_{i\in C}\left(r_i^{k^*} g_r + \left(1-r_i^{k^*}\right)g\right)\right)\right] + \\
&\left[\left(\sum_{(i,j)\in A} x_{ij}^{k^*} t_{ij} + \sum_{i\in C}\left(r_i^{k^*} g_r + \left(1-r_i^{k^*}\right)g\right)\right) + \left(\sum_{(i,j)\in A} x_{ij}^{1} t_{ij} + \sum_{i\in C}(r_i^1 g_r + (1-r_i^1)g)\right)\right] \\
&= 2\left(\sum_{(i,j)\in A} x_{ij}^{1} t_{ij} + \sum_{i\in C}(r_i^1 g_r + (1-r_i^1)g)\right) + 2\left(\sum_{(i,j)\in A} x_{ij}^{2} t_{ij} + \sum_{i\in C}(r_i^2 g_r + (1-r_i^2)g)\right) + \dots + \\
&2\left(\sum_{(i,j)\in A} x_{ij}^{k^*} t_{ij} + \sum_{i\in C}\left(r_i^{k^*} g_r + \left(1-r_i^{k^*}\right)g\right)\right) \\
&= 2\left[T^* + \sum_{k\in K}\sum_{i\in C}\left(r_i^k g_r + \left(1-r_i^k\right)g\right)\right] = 2\left[T^* + (g_r - g)\sum_{k\in K}\sum_{i\in C} r_i^k + ng\right] > k^*(l_0 - e_0)
\end{aligned} \tag{18}$$

where $T^* = \sum_{k\in K}\sum_{(i,j)\in A} x_{ij}^k t_{ij}$ is the minimum total travel time and n is the total number of customers. Vehicle capacity imposes the constraint

$$k^* q_{\max} \geq \sum_{k\in K}\sum_{i\in C} q_i \sum_{j\in V} x_{ij}^k = n \times \frac{\sum_{k\in K}\sum_{i\in C} q_i \sum_{j\in V} x_{ij}^k}{n} = n\bar{q} \tag{19}$$

where $\bar{q}$ is the average customer demand. Because vehicles leave the depot fully charged, the travel distance of each tour has a lower bound $\sum_{(i,j)\in A} x_{ij}^k d_{ij} > \lambda \sum_{i\in C} r_i^k \;\forall k \in K$; summing over all k and rearranging:

$$\sum_{k\in K}\sum_{i\in C} r_i^k < \frac{1}{\lambda}\sum_{k\in K}\sum_{(i,j)\in A} x_{ij}^k d_{ij} = \frac{1}{\lambda} L^*. \tag{20}$$

Combining expressions (18), (19) and (20) and substituting $T^* = \frac{1}{\alpha} L^*$ and the average tour distance $\bar{L} \equiv \frac{L^*}{k^*}$ yields

$$\begin{gathered}
2\left[\frac{1}{\alpha} L^* + \frac{g_r - g}{\lambda} L^* + k^* g \frac{q_{\max}}{\bar{q}}\right] > k^*(l_0 - e_0), \\
\bar{L} > \frac{\frac{1}{2}(l_0 - e_0) - 2g\frac{q_{\max}}{\bar{q}}}{\frac{1}{\alpha} + \frac{g_r - g}{\lambda}}.
\end{gathered} \tag{21}$$

To establish the upper bound, the constraint on tour duration can be extracted from constraints (12) and (13) and expressed as

$$l_0 - e_0 \geq \frac{1}{\alpha}\sum_{(i,j)\in A} x_{ij}^k d_{ij} + \sum_{i\in C}\left(r_i^k g_r + \left(1-r_i^k\right)g_i\right) \forall k \in K. \tag{22}$$

Summing over all $k \in K$ gives

$$k^*(l_0 - e_0) \geq \frac{1}{\alpha} \sum_{k\in K} \sum_{(i,j)\in A} x_{ij}^k d_{ij} + \sum_{k\in K} \sum_{i\in C} \left(r_i^k g_r + \left(1 - r_i^k\right) g\right)$$
$$= \frac{1}{\alpha} L^* + (g_r - g) \sum_{k\in K} \sum_{i\in C} r_i^k + ng. \qquad (23)$$

Tour distance has an upper bound such that $\sum_{(i,j)\in A} x_{ij}^k d_{ij} \leq \lambda + \lambda \sum_{i\in C} r_i^k \; \forall k \in K$; summing over all k and rearranging:

$$\sum_{k\in K} \sum_{i\in C} r_i^k \geq \frac{1}{\lambda} L^* - k^*. \qquad (24)$$

The number of vehicles is at most equal to the number of customers imposing $n \geq k^*$; combining this and expressions (23) and (24) along with the lower solution bound gives

$$k^*(l_0 - e_0) \geq (g_r - g)\left(\frac{1}{\lambda} L^* - k^*\right) + \frac{1}{\alpha} L^* + k^* g,$$
$$\frac{\frac{1}{2}(l_0 - e_0) - 2g \frac{q_{\max}}{\bar{q}}}{\frac{1}{\alpha} + \frac{g_r - g}{\lambda}} < \bar{L} \leq \left(\frac{l_0 - e_0 + g_r - 2g}{\frac{1}{\alpha} + \frac{g_r - g}{\lambda}}\right). \qquad (25)$$

5.2 CRVRP-TW Solution Bounds

The upper bound on $\bar{L}$ for the CRVRP-TW is obtained by exploiting properties of the customer time window constraints. Consider a feasible tour k with m customers. In order for the first customer to be feasibly connected, the travel time must not exceed the latest possible service time such that

$$x_{01}^k t_{01} \leq l_1 - e_0.$$

A connection between the first and second customer must also account for the service time and the possibility of recharging at the first customer:

$$x_{12}^k t_{12} + (1 - r_1^k) g + r_1^k g_r \leq l_2 - e_1.$$

Generalizing these relations for m customers:

$$x_{23}^k t_{23} + (1 - r_2^k) g + r_2^k g_r \leq l_3 - e_2,$$
$$\vdots$$
$$x_{m-1,m}^k t_{m-1,m} + (1 - r_{m-1}^k) g + r_{m-1}^k g_r \leq l_m - e_{m-1},$$
$$x_{m0}^k t_{m0} + (1 - r_m^k) g + r_m^k g_r \leq l_0 - e_m.$$

The above inequalities can be combined into a single expression such that

$$\sum_{(i,j)\in A} x_{ij}^k t_{ij} + \sum_{i\in C} \left[\left(1 - r_i^k\right) g + r_i^k g_r\right] \leq (l_0 - e_0) + \sum_{i\in C} (l_i - e_i). \qquad (26)$$

Summing expression (26) for k^* routes:

$$\sum_{k\in K} \sum_{(i,j)\in A} x_{ij}^k t_{ij} + \sum_{k\in K} \sum_{i\in C} \left[\left(1 - r_i^k\right) g + r_i^k g_r\right] \leq k^*(l_0 - e_0) + \sum_{k\in K} \sum_{i\in C} (l_i - e_i).$$

Defining $\bar{w} \equiv \frac{\sum_{k\in K} \sum_{i\in C} (l_i - e_i)}{n}$ as the average customer time window width and substituting expressions (19) and (24) and $n \geq k^*$ gives

$$\frac{1}{\alpha} L^* + (g_r - g)\left(\frac{1}{\lambda} L^* - k^*\right) + k^* g \leq k^*(l_0 - e_0) + k^* \frac{q_{\max}}{\bar{q}} \bar{w},$$
$$\bar{L} \leq \frac{l_0 - e_0 + \frac{q_{\max}}{\bar{q}} \bar{w} + g_r - 2g}{\frac{1}{\alpha} + \frac{g_r - g}{\lambda}}. \qquad (27)$$

To establish the lower bound on $\bar{L}$, the worst possible tour must be considered under highly constraining customer time windows. Since every customer can be feasibly served from the depot, the worst case is each route consisting of one customer such that

$$L^* \geq k^* \left(\sum_{l \in C} x_{0l} d_{0l} + \sum_{l \in C} x_{l0} d_{l0} \right)$$

For Euclidean distances $\sum_{l \in C} x_{0l} d_{0l} = \sum_{l \in C} x_{l0} d_{l0} \; \forall l \in C$ and the lower bound can be written as

$$\bar{L} \geq 2\bar{r} \tag{28}$$

where $\bar{r}$ is the average customer-depot connecting distance. The bounds on $\bar{L}$ for the CRVRP-TW therefore are

$$2\bar{r} \leq \bar{L} \leq \frac{l_0 - e_0 + \frac{q_{\max}}{\bar{q}}\bar{w} + g_r - 2g}{\frac{1}{\alpha} + \frac{g_r - g}{\lambda}} \tag{29}$$

6. Predicting Average Tour Length

Energy consumption/costs and distance traveled are directly correlated. The estimation of the average distance a fleet of vehicles will need to travel is an important measure with a wide range of applications in fleet planning, facility siting, network design and other logistics problems [11]. A comparison of the trends in fleet characteristic averages with the solution bounds derived in sections 5.1 and 5.2 suggest a strong correlation between the results obtained and theoretical analysis. This is tested by modeling average tour length for both the CRVRP and CRVRP-TW as functions of the bounds. More specifically, the behavior of the observed average tour distance is modeled as a linear function of an expectation value $E[\bar{L}] = \beta_1 \frac{\bar{L}_-}{2} + \beta_2 \frac{\bar{L}_+}{2}$ where $\bar{L}_-$and $\bar{L}_+$ are the lower and upper bounds, respectively and β_1 and β_2 are parameters estimated by regression. The general equations for estimating the average tour distance for the CRVRP and CRVRP-TW, respectively, are:

$$\bar{L}_{CRVRP} \approx \beta_1 \frac{\frac{1}{2}(l_0 - e_0) - g\frac{q_{\max}}{\bar{q}}}{2\left(\frac{1}{\alpha} + \frac{g_r - g}{\lambda}\right)} + \beta_2 \frac{l_0 - e_0 + g_r - 2g}{2\left(\frac{1}{\alpha} + \frac{g_r - g}{\lambda}\right)}, \tag{30}$$

$$\bar{L}_{CRVRP-TW} \approx \beta_1 \frac{l_0 - e_0 + \frac{q_{\max}}{\bar{q}}\bar{w} + g_r - 2g}{2\left(\frac{1}{\alpha} + \frac{g_r - g}{\lambda}\right)} + \beta_2 \bar{r}. \tag{31}$$

Estimations are obtained by pooling the results from the 30 customer instances tested in the previous section. Each customer distribution has a unique average demand $\bar{q}$. The time window widths among all customers are uniform, but are randomly distributed. Table 3 gives estimated parameters and regression statistics for the models tested. In addition to the R^2 value, mean percentage error (MPE) and mean absolute percentage error (MAPE) are given for the estimated models. The MPE determines whether a model under or overestimates the predicted value while the MAPE gives the average deviation of the predicted value from the observed value.

Overall the models have a very high R^2 value and low MPE and MAPE. Because the second estimated parameter in expression (31) was not quite statistically significant at a 95% confidence interval, two additional models were tested. The second model estimates a single coefficient for the expectation value and a third model considers only the relationship with the upper bound. Though the statistical significance improves, correlation and approximation quality suffer somewhat. Overall a tighter and more robust estimation of the lower solution bound would likely allow for better overall estimation.

Table 3: Regression statistics for the estimation of the average tour length

	CRVRP	CRVRP-TW		
Formula	$\beta_1 \frac{\bar{L}_-}{2} + \beta_2 \frac{\bar{L}_+}{2}$	$\beta_1 \frac{\bar{L}_-}{2} + \beta_2 \frac{\bar{L}_+}{2}$	$\beta_1 \left(\frac{\bar{L}_-}{2} + \frac{\bar{L}_+}{2} \right)$	$\beta_1 L_+$
β_1	0.47	0.08	0.52	0.29
t-stat	1.98	2.59	3.30	3.45
β_2	2.16	2.75		
t-stat	2.29	1.94		
R^2	0.997	0.997	0.993	0.992
MPE	-0.24%	-0.34%	-0.22%	-0.14%
MAPE	4.66%	4.55%	6.95%	7.37%
N	1980			

7. Conclusion

This research presents the first formulations of the RVRP. Initial sensitivity analysis shows intuitive behavior for solution characteristics such as fleet size and fleet distance traveled. The impacts of customer time windows are also shown to greatly limit tour distance when vehicle range is constrained and recharging time is long. Lastly, derived solution bounds have proved to be useful for predicting average tour length with reasonable statistical significance and high approximation quality. Future research will focus on expanding the problem to better model vehicle energy consumption and variable customer demands and constraints.

Acknowledgments

The authors extend their sincere thanks to the Oregon Transportation, Research and Education Consortium (OTREC) for sponsoring this research. The authors also thank Myeonwoo Lim, Computer Science Department at Portland State University, for assisting with the computer programming aspects of this project.

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