**EE 520: Random Processes** **Fall 2021**

# Exercises 16

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The below are in-class exercises designed to help solidify your understanding of the material covered in the notes. They will also aid you in completing some homework problems. Please work together with your group to complete as many of these problems as you can.

PN refers to the online textbook by Pishro-Nik available here. Please do not look at the solutions until after you have completed the problem or received hints from me.

**Exercise 1**

Let $X_m$, $m \geq 0$ be a Markov chain. Determine whether $Y_m = X_{2m}$ is also a Markov chain.

**Exercise 2**

Let $Z_k$ be a RP such that for odd values of $k$, $Z_k \overset{\text{i.i.d.}}{\sim} \text{SymBer}(1/2)$, i.e., each $Z_k$ takes values in $\pm 1$ with equal probability, and for even values of $k$, we have $Z_{2k} = Z_{2k-1} Z_{2k+1}$. Is $Z_k$ Markov?

**Exercise 3**

Consider a queue with infinite buffer having transition probabilities defined by

$$\begin{cases} P(X_n = X_{n-1}) = 1 - a \\ P(X_n = X_{n-1} + 1) = a \\ P(X_n = X_{n-1} - 1) = b \end{cases}$$

where $X_n$ denotes the number of items in the queue at time $n$. Find the stationary distribution of the chain assuming $a < b$.