

Exercises 12

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The below are in-class exercises designed to help solidify your understanding of the material covered in the notes. They will also aid you in completing some homework problems. Please work together with your group to complete as many of these problems as you can.

PN refers to the online textbook by Pishro-Nik available here. Please do not look at the solutions until after you have completed the problem or received hints from me.

Exercise 1

Let $X \sim \mathcal{N}(0, 1)$ and $Y = 3X$. Show that X and Y are jointly Gaussian and find their covariance matrix.

Exercise 2

Let X_1, \dots, X_n be RVs and define

$$Y_k = \sum_{i=1}^k X_i, \quad k = 1, \dots, n.$$

Suppose that Y_1, \dots, Y_n are jointly Gaussian. Determine whether X_1, \dots, X_n are jointly Gaussian.

Exercise 3

Let X, Y, U, V be jointly Gaussian with X, Y independent $\mathcal{N}(0, 1)$ RVs. Define

$$Z = \det \left(\begin{bmatrix} X & Y \\ U & V \end{bmatrix} \right).$$

If $[X \ Y]^T$ and $[U \ V]^T$ are uncorrelated, find the conditional density $f_{Z|U,V}(z | u, v)$.