Ex 1-2: See PN.

Ex 3:

$$\mathbb{E}[Y] = \mathbb{E}[3-2X] = 3 - 2\mathbb{E}[X] = 3-4 = -1$$

$$\mathbb{E}[Y^2] = \mathbb{E}[(3-2X)^2] = \mathbb{E}[9 - 12X + 4X^2]$$

$$= 9 - 12\mathbb{E}[X] + 4\mathbb{E}[X^2]$$

Note that $4 = \mathrm{var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \mathbb{E}[X^2] - 4$

$$\Rightarrow \mathbb{E}[X^2] = 8$$

$$\Rightarrow \mathbb{E}[Y^2] = 9 - 12\cdot 2 + 4\cdot 8 = 17$$

$$\mathrm{var}(Y) = \mathbb{E}[Y^2] - (\mathbb{E}[Y])^2$$

$$= 17 - (-1)^2 = 16$$

$$\mathrm{Cov}(X,Y) = \mathbb{E}[(X-\mathbb{E}[X])(Y-\mathbb{E}[Y])]$$

$$= \mathbb{E}[(X-2)(Y+1)] = \mathbb{E}[(X-2)(3-2X+1)]$$

$$= \mathbb{E}[(X-2)(-2X+4)] = \mathbb{E}[-2X^2+8X-8]$$

$$= -2\mathbb{E}[X^2] + 8\mathbb{E}[X] - 8$$

$$= -2\cdot 8 + 8\cdot 4 - 8 = 8$$

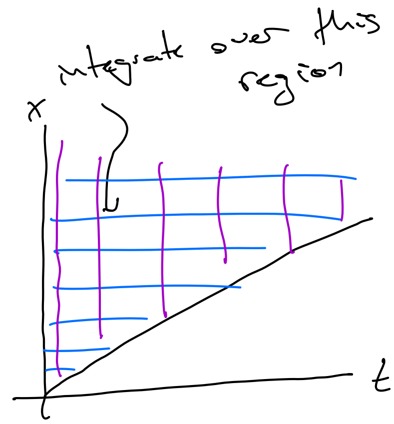
Ex 4: See PN.

Ex 5: See PN, 5.3.1, Example 5.3.2.

Ex 6: See PN.

Ex 7:

think: vertical strips



(a)

$$\int_0^\infty P(X>t)\,dt = \int_{t=0}^{\infty}\int_{x=t}^{\infty} f_X(x)\,dx\,dt$$

$$= \int_{x=0}^{\infty}\int_{t=0}^{x} f_X(x)\,dt\,dx$$ think: horizontal strips

$$= \int_{x=0}^{\infty} f_X(x)\, x\, dx = E[X]$$

(b) $$E[X] = \int_0^\infty P(X>t)\,dt = \int_0^\infty (1-F_X(t))\,dt$$

$$= \int_0^\infty e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2}$$

Ex 8:

Let X_i denote the output of the i^{th} regulator and define

$$Y_i = \begin{cases} 1 & \text{if } i^{th} \text{ regulator greater than } v \text{ volts} \\ 0 & \text{otherwise} \end{cases}$$

Then

$$P(Y_i = 1) = P(X_i > v) = \int_v^\infty \lambda e^{-\lambda x}\,dx$$

$$= -e^{-\lambda x}\Big|_v^\infty = e^{-\lambda v}.$$

Now the prob of 3 successes in 10 trials is a binomial RV with prob of success $e^{-\lambda v}$.

$$P\left(\sum_{i=1}^{10} Y_i = 3\right) = \binom{10}{3} e^{-3\lambda v}\left(1-e^{-\lambda v}\right)^7$$

Ex 9:

From the description, we have

$$H_1: Y = \mu + N$$

$$H_0: Y = N$$

where $N \sim \mathcal{N}(0, \sigma^2)$. We then have

$$f(Y|H_0) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{y^2}{2\sigma^2}\right)$$

$$f(Y|H_1) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right)$$

and the ratio is therefore

$$\frac{f(y|H_1)}{f(y|H_0)} = \exp\left(-\frac{1}{2\sigma^2}(\mu^2 - 2y\mu)\right)$$