Ex 1-10 : See PN.

Ex 11

Intuition may suggest the answer is $\frac{1}{2}$, since the drawn card is either RB or RR. However, it could be either side of the RR card, and hence the RR card accounts for two of the tree possible outcomes. Alternatively, we can use Bayes' rule:

$$P(RR|R) = \frac{P(R|RR)\,P(RR)}{P(R)}$$

$$= \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}.$$

Ex 12

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} > P(A)$$

$$\Leftrightarrow \frac{P(B|A)}{P(B)} > 1 .$$

Ex 13

(a) $P(A) = \frac{1}{2}P(\{DDD\}) + \frac{1}{2}P(\{LLL\}) = \frac{1}{4}$

$P(B) = P(0 \text{ dogs}) + P(1 \text{ dog}) = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$

$P(C) = P(2 \text{ dogs}) + P(2 \text{ lizards}) = \frac{3}{4}$

$P(A\cap B) = P(\text{all lizards}) = \frac{1}{8} = \frac{1}{4}\cdot\frac{1}{2}$

$P(B\cap C) = \frac{3}{8} = \frac{1}{2}\cdot\frac{3}{4}$

(b) $P(A\cap C) = 0 \neq \frac{1}{4}\cdot\frac{3}{4}$

(c) Only when the probability of one pet is zero.

Ex 14

Let E be the event of interest.

(a) The number of possible outcomes is 40^{10}. Finding $|E|$ is difficult. Instead consider the event E^c, in which exactly 10 stops are made. We can either view this problem as ordered or unordered.

<u>Ordered:</u> There are 10 experiments with 40 possible outcomes each, so $|\Omega| = 40^{10}$. Similarly, $|E^c| = \frac{40!}{(40-10)!}$. Therefore

$$P(E) = 1 - \frac{40 \cdot 39 \cdot 38 \cdot \ldots \cdot 31}{40 \cdot 40 \cdot 40 \cdot \ldots \cdot 40}$$

<u>Unordered:</u> Determining $|\Omega|$ is unordered sampling <u>with</u> replacement, which we skipped. To find $|\Omega|$, note that there are 40^{10} possibilities in the ordered case and $10!$ ways to order the 10 people, so

$$|\Omega| = \frac{40^{10}}{10!} \begin{matrix} \leftarrow \text{\# ordered arrangements} \\ \leftarrow \text{\# ways to order people} \end{matrix}$$

In the unordered case, $|E^c| = \binom{40}{10} = \dfrac{40!}{30!\,10!}$. This gives

$$P(E^c) = \frac{\frac{40!}{30!\,10!}}{40^{10}/10!} = \frac{40!/30!}{40^{10}}$$

which matches the ordered case.

(b) Let $E_1 = \{\text{stop on } 39\}$, $E_2 = \{\text{stop on } 40\}$.

We're interested in $P(E_1 \cap E_2)$. However, it's easier to consider

$$P(E_1^c \cup E_2^c) = P(E_1^c) + P(E_2^c) - P(E_1^c \cap E_2^c)$$

$$= \frac{39^{10}}{40^{10}} + \frac{39^{10}}{40^{10}} - \frac{38^{10}}{40^{10}}$$

$$\Rightarrow P(E) = \frac{40^{10} - \left(2(39^{10}) - 38^{10}\right)}{40^{10}}.$$